Non-uniform Growth and discrete Conformal Mappings

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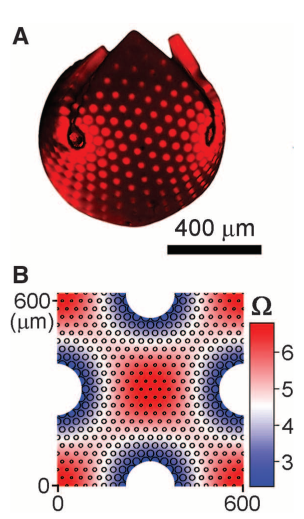
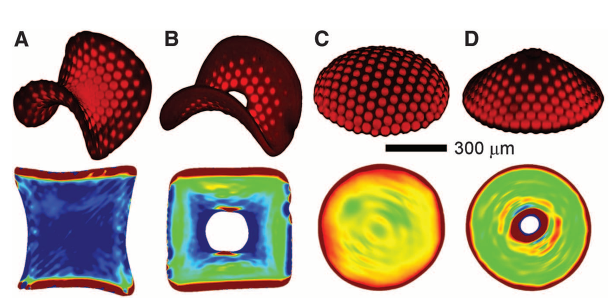
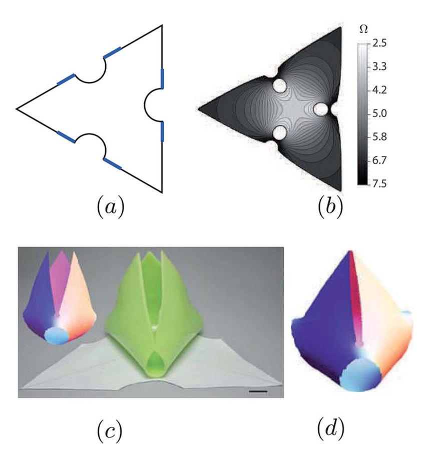
MOTIVATION

- Non-uniform growth has been used as a way of creating different kinds of shapes.
- It is possible to experimentally recreate growth processes by manipulating area ratios between different surfaces.
- It is desirable to find optimal growth patterns that would yield a desired shape while keeping the area distortion as small as possible.



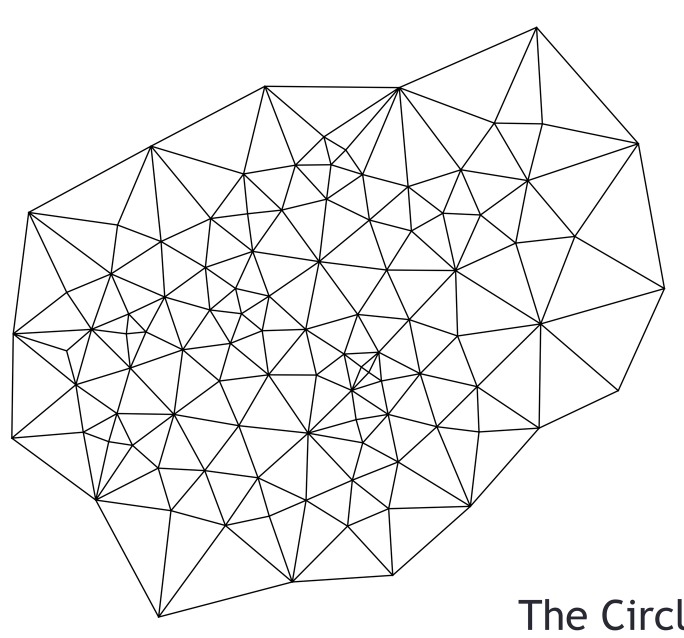




BACKGROUND

We start with a triangulation of a topological surface

We find a *Circle Packing* of tangent circles.



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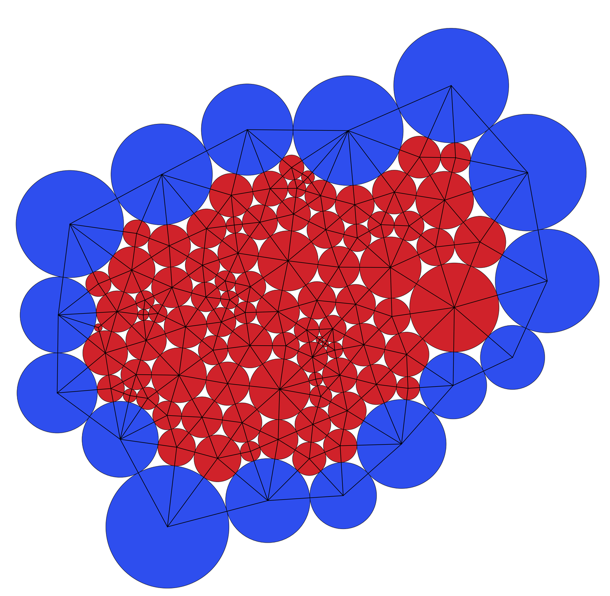
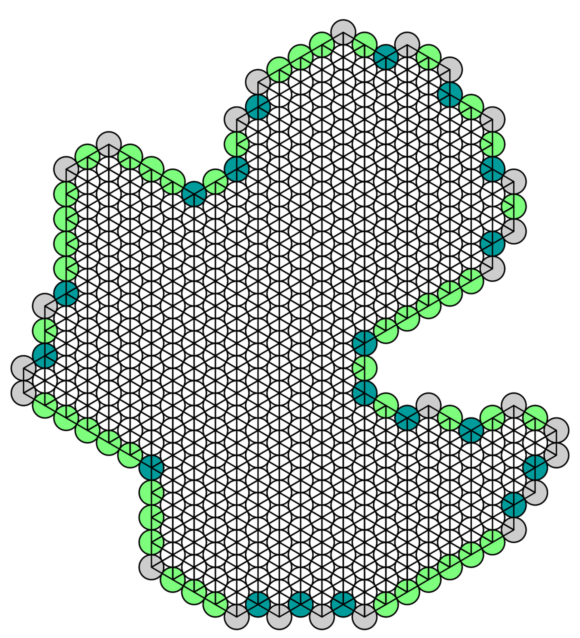


The Circle Packing concept can be used to construct discrete conformal mappings



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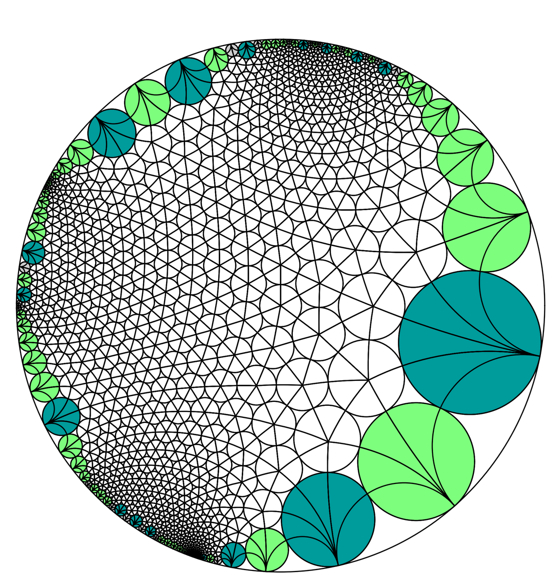
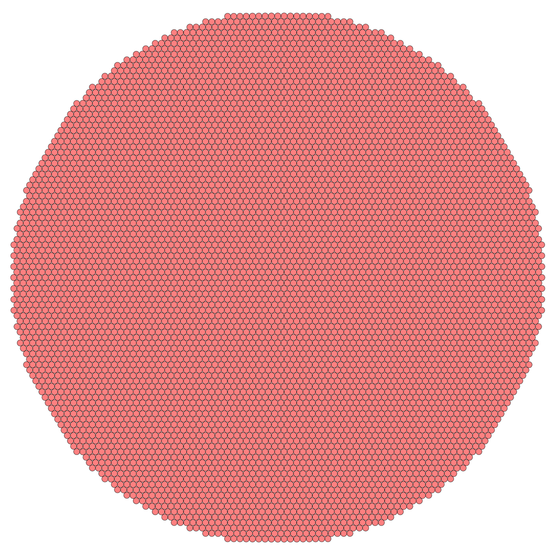


AREA DISTORTION



Stereographic Projection

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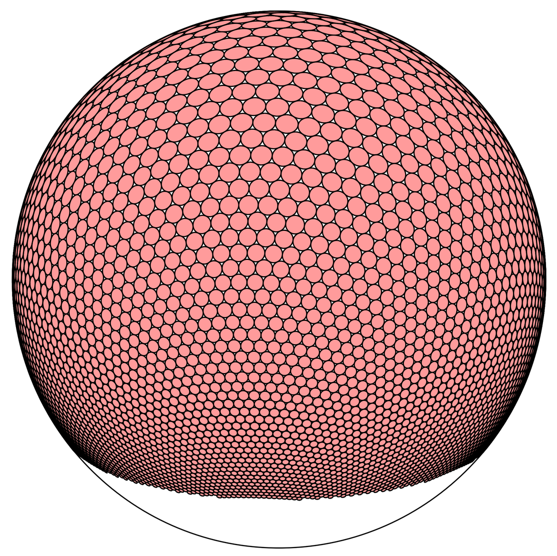
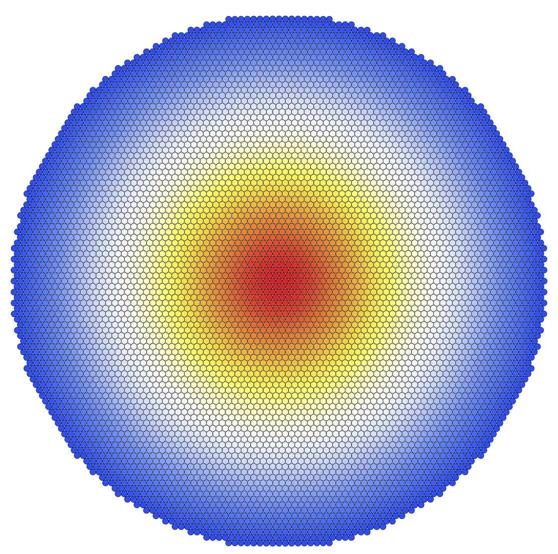




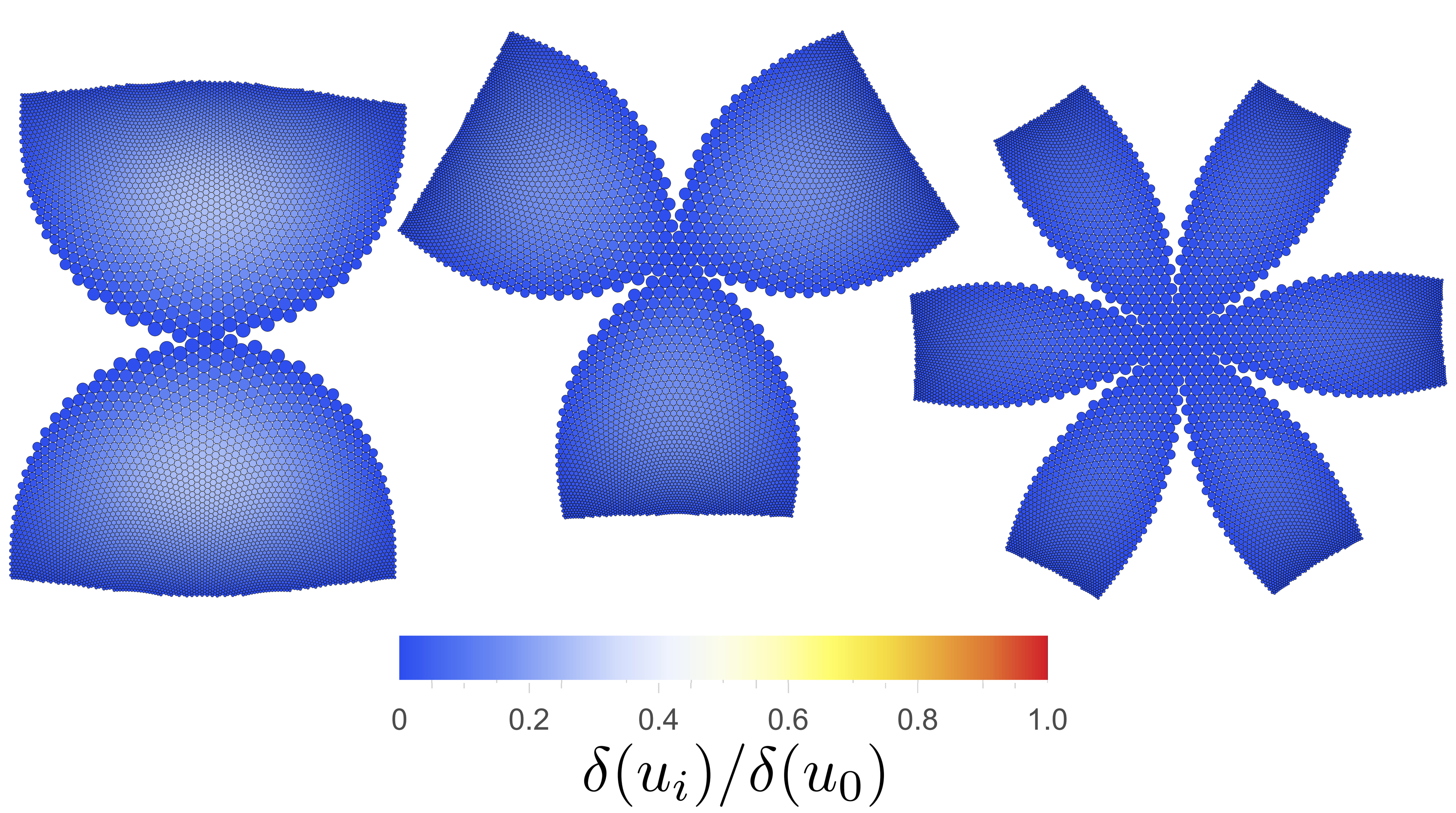
Infinitesimal Circles are distorted according to a conformal factor.

$$\Omega(x,y) = \frac{4}{(1+x^2+y^2)^2},$$

Close to the north pole of sphere the distortion is bigger by a factor of 4.

CHEBYSHEV'S PRINCIPLE

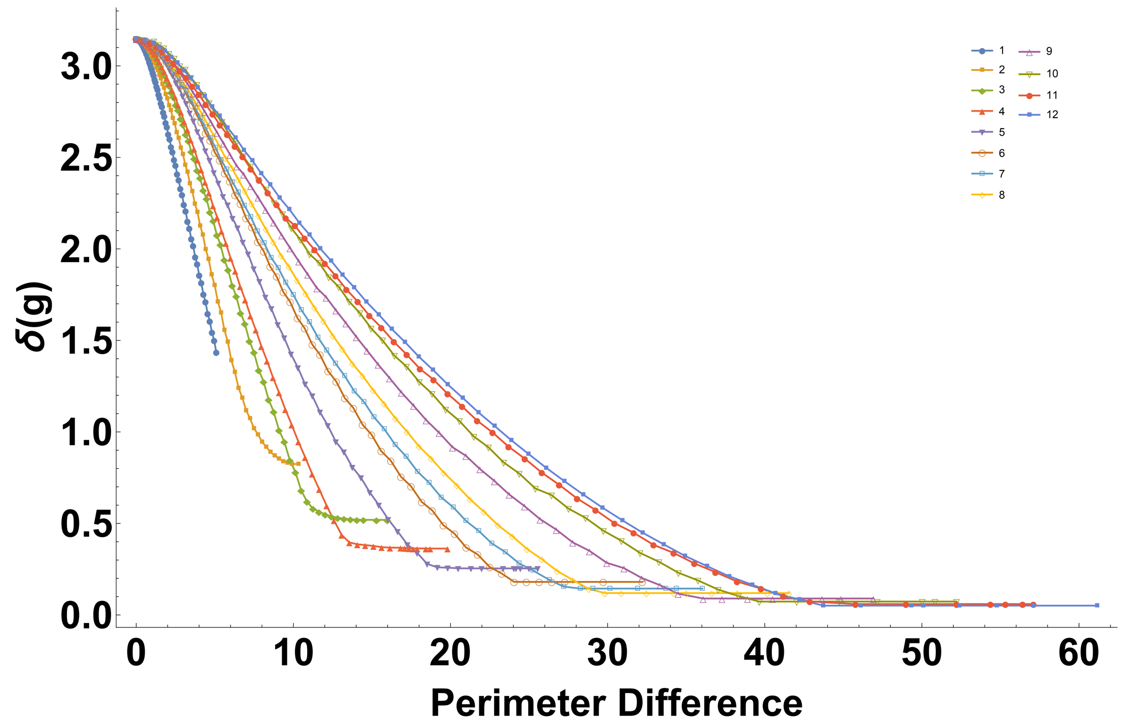
- We want to construct discrete conformal transformations that minimize the area distortion when going from one circle packing to another. The area distortion can be reduced by making cuts.



- According to Chebyshev principle, this map is characterized by the property that its conformal factor along the boundary is a constant.

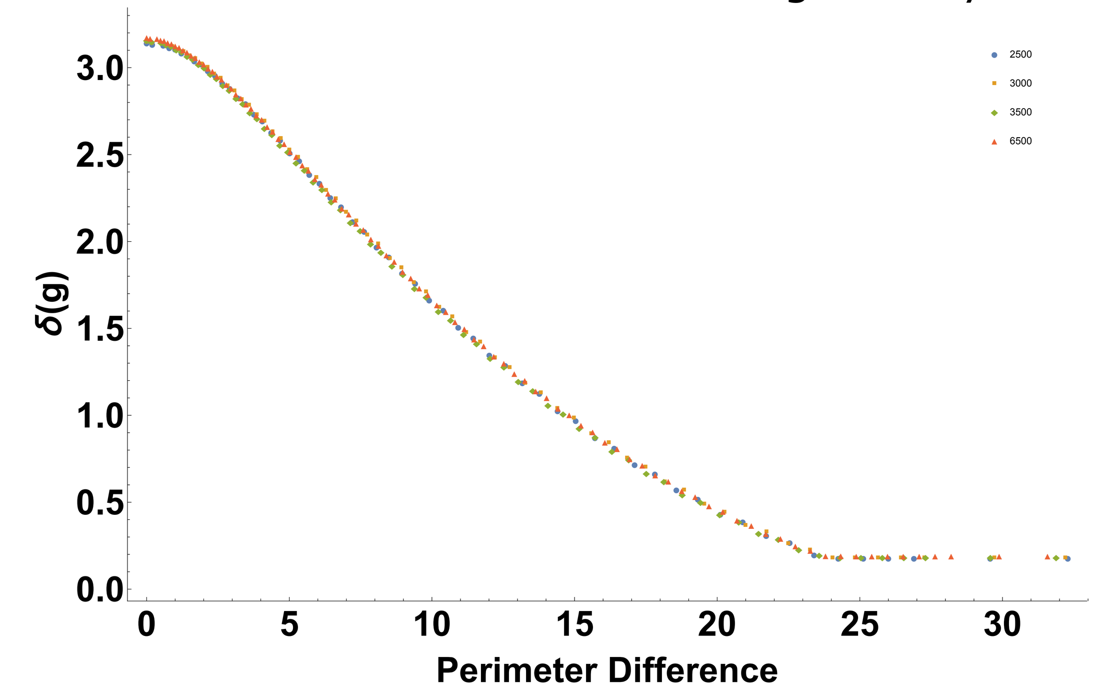
AREA DISTORTION

- We can see how the area distortion decreases by different amounts as we introduce more radial cuts and vary their length.



SYSTEM SIZE DEPENDENCE

- Furthermore, it is possible to see that there is a continuum limit when we fix the number of cuts and change the system size.



PERSPECTIVES

- We want to find efficient ways of making cuts on contact networks in order to minimize the area distortion as much as possible.
- The ability of choosing target area ratios before making any kind of cuts is also under investigation since it could bring more freedom when designing specific shapes.

REFERENCES

(1) N. Bende, R. Hayward and C. Santangelo (2014). Nonuniform growth and topological defects in the shaping of elastic sheets. ***Soft Matter***, 2014, 10, 6382.

(2) J. Kim1, J. Hanna, M. Byun, C. Santangelo, R. Hayward (2012), Designing Responsive Buckled Surfaces by Halftone Gel Lithography. ***Science***, 2012, 335, 1201.