Quantum Walk in Degenerate Spin Environments

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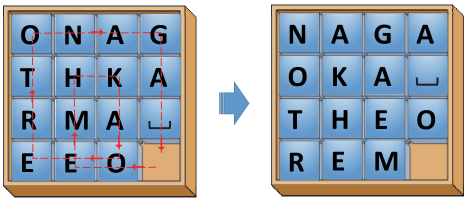
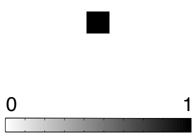
We consider the propagation of a single hole in a dynamically passive spin environment. This canonical problem has important connections to a number of physical systems, from quantum crystals to strongly correlated electronic systems, and is perfectly suited for experimental realization with ultra-cold atoms in an optical lattice. At the short-to-intermediate timescale, the propagation turns out to be neither diffusive, nor ballistic (except at the beginning). We discuss possible scenarios for long-term evolution that could be explored with an unprecedented degree of detail in experiments with single-atom resolved imaging.



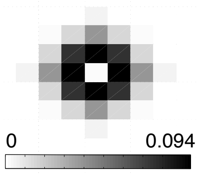
Hole propagation in a dynamically passive environment amounts to a quantum mechanical realisation of the N-puzzle game, where a particular order is sought with as few tile movements as possible.

METHODS: We expand the wave-function at a given time t in a stochastic series and compute it using Monte Carlo sampling. From the wave-function we obtain the time-dependent probability distribution of the hole.

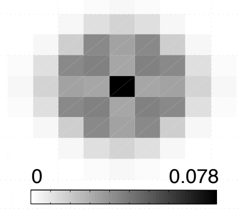
SPATIAL DISTRIBUTION



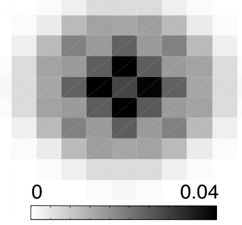
Time $t = 0$.
Particle spin $N = \infty$.
The system is prepared with the hole situated in the origin.



Time $t = 0.9$.
Particle spin $N = \infty$.
The probability of finding the hole in the origin is close to zero due to interference.

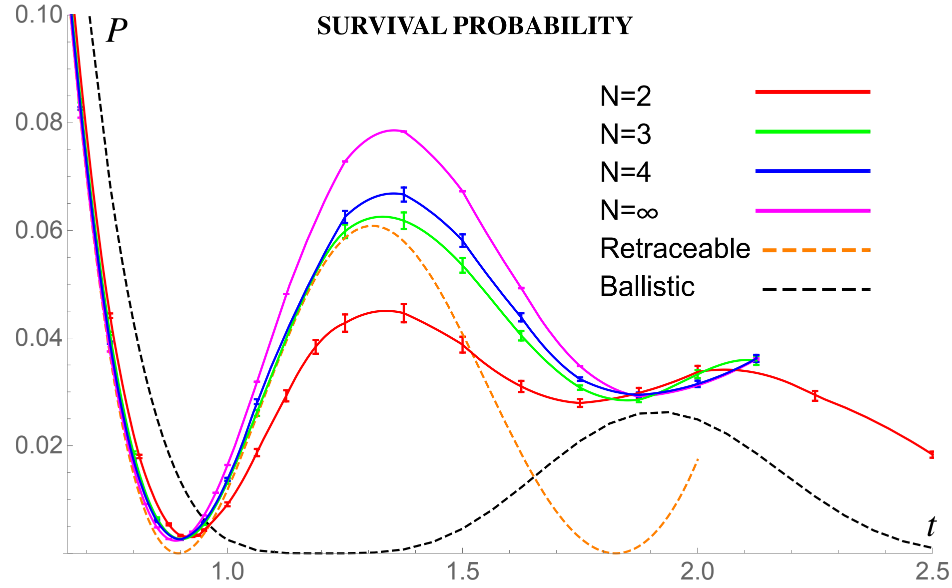


Time $t = 1.375$.
Particle spin $N = \infty$.
The hole is now most likely to be found in the origin.



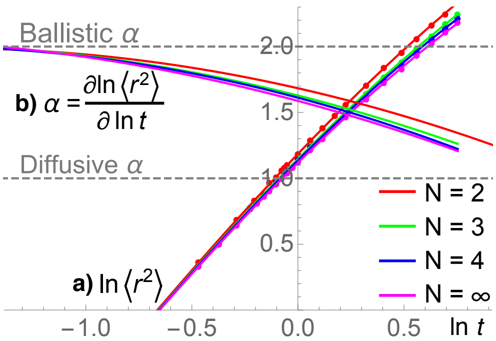
Time $t = 1.875$.
Particle spin $N = \infty$.
The origin once again features a local minimum, though not close to zero.

SURVIVAL PROBABILITY



Probability of finding the hole at the origin as a function of time. Particle spin is N = 2, 3, 4, ∞. The self retraceable approximation due to Brinkman & Rice, as well as the ballistic case are included for comparison. The first minimum is only a fraction of a percent. For larger t, interference continues to create oscillation in the probability, but perfect destructive interference is no longer possible.

TRANSPORT PROPERTIES



(a) Logarithmic plot of mean square displacement. **(b)** MSD exponent α as function of *ln t*. The initial propagation is ballistic. However α is time dependent, indicating that we are in a cross over region between different modes of propagation. The precise nature of the propagation at large t (specifically if the motion is diffusive) remains an open question.

SUMMARY OF RESULTS

- Initial propagation is ballistic and dominated by self-retracing paths that preserve the spin environment.
- At intermediate time-scales, the system is in a crossover region where the propagation is not ballistic, but destructive interference continues to play an important role for the mobility and spatial distribution, while the MSD exponent is still greater than the diffusive value of *1*.

OPEN QUESTIONS

- What is the nature of propagation at larger time scales? In particular, is there a diffusive regime?
- What is the role of ferromagnetism in this problem? According to the Nagaoka theorem, low energy states exist where the hole is delocalised in a ferromagnetic "bubble". This should imply states of extremely low mobility.
- What is the long term impact of the hole on the spin environment? In particular, can it *create* magnetic regions?

CONCLUSION

- This work represents the first serious attempt at understanding the nature of hole propagation in a spin environment where the trajectory is "recorded" by the environment. We find this propagation to be distinctly different from both classical random walks and coherent propagation.
- Simulations with deliberately designed initial conditions can give insights into the role of ferromagnetism for hole propagation.
- Experiments with atoms trapped in optical lattices are already at the planning stage at Harvard.