Mathematics & Statistics
NEWSLETTER
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UMassAmherst
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MATH FOR ALL

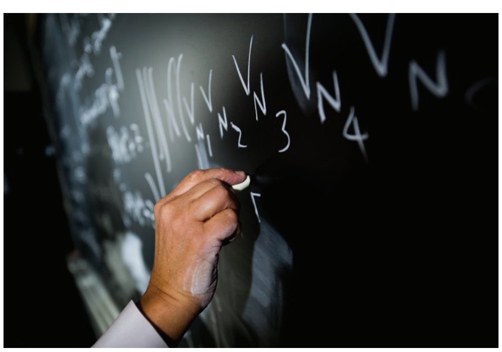
New faculty member **Annie Raymond** hopes to help break down stereotypes by using Instagram to feature a different mathematician each week. In a recent interview she discusses how she got the idea and why she chose Instragram as the platform. "A few months ago, I read a wonderful article in the *Notices of the American Mathematical Society* highlighting the careers of many great female mathematicians, including our very own **Andrea Nahmod**. It was immensely inspiring. Upon finishing it though, I realized that it was unlikely that any of the undergrads in my class saw it—even though reading it would be even more beneficial for them than for me.

"There are so many great projects out there that promote women in mathematics, but few of them actually reach younger women at the time when they are deciding what to study or what kind of career to pursue. This is especially problematic given that most math departments will tell you that they would like to be more gender-balanced—at every level—but they are simply not getting enough applications from women. Clearly, we need to do a better job at reaching out to younger generations who have yet to decide whether they want to be mathematicians. This is a nice thought, but the problem is obvious: how do you reach these young women? Given that close to 60% of 18-29 year old internet users have an Instagram account, and that women account for about two thirds of Instagram users, Instagram seemed like a great platform to launch a project targeting younger women.

Article continues on page 13

PUTTING A NUMBER ON TRUST

Seeking to answer the question of how much one should trust the predictions made by models and algorithms, and whether they are trustworthy enough for design and decision-making tasks, professors **Markos Katsoulakis** and **Luc Rey-Bellet**, with applied mathematics professor Paul Dupuis at Brown University, recently received a three-year, $900,000 grant from the Air Force's Office of Scientific Research to develop mathematical tools to assess and improve the predictive performance of complex mathematical and computational models. The UMass component is $600,000.



Katsoulakis explains that probabilistic computational models are currently a primary discovery tool for understanding and predicting complex phenomena in weather, traffic patterns, social networks, finance, chemistry, life sciences and other fields. Though potentially powerful, computational models are susceptible to errors and uncertainty, since they have to model highly complex systems with millions or even billions of variables and parameters, and must be informed by available data across different scales.

Article continues on page 21

SIMONS FELLOWSHIP FOR ALEXEI OBLOMKOV

Associate Professor Alexei Oblomkov was named a Simons Fellow for the year 2018. While a Research Professor on sabbatical at MSRI for the Spring semester as a part of its *Enumerative Geometry Beyond Numbers* program, he was working on the Donaldson-Thomas and Gromov-Witten theories of 3-folds, beyond the Calabi-Yau case where physics predicts a gauge/string duality. Alexei's other current project relates knot homology to 3D quantum topological field theories.

The Simons Foundation supports a one-semester extension of Alexei's sabbatical leave through Fall 2018, allowing him to collaborate with Andrei Okounkov (Columbia) and with Rahul Pandharipande (ETH), and to give lectures in New York and in Zürich on these projects.

MATHEMATICS & STATISTICS NEWSLETTER

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The Department of Mathematics and Statistics publishes its annual newsletter for alumni and friends. You are important, and we want to hear from you! Please contact us at dept@math.umass.edu to share your news, let us know how you are doing, and learn ways to become involved with the Department. Our website is a valuable resource for current happenings and news, so we encourage you to visit us regularly at **www. math.umass.edu**

On a more somber note, we mourn the passing on 2 July 2018 of our dear colleague Richard S. Ellis, an internationally regarded probabilist, and a good friend to many of us on campus and in the community. It's an immeasurable loss to our Department. For many years Richard edited the annual Newsletter and monthly News Briefs, and I — your current editor — enjoyed working with him on both for much of the past decade. As I re-drafted the letter soliciting briefs this September, I realized it was based on Richard's traditional text, passed down from him to me several years ago. Both this Newsletter and the News Briefs have become Department traditions, shaped by each of us who has taken on the role of editor, but Richard's influence looms large. He and I shared many common interests outside mathematics, particularly an intellectual curiosity about languages and literature, and both of us have been deeply committed to carry on the tradition of publishing. Richard's dedication to this enterprise is evinced by his last email to me: "How's the Newsletter coming along? Just curious...." It was sent on the evening of 1 July 2018. –*Rob*

NEW CHALLENGE PROBLEMS

Here is a selection of some of the more challenging problems from the 2018 Jacob-Cohen-Killam prize exam. Three additional prize exam problems are included in the Awards Dinner article on page 14.

Problem 1. Compute the volume of the 2018–dimensional simplex

$\{(t_1, \dots, t_{2018}) \in \mathbf{R}^{2018} : 0 \le t_1 \le \cdots \le t_{2018} \le 1\}$.

Problem 2. Given numbers a_1, a_2, a_3, a_4, suppose a_n is defined recursively as

$$a_n = (a_{n-1} + a_{n-2} + a_{n-3} + a_{n-4})/4.$$

Show that the sequence $\{a_n\}$ converges and calculate $L = \lim_{n\to\infty} a_n$.

Problem 3. Consider a regular tetrahedron with edges of length 1. What is the length of the shortest loop on its surface which surrounds two vertices?

Problem 4. How many 3×3 arrays are there such that (i) entries include all the digits from 1 through 9, and (ii) entries in every row and column are in increasing order?

Problem 5. Let E_n be the coefficients in the expansion: $\sec x + \tan x = \sum_{n=0}^{\infty} E_n \frac{x^n}{n!}$.

Show that they satisfy $E_0 = E_1 = 1$ and for $n \ge 1$

$$2E_{n+1} = \sum_{k=0}^{n} \binom{n}{k} E_k E_{n-k}.$$

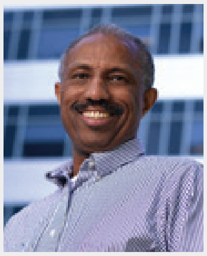
Problem 6. Let A_n be the set of all subsets S of $\{1, 2, \dots, n\}$ such that S contains the size $|S|$ of S. Let B_n be the set of all S in A_n which are *minimal*, i.e., if S contains some subset T in A_n then $T = S$. Show that the number b_n of elements of the set B_n is the n^{th} Fibonacci number F_n (here $F_1 = 1 = F_2$ and after that $F_n = F_{n-1} + F_{n-2}$).

Problem 7. Evaluate $\int_0^{\pi/2} \frac{1}{1+(\tan x)^{\sqrt{2018}}}\,dx$.

Send solutions, comments, or feedback via email to Professor Franz Pedit <pedit@math.umass.edu> with the subject line "Challenge Problems 2018."

Last year's solutions are on page 17.

DEPARTMENT HEAD'S MESSAGE



I'd like to welcome you to our Newsletter as the new Department Head. I officially started on 21 January 2018. It's a true honor – and somewhat daunting – to follow in the footsteps of **Farshid Hajir**, who now serves as Vice Provost of Academic Affairs and did a fantastic job in his 3 years as Head. I take over a department that's risen tremendously in stature since I came to UMass in 1987. We've made some exceptional hires and improved our graduate and undergraduate programs. Our graduates have gone on to do outstanding things. We're very proud of our alumni.

A particular area of growth is our number of undergraduate majors, which continually exceeds expectations. More and more of our incoming students recognize the benefits of majoring in math. We have almost 1000 majors in 7 areas: Actuarial Science, Applied Math, Computing, Pure Math, Statistics, Teaching and an Individual concentration. Mathematics is also an essential part of any 21st century education, as over 15,000 UMass students take our classes each year.

Thanks to the hard work of our search committee, our tenure track position in Math Biology was filled by our top candidate: Assistant Professor **Brian Van Koten** is an expert in applied probability and numerical analysis, performing computer simulations of molecular systems with applications in biology, chemistry and materials science. We also had a very successful year in hiring 8 new outstanding Visiting Assistant Professors: **Siddhant Agrawal**, **Panagiota Birmpa**, **Noriyuki Hamada**, **Kuan-Wen Lai**, **Jonathan Simone**, **Zahra Sinaei**, **Zheni Utic** and **Jiayu Zhai**. Our new Marshall Stone VAP, **Laura Colmenarejo Hernando**, will join us in Spring 2019.

Our Newsletter, and the on-line Faculty News Briefs at www.math.umass.edu, provide an opportunity to highlight some of the outstanding work of our faculty in pushing the envelope of knowledge in mathematics and statistics each year: Assistant Professor **Nestor Guillen** spoke at the Austin workshop "Gerrymandering Steals Elections: Learn how It's Done and How to Stop It," and was part of a panel on the topic including US Representative **Jim McGovern** this year; Associate Professor **Paul Hacking** was invited to lecture at the 2018 International Congress of Mathematicians; Associate Professor **Alexei Oblomkov** was awarded a Simons Fellowship; Professors **Markos Katsoulakis** and **Luc Rey-Bellet** received a major Air Force grant to develop performance guarantees in predictive modeling; Professor **Andrea Nahmod** was part of a feature honoring women in the *Notices of the AMS*; and University Distinguished Professor **Panos Kevrekidis** published 4 papers in the *Nature* family of journals.

Our Department hosted 3 conferences this past year: Associate Professor **Anna Liu** was among the organizers of the 32nd *New England Statistical Symposium* bringing together nearly 250 statisticians to discuss emerging issues in April 2018; Professor **Andrea Nahmod** helped organize *Women in Partial Differential Equations* in March 2018; and Associate Professor **Hongkun Zhang** co-organized *Mathematical Physics Perspective of Billards and Dominoes* celebrating the 70th birthdays of Pavel Bleher and Leonid Bunimovich in September 2017.

UMass Amherst undergraduates **Jonah Chaban**, **Artem Vysogorets**, and **Jimmy Hwang** worked as a team, led by Assistant Professor **Matthew Dobson**, to produce the top-scoring project among 7 competing teams at the first ever Student Competition Using Modeling (SCUDEM) in Fall 2017. Three more SCUDEM teams led by Dobson also did very well in Spring 2018, receiving meritorious mention and outstanding mention.

We are very sad to have lost an important member of our faculty, **Richard S. Ellis**, who passed away in July 2018. Richard joined the department in 1975, and also taught classes in Judaism and the Torah as an adjunct professor of Judaic and Near Eastern Studies. He was an internationally regarded expert in probability and large deviation theory, who also cared very much about the well-being of his students and colleagues, organizing sessions to relieve stress both inside and outside of the classroom. Richard will be truly missed.

We are also sorry to report the retirement of two important members of our faculty, **Michael Lavine** and **Bill Meeks**, in May 2018. Professor Lavine, a well-respected Bayesian statistician and a leader in our statistics group, had been a department member since 2008. Professor Meeks held the prestigious George David Birkhoff chair since joining the department in 1986; he is a world-renowned differential geometer and topologist, with a distinguished reputation in the area of minimal surfaces.

The support of our alumni and friends is invaluable to us. We want to know how you are doing. Please send us your news, or stop in for a visit.

– Nathaniel Whitaker

MULTIPLE SCALE MODELING FOR PREDICTIVE MATERIAL DEFORMATION ANALYSIS

Motivation. Material deformation and stress-strain is an active area of mathematical modeling relevant to industrial and research-oriented materials science. It is important to take variations in material properties into account in these models. Multi-scale models that incorporate inhomogeneity were studied and modeling frameworks that address this need were created and tested. Incorporating variations in material properties at the micro scale resulted in significantly different predictions of material deformation under similar loading. Variations in material properties were accounted for through averaging over stresses in representative volume elements (RVEs). This project was a collaborative effort by students, **Rachel Aronow**, **Aaron da Silva**, **Rose Dennis**, **Abdel Kader Geraldo**, **Dean Katsaros**, **Melissa Sych**, and **Richard Touret**, under the guidance of Professor **Qian-Yong Chen**.

Background. In material science, the deformation of materials under tension is an important problem with widespread application. Using the stress/strain equations as the basis for the model, material properties are encoded, and the model is solved to predict behaviors of real-life materials. The material properties are encoded via the material's Poisson ratio, and its young's modulus. Young's modulus is defined mathematically as the ratio of the tensile stress to the strain. Poisson's ratio is the ratio of the transverse strain to the axial strain. Simply, Young's modulus is the measure of material stiffness, and Poisson's ratio the tendency of the material to expand perpendicular to the direction of compression.

It is important to note that a material may not have the same Poisson ratio or Young's modulus in different parts of the material. We say that this is an inhomogeneous material. Commonly, deformation models assume homogeneity in the material being modeled. The homogenous version of these stress/strain equations is much easier to solve. However, this assumption leaves important characteristics of the material out of the model. This motivates taking a multi-scale approach to modeling the material.

The multi-scale approach incorporates the inhomogeneity of a material via combining a macro and micro-scale set of equations. Both equations assume homogeneity, but the micro-scale equations have different property coefficients corresponding to their location. Representative Volume Elements (RVEs) is the name given to the micro-scale units. This project investigated the differences in modeling results when a multi-scale approach is taken.

Tissue Mechanical Testing. To investigate the effects of multiscale modeling, we chose to focus on the problem of a ligament under uniaxial tension testing, as performed by Chokandre et al. in 2015. In particular, ligaments are an inhomogeneous material. The uniaxial tension test performed in this study is described as follows: a 1 mm x 4.46 mm dumbbell-shaped sample of tissue from an MCL (0.53 mm thick) is clamped at both ends and stretched slowly, such that the internal forces in the tissue remain balanced (quasi-static equilibrium). At each time step, the force applied to the tissue and the tissue's resulting displacement is recorded.

To model this experiment, we used the plane stress/strain equations. These equations are given by:

$$\nabla\cdot(\sigma) = F \qquad \sigma = D\epsilon \qquad \epsilon = \nabla\cdot \mathrm{U}$$

where $U=[u,v]$ is displacement, $\epsilon=[\epsilon_x,\epsilon_y,\tau_{xy}]$ is strain, $\sigma=[\sigma_x,\sigma_y,\tau_{xy}]$ is stress, $F=[F_x,F_y]$ is the internal body force (set to zero), and D is a coefficient matrix describing the elasticity of the material. D is dependent on E and v, and is easy to calculate for a homogenous material. These equations are derived from the following assumptions:

1. The problem is two-dimensional
2. The material is homogenous and its elasticity can be described fully by two parameters: Young's Modulus, E and Poisson's Ratio, v
3. The material is in equilibrium (stresses are balanced throughout the material)

Using MATLAB and the open source toolbox FEATOOL (https://www.featool.com/), we defined a 1 mm x 5 mm rectangular region as our ligament. With the data from Chokandre et al. (2015) in hand, we calculated an average Young's Modulus (slope of the stress vs. strain curve) of the MCL (E=36 MPa) and used this to describe the elasticity of our simulated material. For a Poisson's ratio, we adopted a typical value for an MCL (v=0.02) from Sweigart & Athanasiou (2005). Assuming quasi-static equilibrium, we set the initial displacement of the material to be 0 and applied a forcing of 1.3 N in the positive y-direction to the top edge (Figure 1). To simplify the model, we assumed symmetry and set the bottom edge to be held fixed. These conditions were enforced via boundary conditions. Solving the plane stress equations via the finite element method results in

the displacements displayed in Figure 2. We observe that the top edge of the region is displaced 0.27 mm, compared to a displacement of ≈0.45 mm observed in the data set.

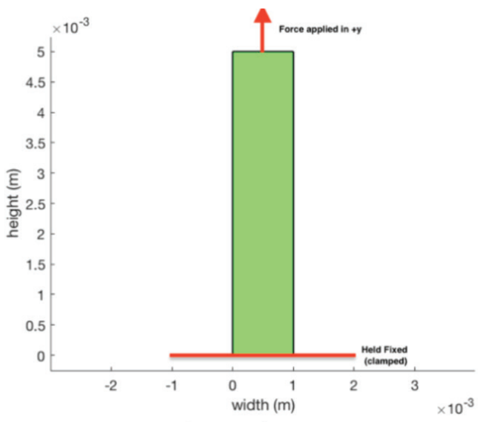


Figure 1.

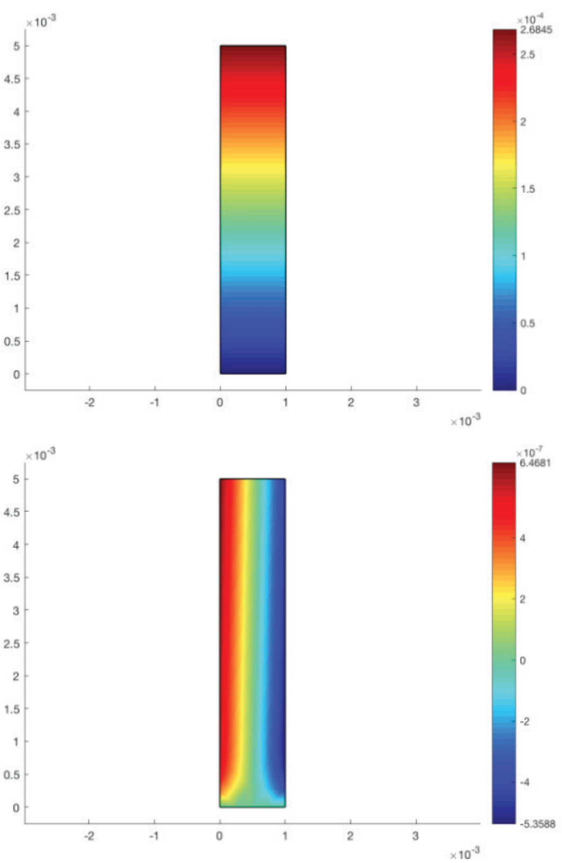


Figure 2. (y displacement left, x displacement right)

Multiscale Modeling Approach. In order to solve the force balance equations, we had to make the assumption that the entire ligament is homogenous. However, some of the most exciting potential applications of biomechanics apply to problems involving in-homogenous materials. For example, we consider a ligament with an "injury" point, in order to incorporate a multi-scale approach to the problem.

One Macro-Region. For a benchmark, we first solved the problem for the simple homogenous case. As in Section 2, we used the average values of E and ν across the whole body. Again, the initial displacement and internal forcing (body force) were set to 0, and a 1 N upwards forcing was applied to the top edge while the bottom edge was held fixed. Then the plane stress/strain equations were solved using the finite element method. These steps were repeated until the material stabilized its shape. The solution was then interpolated for the entirety of the region using MATALB's scatteredInterpolate function. The resulting deformation of the material is shown in Figure 3. Qualitatively, the material behaves as expected under tensile stress.

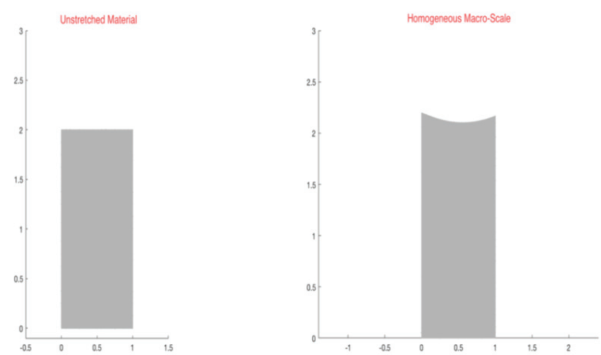


Figure 3.

For our second model, we introduced four RVEs, located at the Gauss points of the rectangular macro-region. The RVE in the top right corner was given a lower Young's Modulus and greater Poisson's ratio than the rest of the material, to indicate a point of poor stability (see insert in Figure 4). We then applied a multi-scale approach based off Barocas (2007), and summarized as follows: We first solved one iteration of the macro-problem as described in Section 3.1. We then solved the plane stress equations in each homogenous RVE for the stress resulting from the external loading. For this step, we used boundary conditions interpolated from the closest macro-region boundary to the RVE. Next, we calculated Q, as introduced in Barocas (2007). The Q factor is defined by the volume average of $Q_{RVE(k)}$ where k refers to the indexing of the RVEs and $Q_{RVE(k)} = \frac{1}{Vol_{micro}} \int \left(stress_{RVE(k)} - AvgStress_{macro}\right) d\mu(x)$. We then resolved the plane strain equations in the macro-region, but with the internal forcing set to Q instead of 0. This captures the effect of inhomogeneity on the internal forcing of the material, and thus how it responds

to an external forcing. We continued to switch between calculating Q in the micro-scale problem and balancing forces in the macro-scale problem until the solution stabilized. The result of this simulation is shown in Figure 4.

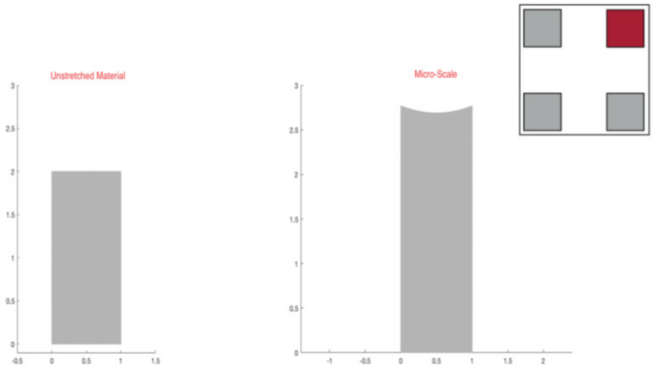


Figure 4.

Multiple Macro-Units with a Microscale. To include finer detail in the model, we introduced a grid of four macro-regions, each containing four RVEs at their Gauss points. We therefore have sixteen locations to introduce inhomogeneity. The top and bottom edges of the ligament, where the tissue is anchored to the bone, were set to be slightly less elastic than the overall material, and the injury point was set to be more elastic, representing an instability point in the fiber alignment. Starting with the top left macro-unit, we complete one iteration of the method described in Section 3.2. We then repeated the process for the top right macro-unit, with the additional boundary condition of the stress along the right edge of the top-left macro-unit being equivalent to the stress felt by the left edge of the top-right macro-unit. Continuing clockwise, we then solved the problem for each of the bottom units in a similar manner. This process was then repeated until the solution stabilized.

Results & Discussion. A comparison of the three simulations is shown in Figure 5. Incorporating the microscale in the second simulation affects the magnitude of the deformation, but not its shape. This is an issue inherent to our method. The microscale is represented by the Q factor, which behaves like a weighted average of the relative stress felt in each RVE. This term was then used as an internal forcing term, and so a positive Q value forces the body in the same direction as the external force, meaning the same magnitude external force can displace the material further. Therefore, this model does not capture any information about the location of the fine details, just the existence of a more flexible region within the tissue.

The model with multiple macro-units however, does demonstrate a change in both the magnitude and shape of the deformation. The lopsided tissue shows signs of the location of the injury point. Although the effects of the micro-units are still averaged together to find Q, there is now a Q factor for each macro-region, therefore holding on to structural details within each macro-unit. Thus, the introduction of multiple macro-units is better at encoding fine details, without losing all the details to averaging.

One of the challenges to multi-scale modeling is the lack of a rigorous benchmarking method. Our results show that incorporating a micro-scale changes the shape of the deformation, but we do not possess the data sets to determine which model is "best."

Furthermore, the way the microscale stresses are incorporated into the macro scale should be examined in future work. The Q factor is a counterintuitive way to do this, as it is an averaging process. The degree to which the microscale is detailed can become computationally impractical very quickly, so some sort of approximation is necessary. However, it is not clear whether there is a better approximation than the Q factor.

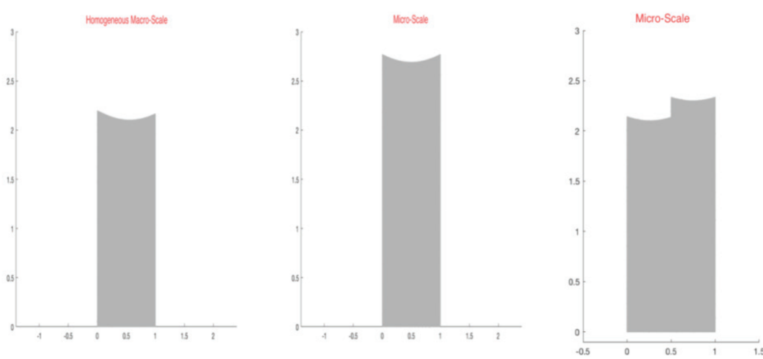


Figure 5.

Conclusion & Future Work. Incorporation of the microscale causes different deformation behavior. This reflects the importance of taking into account the heterogeneous nature of many materials. For more comprehensive studies to be productive, the benchmarking of these models needs to be more rigorous. Benchmarking should not necessarily be reliant on imaging, as comparison of real images to experimental images can be ill-defined. The model could be made more robust in future studies by using fiber equations instead of stress-strain modeling. Here, microscale stresses would be calculated at cross-linking sites in the micro-scale, and these stresses would be factored into the plane stress at macro-scale, a computationally expensive approach.

ELECTROMYOGRAPHY CLASSIFICATION USING RECURRENT SYSTEMS

I. MOTIVATION

Classifying Electromyography (EMG) data is important for many application domains, such as prosthetics or the remote control of robots, but the current state of the art is stuck in a tug of war between interpretability, accuracy, and generalization to new data sets. This Applied Math Master's Project compared methods typically used to achieve state-of-the-art performance on this EMG classification task with newer methods which might better exploit the time-series nature of the data, possibly at the expense of interpretability. The project was a collaborative effort by students **Connor Amorin**, **Gabriel P. Andrade, Chris Brissette**, **Matthew Gagnon**, **Brandon Iles**, **Jimmy Smith**, **and Lance Wrobel**, under the guidance of Professor **Qian-Yong Chen**.

II. EMG DATA

EMG is a technique for recording the electrical activity produced by the neurons in skeletal muscles. The signals produced by EMG are collected from electrodes either placed on the surface of the skin or inserted directly into the muscle. Though measurements are less reliable due to noise introduced from a myriad sources, surface EMG data is more readily available since it is far less invasive. For these reasons, in what follows, we will focus on these surface EMG recordings; an example of a raw signal is shown in Figure 1.

EMG data has a number of issues of which we must always be mindful. These factors force us to place significant trust in whomever collects the data, making it very hard to assess how best to work with the data. As was already mentioned, the data is necessarily noisy. Furthermore, the subject's recorded response to stimulus is understood to be encoded in changes of amplitude and also of frequency[1]. When these changes occur, and how they change, however, depends on the stimulus itself,

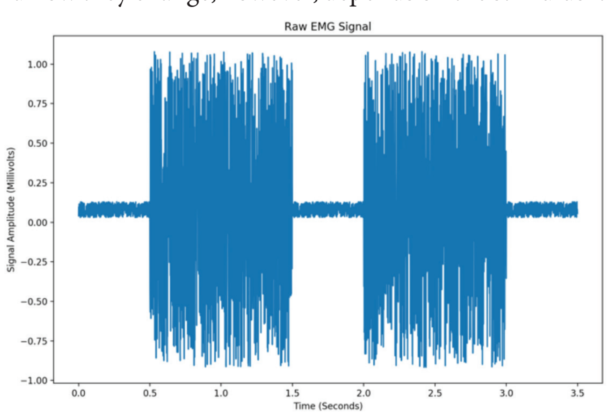


Figure 1: An example of an unfiltered EMG signal.

the "intensity" of the action, the state(s) of the subject prior to taking the action, and so forth. Further complications arise due to measurements being highly dependent on electrode placement and differing stimulus response from individual subjects.

III. THE DATASETS

Our goal is to compare methods for classifying EMG with accuracy, ease of generalizing to a new data set, and interpretability of the method in mind. Therefore we will be discussing:

1. The University of California Irvine Machine Learning Repository's hand movement data set

2. Dr. Rami Khushaba's (from the University of Technology Sydney) finger movement data set

The first data set (hand movement) contains signals from 5 subjects (3 female, 2 male) performing 30 trials for each of 6 specific hand movements (6-second-long trials). The 6 hand movements used in this experiment can be seen in Figure 2.

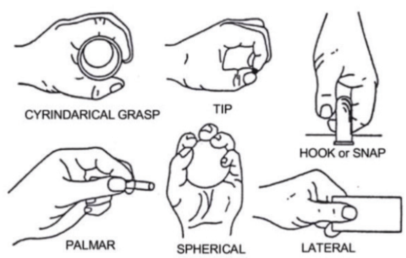


Figure 2: Different hand motions used for classification.

The second data set (finger movement) contains EMG signals from 8 subjects (6 male, 2 female) performing 3 trials for each of 14 specific finger movements (20-second-long trials). The finger movements used in this experiment can be seen in Figure 3.

Both of these datasets are popular benchmarks in the literature and differ quite drastically with regards to data collection (e.g. machines used, sampling rate). By using these data sets in our evaluation of EMG

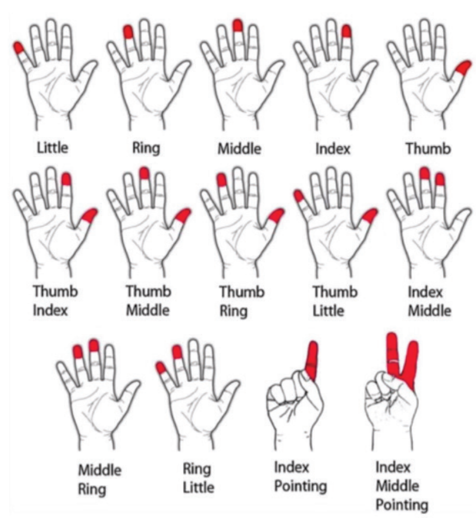


Figure 3: Different finger motions used for classification.

classification techniques, we were able to test the three factors we are interested in simultaneously.

IV. CLASSICAL MACHINE LEARNING

To strike a balance between accuracy, interpretability, and generalization, a bulk of the literature on EMG classification focuses on properly preparing the data and then using this to train classification models we understand well. Preparation consists of preprocessing and feature extraction which ideally make the information received from our learning algorithm easier to interpret. Unfortunately, this type of approach is fundamentally limited by the method that one chooses for preparation. We review common preprocessing and feature-extraction approaches used in the literature along with several classification techniques. In the interest of space, we do not include everything we did using these methods, but instead describe models which best give the basic idea of the general class of classifiers we experimented with[2].

1. Data Processing

Due to noise from electronics and external sources, EMG signals should undergo preprocessing steps to try and strip away "abnormalities". A common first step is to filter the EMG signal with the Butterworth Filter $H = 1/\sqrt{1 + \left(\frac{w}{w_c}\right)^{2n}}$ where w represents the angular frequency, w_c is the cutoff frequency represented as an angular value, and n is the number of elements in the filter. Next is rectification of the EMG signal to remove the negative amplitudes, either by half rectification, or by full rectification. This is followed by smoothing of the signal in some way (we used a moving average of subsets of the signal). Finally, we partition this "cleaned" form of the EMG signal into subsets called windows used for feature extraction or classification. This process is demonstrated in Figure 4.

We experimented with multiple window sizes on our data sets and ultimately concluded that, with feature extraction[3], (i) windows of 512*ms* with a 256*ms* overlap between windows worked best on the hand data and (ii) windows of 128*ms* with a 64*ms* overlap between windows worked best on the digit data.

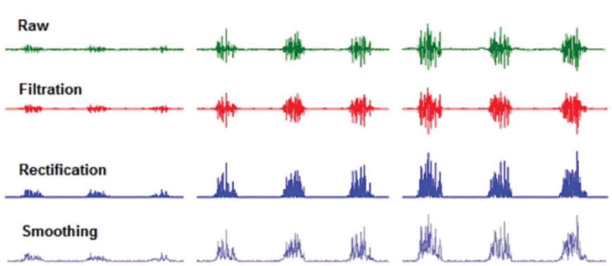


Figure 4: Four preprocessing steps EMG data goes through prior to classification.

Name	Domain	Formula
Root Mean Square	Time	$RMS = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}}$
Mean Absolute Value	Time	$MAV = \frac{1}{n}\sum_{i=1}^{n}\lvert x_i\rvert$
AR Coefficients $(\beta_1, ..., \beta_p)$	Frequency	$X_t = c + \sum_{i=1}^{p} \beta_i X_{t-i} + \epsilon_t$
Waveform Length	Time	$WL = \frac{1}{n}\sum_{i=1}^{n-1}\lvert x_{i+1} - x_i\rvert$
Wilson Amplitude	Time	$WAMP = \sum_{i=1}^{n-1} I(\lvert x_{i+1} - x_i\rvert \geq h)$
Sign-Slope Changes	Time	$SSC = \sum_{i=2}^{n-1} I((x_i - x_{i-1}) \cdot (x_i - x_{i+1})) \geq h)$
Zero Crossing	Time	$ZC = \sum_{i=1}^{n} sgn(-x_i \cdot x_{i+1})$

Figure 5. Features used for Classification. In this table, I refers to an indicator function: either 1 when its argument is true or 0 otherwise; h is some arbitrary threshold value.

2. Feature Extraction

Though preprocessing certainly "cleans" the signal and relieves a large amount of uncertainty surrounding it, we are still left with a long non-stationary time series that may contain inconspicuous redundant information. We use feature extraction methods to attempt uncovering those redundancies in a more succinct form while abstracting away properties that emerge due to time. Most classic machine-learning models are not designed with ordered data in mind, and they struggle with high-dimensional input; feature extraction simultaneously acts to reduce dimensionality of the data while also packaging data in a form more suitable for training.

Using the same windows described above, we calculate features of a preprocessed signal and append the results to get a "feature vector" that describes multiple aspects of the signal. In this project, multiple combinations of time-domain features were used along with autoregressive model coefficients. Figure 6 contains details about the most useful features employed.

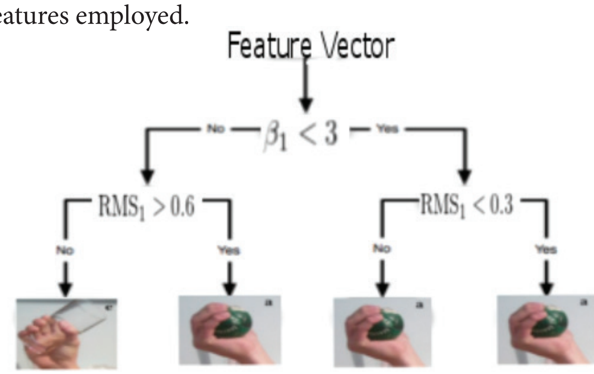


Figure 6. A pictorial representation of a simple tree for hand data classification.

3. Support Vector Machine

A popular and well understood classifier used for EMG is the support vector machine (svm). This model attempts to split an m-dimensional feature space of training data with (m-1)-dimensional hyperplanes. These hyperplanes carve out regions in the feature space that we associate with classes by maximizing the margin between the two closest points on either side of the hyperplane during training[4]. This creates the maximal separation between classes seen in training and allows for the most "flexibility" assuming the test data behave similarly.

More formally, if the hyperplane is defined by

$$(n \cdot x) - b = 0$$

with x as our training data, then we have the optimization problem of minimizing $|| n ||$ subject to

$$y_i(n \cdot x_i - b \geq 1), \forall i \in [m]$$

where $y_i \in \{-1, 1\}$ is the class of training sample x_i. The n and b solving this problem determines the maximum-margin hyperplane.

4. Classification Tree

Another common – and certainly easy to interpret – classifier used in the literature is the classification tree. These models essentially step through each element of the input vector, making a simple decision at each element, and ultimately settling on a class it assigns to said input. We trained these using the popular greedy algorithm known as CART, which splits the feature space recursively using training data in order to create a decision tree for classifying new data points.

This algorithm starts at the root of the tree (i.e. some specific element of the input vector) and, for each feature in the samples' feature vectors, chooses a "best" splitting point for defining a class from the range that said feature exists in; we define this as the point that minimizes the sum of the *Gini impurities*[5] across the two children nodes. Once the algorithm has done this with the entire training set, it then chooses the best split point at each feature by using the "purest" split among those points it found. This simple process is repeated until some stopping criterion is met (e.g. the existence of a node with zero impurity).

A major problem with classification trees is that they tend to overfit the training data it is given. To deal with this problem as well as reduce variance, we can use what is known as bagging, since the computational burden of producing a tree is minimal. Tree bagging (also known as bootstrap aggregating) takes n subsets of training samples with replacement from the complete set of training data and trains a classification tree using these subsets. When we have input we hope to classify, we simply run it through every tree and then use the class predicted by the majority of the trees. Of course, this method is rather naïve and there tends to be correlation between trees due to certain features having higher predictive power for certain classes. We again rely on the simplicity of this model to tackle the problem by creating a "random forest" of trees created from random subsets of the features themselves (with bagging done on each subset of features).

V. NEURAL NETWORKS

Neural networks are a popular form of machine learning based loosely on connections in the brain. They have gained traction recently due to their success in areas such as computer vision, natural language processing, and time series classification and forecasting. Neural networks consist of layers of nonlinear functions composed with trainable linear transforms and as with classic machine learning, they ultimately aim to approximate statistical distributions hidden in underlying data. These models have next to no interpretability, but they generalize trivially to new data sets and can drastically increase accuracy when enough data are available.

1. Neurons and Connections

Neural networks are composed of two main components neurons and synapses. Synapses are weighted connections between neurons that together form the linear transforms between nonlinear-neuron layers.

We can think of a neural network as a series of matrix multiplications followed by nonlinear transforms on vectors. That is to say, our input vector $X = (x_1, x_2, ..., x_n)$ is operated on by the synapses a_{ij} composing a matrix of weights

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

and then each of the resultant vector's elements is fed into some nonlinear function. We repeat this process through a predetermined number of layers until we have a vector of the same dimension as we have classes. It is very important that we note $m = n$ in general, as neuron layers can contain different amounts of neurons.

2. Training

Training of the neural network hinges on altering the weights in between layers. A training set consists of data

for which we know the expected outputs. To train we alter weights between layers using a method of computational gradient descent known as back-propagation. The goal of backpropagation is to recreate expected input-output pairs on our training set. Back-propagation consists of two steps: the forward pass and the backward pass. The forward pass requires calculating outputs of our network over the training set, and the backward pass calculates gradient descent over our loss function via the chain rule through our network. This first requires a notion of "distance" which is provided by our loss function. A loss function tells us how close we are to our expected output on the training set.

The specific loss function we use for our neural networks is cross entropy. Cross entropy is a value from information theory which measures the average amount of information needed to identify an event from an underlying set. It takes two input probability distributions, p and q, and is calculated as:

$$H(p,q) = -\sum_{p} p(x)\log\left(q(x)\right)$$

Naturally, this makes sense as a loss function since we are looking to determine an event based on the smallest amount of information possible. The value is closely related to the Kullback-Leibler Divergence,

$$D_{KL}(P||Q) = \sum_{i} P(i)\log\frac{P(i)}{Q(i)}$$

which, while not symmetric, and thus not a true metric, determines the "distance" between probability distributions.

The actual calculation of the gradient descent can be done in a few ways; here we focus on stochastic gradient descent, which is what was used. The method consists of taking portions of the training set, calculating loss for them, averaging them, and then calculating gradient descent for this average. This is opposed to standard gradient descent which does the same for full expectations of the training set. With stochastic gradient descent we can, in many situations, avoid settling into a local, as opposed to a global, minimum. This motivates our use of the method.

3. Recurrenty Neural Networks

What we have described is the theory around the "feed-forward" neural network. This system has a large shortcoming when it comes to time-dependent data. Feed-forward networks are poor at finding correlations between disparate events in time. To remedy this, we give our network a notion of memory: this constitutes the recurrence. Recurrent neural networks (RNNs) are feed-forward networks where the output from the last hidden layer is included in the input during the next iteration of the network. To train, we take time-series data and partition it. These partitions are fed in to the network successively and the hidden layer output is fed into the network again for the successive calculation. In a sense this can be seen as a very deep feed-forward network, but in time.

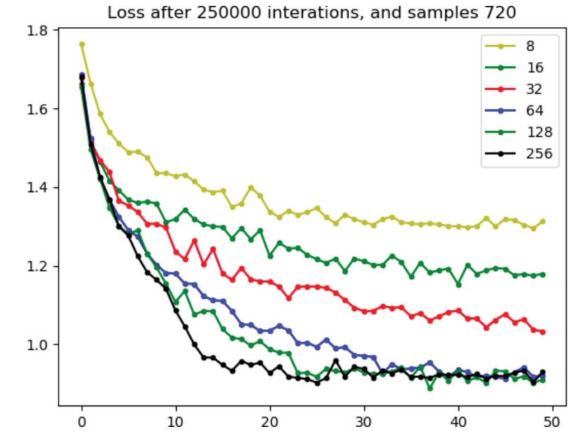


Figure 7. Each color represents different numbers of neurons in the hidden layer. The floor of the loss seems to change predictably with the decreasing size of the hidden layers. Once the hidden layer size is 64, we seem to have found the minimum of this loss function that is possible by only changing this parameter.

In fact, training is done the same way training a feed-forward network is done. The main difference here is that during gradient descent partial derivatives are in time. These networks are particularly deep, though, and as such are very susceptible to both vanishing gradient and gradient blow up. Respectively, these entail our gradient all but vanishing to zero, and our gradient becoming too large and changing our loss drastically in a single iteration. Both of these are important to avoid, and make training RNN's far harder than feed-forward networks.

The recurrent neural network used in our project was based off a simple single layer network, and was programmed in the Python package PyTorch. A set of autoregressive coefficients were set as the input layer for the recurrent network. Usually, an important aspect of the architecture of a recurrent network is the ability to take in variable input sizes, but with this feature input system, it is not entirely necessary.

We used cross entropy loss in conjunction with stochastic gradient descent for training. A total of 250, 000 iterations of training were done, which equates to roughly 250 passes through randomly generated training sample sets. In order to decide an optimal hidden layer size for classification, we used a variety of different sizes. Using the set of values [8, 16, 32, 64, 128, 256] for hidden layer size, the recurrent network was trained and tested on the same data in order to compare accuracy between sets. Overall, there was large improvement

on loss and accuracies for all increases in hidden layer size, with an exception of hidden layers of size 64, 128, and 256 which were comparable in loss and test results.

VI. RESERVOIR COMPUTING

Reservoir computing is a paradigm from computational neuroscience that has been gaining traction in recent years due to the computational efficiency and accuracy of the models it produces. The basic idea is to use an RNN without trained weights as a nonlinear, time-dependant filter for the input signal and to use the new time-series of states from some predetermined subset of neurons in this filter as the input to a simple classification model (typically a regression). Due to how recently these models were discovered, there is no hard consensus on what exactly qualifies as a reservoir computer, but it is generally accepted in the literature that any reservoir computing model has input neurons, a reservoir containing many recurrently connected neurons, and a set of readout neurons whose states are used for classification. An example of this is found in Figure 8. These models are arguably the least interpretable of any of them, but generally require much less data than RNNs.

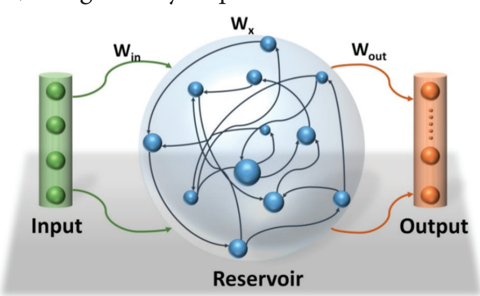


Figure 8. The basic structure of reservoir computing models. Only W_{out} is trained corresponds to training the classifier being used.

As opposed to an neural network, a reservoir computer does not change internal weights and instead trains only the classifier itself. Furthermore, individual points from the signal are fed into the reservoir sequentially. By constructing a model in this way, we rely on the fact that a reservoir is a highly coupled dynamical system with unique solutions that essentially behave like a preprocessing step (input datum act like perturbations which drive the high dimensional dynamics). Not much is known about what makes a reservoir "good," but they tend to perform best when they allow the effects of previous signal points to echo without amplifying themselves. The time series of states of the readout neurons are bundled together and used to train a classifier in the hopes that the dynamics induced by new data are comparable.

In this project, we developed a modular code base using PyTorch for quickly specifying reservoir computing models (we could quickly change topology, weight sampling distribution, size, etc.). Using this we experimented with multiple reservoir network structures, but focused on two simple topologies since they emphasize extremes of modern reservoir design found in the literature:

(i) a fully connected graph of nodes and with weights sampled from a normal distribution and (ii) a cyclic graph of nodes with fully connected input, fully connected output, and weights sampled from a simple uniform distribution. Both use simple *tanh* neurons which transform their input as follows:

$$\tanh\left(I(t) + \sum_{v \in N(x)} v w_{v,x}\right)$$

Here $N(x)$ is the open neighborhood of node x, $w_{v,x}$ is the weight from node v to x, and $I(t)$ is the input data at time t. We should note that since noise should not present itself in a consistent way, Reservoirs should intuitively be good at removing the effects of noise, as such we do not feature extraction nor preprocessing on the experiments involving reservoirs[6].

Model	Features	Accuracy
True Random	N/A	16.6%
FC Reservoir	None	37.7%
SVM	AR(6)+RMS	40.1%
Ring Reservoir	None	43.3%
RNN	ARMA(1,1)	54.5%
Classification Trees	AR(6) + RMS	91%

Figure 9. Comparing classification methods for hand data

Model	Features	Accuracy
True Random	N/A	6.6%
SVM	AR(11)+RMS	22.9%
Classification Trees	AR(6)+MAV+WAMP	55%
LSTM	None	60.7%
Ring Reservoir	None	70.3%
FC Reservoir	None	82.5%

Figure 10. Comparing classification methods for digit data

VII. RESULTS & CONCLUSIONS

For classification of hand data, classical machine learning achieved by far the best results, while the reservoirs struggled. A comparison of the methods described above can be found in Figure 9. In the classification of the digit data, we had drastically different different performances between our models. On this dataset, the reservoirs performed the best while the classic machine learning methods yielded poor results despite careful feature engineering.

This work suggests that feature extraction methods in conjunction with simple machine learning models are good when computational resources are limited and

interpretability is key. By contrast RNNs are good when there isn't a human capable of tuning a model's features to the dataset in question or when data becomes abundant. Reservoir computing models share some of the pros and cons from both of these approaches since they do not required a skilled human nor an abundance of data, but they require some intuition about dynamic systems and luck. As one would expect, there is no "best" approach for balancing interpretability, accuracy, and generalization to new data sets.

[1]It is rather obvious in Figure 1 that there are windows of time where the signal is in phases which correspond to the subject being recorded at rest vs. while performing an action.

[2]Detailed explanations and all of our classification results can be provided upon inquiry to Qian-Yong Chen: qchen@math.umass.edu

[3]We also experimented without feature extraction, but the results were not far from "random" classification as expected.

[4]For this reason they are aptly named maximum-margin hyperplanes.

[5]For N classes and p_i describing the fraction of items labeled with class i on the node p, we define a Gini impurity by $\sum_{i=1}^{N}(p_i(1-p_i)) = 1-\sum_{i=1}^{N} p_i^2$

[6]Interestingly we found that, for essentially every network topology we tried, preprocessing the data tended to have a negative effect on the Reservoirs' ability to generalize.

HACKING LECTURES AT 'OLYMPICS OF MATHEMATICS' IN RIO DE JANEIRO

Associate Professor **Paul Hacking**, with his collaborator Sean Keel from the University of Texas at Austin, was invited to deliver a lecture at the 2018 International Congress of Mathematicians (ICM) held August 1-9 in Rio de Janeiro. With Keel unable to attend, Hacking delivered the lecture to members of the Algebraic and Complex Geometry section on 7 August.

The ICM, held roughly every four years since 1897, is one of the premier forums for presenting and discussing significant mathematical discoveries. Some call it the Olympics of mathematics; the gold Fields Medal that is awarded there Hacking describes as "the Nobel Prize of mathematics.

"It's an unusual honor to be invited to give a speech at this international meeting. I very much appreciate the recognition of my peers. I'm looking forward to it, but it is a bit daunting. Many of my colleagues will be there and I look forward to catching up with them." The last time UMass Amherst was represented by a speaker at the ICM was 32 years ago in Berkeley, where **Bill Meeks**, now a distinguished professor emeritus, had the honor.

Hacking's research area is algebraic geometry, one of two primary methods scientists use to study and define shapes. He explains, "Differential geometry uses the tools of calculus to solve geometric problems, whereas in algebraic geometry we use abstract algebra instead."

"Geometry is often intuitive, and it's easy for us to visualize the difference between the surface of a donut and a sphere," he adds. "But to absolutely pin it down, you need to develop a language to rigorously describe these objects and how a ball is different than a donut. Mathematical language will nail it down precisely."

Hacking has worked with two main colleagues, Mark Gross at Cambridge, U.K., and Keel at UT Austin, plus Maxim Kontsevich of the Institut des Hautes Études Scientifiques, near Paris, to produce several papers and a survey of this research area over the past five years.

One field where algebraic geometry is useful is theoretical physics, Hacking says. String theory, which seeks to describe the fundamental forces of nature and how the universe operates, asserts that rather than the three dimensions plus time we are familiar with, there are instead 10 dimensions, and six of them are very, very small, in the quantum arena and not visible to the naked eye.

"Imagine a garden hose seen from a long distance away that appears to be one-dimensional, but as you get closer you see another dimension," he explains. "String theory says if you were able to look at smaller scales you'd see extra dimensions. At very small scales, quantum mechanics plays a role in physics and weird phenomena will be explained by studying these six extra dimensions. There is a geometry of the very small object that would explain quantum behavior. This six-dimensional object is called a Calabi-Yau (C-Y) manifold, and it's one of the objects we study in algebraic geometry."

Understanding quantum physics requires precisely describing the shapes of these manifolds, Hacking says. "The other key idea is that the elementary particles of quantum physics are not points but little loops of string. This smooths out the interactions and makes the mathematics possible. Particle interaction, then, rather than an instantaneous collision, is gradual." This leads on to mirror symmetry, which refers to paired C-Y manifolds, related in physics but mathematically very different, he adds.

Hacking, who came to campus in 2009 from the University of Washington, earned his undergraduate and advanced degrees at the University of Cambridge and completed a postdoctoral fellowship at the University of Michigan.

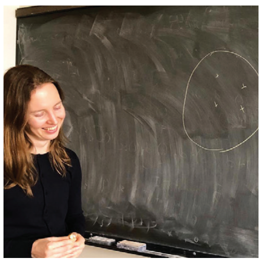
MATH FOR ALL continued from cover

"I chose a format that fits this particular platform: every week, I feature someone new, and every day there is a different picture and quote from an interview I did with them. Throughout the week, we get to know this person in many ways: why and how they ended up in mathematics, what they love about it, what their research is about, as well as any issues they have had to deal with."

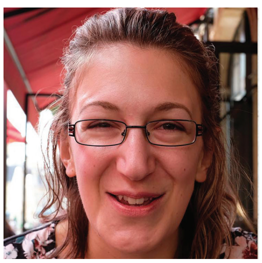
When asked how she selects people to feature, Raymond said, "I mostly feature female mathematicians at different stages of academia. Some of them are friends and colleagues I have known for a long time and others are strangers I meet when I go give talks at conferences or at other universities.

"As an undergrad at MIT, I only had two math classes that were taught by women: neither were professors and both were there very temporarily. Had I started my undergrad the year before or after, I might not have had a single math course taught by a woman. This is troubling: I doubt that I would have stayed in math had I not met these women. Until then, I couldn't picture a future in mathematics—I couldn't recognize myself in my male professors, and it was hard to imagine that someone like me could belong among them.

"I strongly believe the first axiom of Federico Ardila's *Todos Cuentan* which states that 'mathematical talent is distributed equally among different groups, irrespective of geographic, demographic, and economic boundaries'.







"Math is simply for anyone who enjoys tackling challenging problems—nothing else matters"

"As I started working on the project, I realized that there are even less initiatives promoting men of color in mathematics than there are for women—even though, just like women, they are still underrepresented in math and have access to very few role models who look like them. Many of my first followers were also young men of color, so I decided to also feature wonderful mathematicians who look like them."

Who do we picture when we picture a mathematician? "This is a question that has haunted me since seeing Francis Su give his amazing speech Mathematics for Human Flourishing.

"I have personally seen everybody from little girls to grandmas get positively giddy about math. I have stood in front of crowds of teenagers in poor high schools and rooms full of prisoners that erupted in cheers because of math. And yet, these are not the people we think of when we think of mathematicians.

"The goal of my project is to literally show everyone that the first axiom is true. How? By sharing stories from amazing mathematicians and computer scientists, presenting the fascinating work they do, discussing the struggles they have faced and celebrating their diversity.

"One topic that came up many times in interviews is how hard work is so much more important than being 'brilliant' in mathematics. There is this perception that one has to be a genius to be a mathematician, and this is very far from the truth. I want people to know the average mathematician is not a genius constantly working alone.

"Another theme that kept coming back in interviews is how much of a social field mathematics truly is. Most research nowadays is done in collaborations, and most interviewees mentioned how the friendships emerging from these are one of the best part of being a mathematician. The image of the lonely mathematician must also be shattered."

OUTSTANDING STUDENTS HONORED AT 2018 AWARDS DINNER

On 11 April 2018, the Department of Mathematics & Statistics celebrated the meritorious activities of our top students at our annual Awards Dinner. This evening commends the winners of the Jacob-Cohen-Killam Mathematics Competition and the M.K. Bennett Geometry award, as well as our REU participants, members of the Putnam Competition team, and other students deserving special mention. Together with the family and friends of the awardees, we were joined by alumni **James Francis** '86 and **Roy Perdue** '73.

The evening began with refreshments and dinner. Several problems included on the program inspired lively discussions, both mathematical and tangible. The awards portion of the event opened as our guests were attacking their crème brule, with greetings from our newly minted department head **Nate Whitaker**.

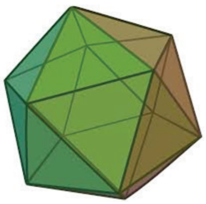
This year the department had a bustling crowd of REU students, thanks in large part to the continued generous support of **Joan Barksdale** '66. Professor **Matthew Dobson**, the fearless leader of our REU program, recognized the following students for their research contributions: **Gabriel Clara**, **Shelby Cox**, **James Hagborg**, **Augustus Ijams**, **Edward McCormick**, **Gregory McGrath**, **Kai Nakamura**, and **Nghia Nguyen** in Pure Math**; Mahesh Dhulipala**, **Sydney Hauver**, **Xinyi He**, **Ji-Hun Hwang**, and **Wenbo Xie** in Applied Math; and **Swastika Khunjeli** and **Hanfei Zhang** in Statistics. Kai and Gregory will start PhDs in pure math at Georgia Tech and UC Santa Barbara, respectively, in the fall, while Sydney will begin a career in the US Air Force Space Command.

The **M.K. Bennett Geometry Award** is presented to the student who exhibits the best performance in our Geometry course Math 461. The award honors the memory of Professor **Mary Katherine Bennett**, who earned the first PhD in our department in 1966. After teaching at Dartmouth College, she returned to UMass Amherst for the rest of her career, where she encouraged interest in geometry and high school teaching among undergraduates. The course she developed now covers Euclidean, spherical, and hyperbolic geometry.

Professor **Paul Hacking** presented the winner **Benjamin Levine** with his award**.** Benjamin plans to work in cybersecurity, combining tools from abstract algebra and computer science.

Alexander Fischer, **James Hagborg**, **Patrick Lei**, and **Artem Vysogorets** were recognized for competing in the 2017 **Putnam Exam**. The team placed 46th out of 575 institutions. James's performance was truly outstanding, placing in the top 7% of contestants. He will begin a PhD in Pure Mathematics at UC Berkeley this Fall.

The program for the dinner included the following problem from the exam:



A regular icosahedron is a polyhedron with 20 triangular faces and 30 edges. How many different ways are there to paint each edge red, white, or blue, such that each of the triangular faces has two edges of the same color and a third edge of a different color?

The **Jacob-Cohen-Killam Mathematics Competition** is named in honor of the memories of Professors **Henry Jacob**, **Haskell Cohen,** and **Eleanor Killam**. These three faculty members spurred interest in mathematics among undergraduates through annual mathematics contests.

The competition is open to first and second year students. Each year around twenty contestants attempt to solve ten challenge problems dreamed up by our faculty members. This year the competition was again generously funded by **John Baillieul** '67, **Roy Perdue** '73 and **James Francis** '86**.**

Professor **Ivan Mirkovic** awarded this year's first prize of $1600 to **Patrick Lei**, the second prize of $1200 to **Nishad Ranade**, and the joint third prize of $400 each to **Mark Xiang** and **Shirui Cao**. Professor Mirkovic explained that the JCK problems, while only requiring basic mathematical knowledge, force the students to think creatively and are designed to connect with important research topics in higher mathematics (this year, the Feynman diagrams of quantum physics and Young tableaux of combinatorial representation theory played a role).

The program for the dinner featured the following problem from this year's JCK competition.

If you randomly break a stick in two places, what is the probability that the three pieces are the sides of a triangle?

On a related note, Professor Matthew Dobson recognized the team of **Jonah Chaban**, **Ji-Hun Hwang**, and **Artem Vysogorets** who won the SCUDEM (Student Competi-


Yankai Mark Xiang (Jacob-Cohen-Killiam Awardee) with Professor Paul Hacking


Diana Hansen (Leadership Award) with mother Joyce


Artem Vysogorets (Putman Exam) with father Mikhail


James Hagborg (Lincoln Scholarship Awardee)

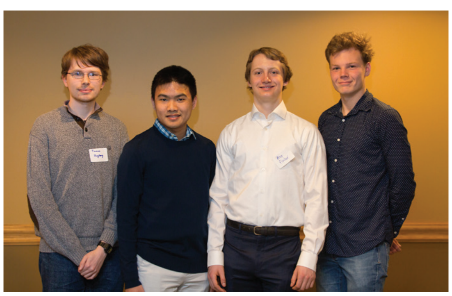

Molly O'Neil (Leadership Awardee)

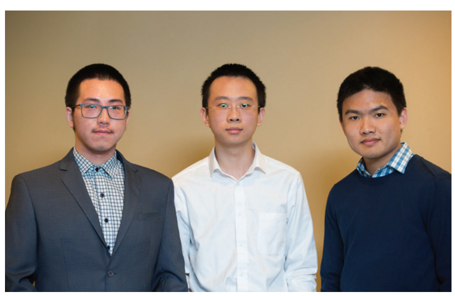

Edward McCormick (REU Participant)


Gregory McGrath, Kai Nakamura, James Hagborg, Edward McCormick, Shelby Cox, Augustus Ijams, Sydney Hauver and Wenbo Xie (REU)


Kai Nakamura, Shelby Cox, Gregory McGrath and Stefan Grosser (Honors Research Thesis)


Jessica Toupin (Teaching Concentration Award) with Professor Farshid Hajir


Gregory Hagborg, Patrick Lei, Alex Fischer and Artem Vysogorets (Putnam Exam)


Yankai Mark Xiang, Shirui Cao, and Patrick Lei (Jacob-Cohen-Killiam Competition Award)


Michael Shliselberg, Kristina Yamkovoy and Nathan Rose (Don Catlin Award)


Whitney Bradley, Molly O'Neil and Diana Hansen (Leadership Award)


Jake Lagerstrom (Undergraduate Advising)


Sidney Hauver (REU) with Department Head Nate Whitaker

tion Using Differential Equations Modeling) Competition in Fall 2017. In the modelling spirit, the following bonus problem was included in the dinner program:

A circular table with four legs is placed on uneven ground. Show that after rotating the table through some angle the table does not wobble.

We are fortunate to have several awards made possible by generous donations.

UNDERGRADUATE AWARDS

Professor **Nate Whitaker** presented the Leon Emory Lincoln and Robert Bradley Lincoln Scholarship to **James Hagborg**, and the Don Catlin Award for Outstanding Achievement in Applied & Computational Mathematics to **Nathan Rose**, **Michael Shliselberg**, and **Kristina Yamkovoy**. Kristina will begin a PhD in Biostatistics at Boston University in the Fall.

Professor **Hongkun Zhang** presented the Bob and Lynne Pollack Award for Outstanding Academic Achievement in Actuarial Science to **Aleigha Correia** and **Jeffrey Spahl**. Professor **Eric Sommers** presented the Steve and Geni Monahan Student Leadership Award to **Whitney Bentley**, **Diana Hansen**, and **Molly O'Neil**.

Professor and former head **Farshid Hajir** presented the awards for Outstanding Academic Achievement in the Teaching Concentration to **Brandon Nowakowski** and **Jessica Toupin**.

Professor Hajir also noted that this year we lost one of the department's most cherished and faithful supporters, Professor Emeritus **Eleanor Killam**, who will be sorely missed by all who knew her. Professor Hajir fondly remembered Eleanor's tireless energy and enthusiasm, often meeting her early on Saturday mornings in the department during his busy tenure as head [*see Remembrance on page 24*].

Finally, Professor **Nate Whitaker** recognized the outstanding academic achievements of seniors **Samantha Baturin**, **Matthew Bissaillon**, **Jonah Chaban**, **Anya Conti**, **Shelby Cox**, **Jillian Davidson**, **James Hagborg**, **Gregory McGrath**, **Anthony Rentsch**, and **Jared Yeager**. Shelby has won a prestigious National Science Foundation graduate research fellowship to pursue a PhD in pure mathematics at University of Michigan [*see Shelby's Student Spotlight on the back cover*]. Anya will begin a masters in Statistics at the London School of Economics in the Fall, while Anthony will begin a masters in Data Science at Harvard.

GRADUATE AWARDS

Graduate program director Professor **Tom Weston** recognized the winners of the graduate student awards: **Zhijie Dong** and **Haitian Yue** won the Distinguished Thesis Award, while **Michael Boratko** received the Distinguished Teaching Award.

Haitian will begin a postdoc at the University of Southern California in the Fall, while Zhijie is planning to work with distinguished Professor Hiraku Nakajima at Kyoto University in Japan.

The evening ended with closing remarks by Physics Professor **Mark Tuominen**, Associate Dean of the College of Natural Sciences. He spoke warmly of the rousing evening and congratulated the students again on their achievements, saying that the work itself is the real reward – though it is nice to celebrate their work as well!

Haitian Yue (Distinguished PhD Thesis)

Jonah Chaban and Artem Vysogorets (SCUDEM)

Graduating seniors with outstanding academic achievement: (front) Annya Conti, Samantha Baturin, Shelby Cox, Jonah Chaban (back) Matt Bissaillon, James Hagborg, Anthony Rentsch, Anthony Yaeger, Gregory McGrath

Awards Dinner photo credit: Vivian B Photography

SOLUTIONS TO LAST YEAR'S CHALLENGE PROBLEMS

Since the Problem Master was on sabbatical leave, this year's solutions, already far past the deadline, will necessarily be a terse affair—more hints than worked out problems.

Problem 1. *You start with an empty bucket. Every second, you either add a stone to the bucket or remove a stone from the bucket, each with probability 1/2. If you want to remove a stone from the bucket and the bucket is currently empty, you do nothing for that second (still with probability 1/2). What is the probability that after 2017 seconds the bucket contains exactly 1337 stones?*

The Problem Master put this question to a group of 9th graders at a Waldorf School. Whereas neither of the college students participating in the JCK exam nor expert faculty had much of an idea what to do with the problem, the 9th graders immediately did the following experiment. They went to the board and wrote down

1 sec: 1 0
2 sec: 2 1 0
3 sec: 3 2 1 2 1 0 1 0
4 sec: 4 2 2 0 2 0 1 0 3 1 1 0 2 0 1 0

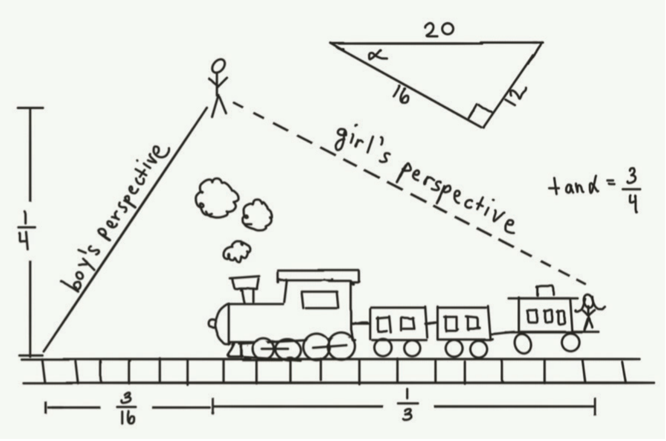
and then a somewhat longer list of possible pebble accumulations in the bucket after 5 seconds. Since the problem asks for the probability that a certain number $N = 1337$ of stones are in the bucket after $n = 2017$ seconds, we need to count how often that number N appears after n many seconds. The 9th graders tallied up their list and obtained for $n = 5$ the following result: 5 and 4 appear once, 3 and 2 appear five times, 1 and 0 appear ten times. The numbers 1, 3, 5, 10 quickly caught their attention and soon the word "combinations" and the symbol $C_{k,n}$ came up. Apparently they had learned in their math class that those numbers come up when selecting (without paying attention to the ordering) k things out of n things for $k = 0, \ldots, n$, usually notated by the binomial coefficients

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

From here it is pretty clear what to do: for odd n, which we are interested in (the even case is similar), rearrange the numbers $0,\ldots,n$ into the list $n, n-2, n-4, \ldots, 0, 2, \ldots, n-1$. Then the k^{th} number in that reordered list occurs exactly $\binom{n}{k}$ many times for $k = 0, \ldots, n$. In the case at hand $n = 2017$ and we are interested how often 1337 pebbles will be in the bucket. Since $1337 = 2017 - 680$, the number 1337 appears at the $k = 340^{\text{th}}$ place in the reordered list. Thus, the odds to have 1337 pebbles in the bucket after 2017 seconds is $\binom{2017}{340}$ out of 2^{2017}. The careful reader is invited to provide a formal proof that the pattern, which those 9th graders observed, is in fact accurate.

Problem 2. *Suppose you are standing in a field near a straight section of railroad tracks, just as the front of the train locomotive passes the point on the tracks nearest to you, which is 1/4 mile away. The train is 1/3 mile long and traveling at 20 mph. How slowly can you run and still catch the train? In what direction should you run?*

Even though the problem sounds like some kind of min/max problem, and thus could be done by a standard calculus argument, the Problem Master consulted with the same group of 9th graders, whom had no idea of calculus. They came up with the idea to look at the situation from a person in the caboose: she would see a guy out there in the fields, 1/3 mile to the left of the caboose and 1/4 mile away from the train tracks, running at 20 mph from the left to the right parallel to the train tracks. Obviously this is not the best plan if the guy wants to join the girl in the caboose. What he must do is to correct his course and angle for the caboose and, since cool dudes don't necessarily want to be in a hurry, run as slowly as possible. Here is what this looks like from the girl's perspective and the boy's perspective:



From elementary triangle geometry, we deduce that the boy has to run at 12 mph at an angle of $\tan^{-1}(3/4) \approx 36.8$ degrees from the vertical in the direction the train moves. He will intercept the caboose 3/16 of a mile from where the locomotive was when he started running. Now you could argue that the boy could have done different course corrections than the one indicated in the drawing. Yes, but . . . recall, he is a cool dude!

Problem 3. *Find all real solutions of the equation.*

$$2(2y-1)^{1/3} = y^3 + 1$$

By introducing $x = (y^3 + 1)/2$ the equation is rewritten as $(2y - 1)^{1/3} = x$, i.e., $2y - 1 = x^3$. So, our equation becomes a *symmetric* system

$$2y-1=x^3 \text{ and } 2x-1=y^3$$

The difference of the two equations says that $2(y-x)$is $x^3-y^3 =(x-y)(y^2+yx+y^2)$, So either $y=x$ or y^2+xy+x^2 is -2. However, the second possibility has no solutions since $y^2+xy+x^2 =(y+\frac{1}{2}x)^2+\frac{3}{4}x^2 \geq 0$. So, the solutions satisfy $x = y$ and this reduces the system to $2y - 1 = y^3$. The last equation has a solution $y = 1$, so it factors as $0 = y^3 - 2y + 1 = (y - 1)(y^2 + y - 1)$ with solutions $y = 1$ and $y = \frac{1}{2}(1 \pm \sqrt{5})$.

Problem 4. *Suppose there is a circular road with n gas stations having in total just enough gas for your car to go around the circle once. Your cars gas tank is empty. Prove that there is some gas station at which you can start so that you make it around the circle without running out of gas.*

We list the gas stations by $x_1,\dots,x_n$ and plan to drive following this order. Then we define the excess $e(x_k)$ to be the difference between the amount of gas at station x_k and the amount of gas needed to get to the next station x_{k+1}. We denote $x_{n+1} = x_1$ since we are on a circle. We know that the combined amount of gasoline from all n stations gets the car exactly once around the circle, therefore we must have

$$\sum_{k=1}^{n} e(x_k) = 0\,.$$

You obviously would not want to start at a station x_k with negative $e(x_k)$. But starting at a station with positive $e(x_k)$ will, in general, not work either. There could be too many stations down the circular road with negative excess gas. So we carry out the following "deleting gas stations" algorithm: if $e(x_k) > 0$ then we add

$$e(x_k) + e(x_{k+1}) + \cdots + e(x_l)$$

up to the next positive excess $e(x_{l+1})$. We thus obtain a new shorter list of excesses with which we repeat the same "deleting gas stations" algorithm. Eventually the list will consist of only one member with zero excess. This is the gas station from whence to start.

Problem 5. *Show that for any positive integer m*

$$m^{2017} = \sum_{k=1}^{2017} \binom{m}{k} x_k$$

with integers x_k. Here we set $\binom{m}{k} = 0$ if $k > m$.

There's nothing special about the power 2017, so you might experiment with small powers of m, and then try to guess an explicit formula for the integer coefficients x_k. A sophisticated way to find such a formula is to view this expansion for $f(m) = m^{2017}$ as analogous to the Taylor expansion where $\binom{m}{k}$ replaces m^k. The *difference* operator δ defined by $(\delta f)(m) = f(m+1) - f(m)$ is analogous to the derivative, and one checks $x_k = (\delta^k f)(0)$ are these integers!

A more elementary way to do this is to start with the sequence a^0 with $a^0_m = m^N$ and pass to sequences a^s of successive differences by $a^s_m = a^{s-1}_{m+1} - a^{s-1}_m$. (Of course, we are secretly applying our operation δ.) Experiments show that the $(N+1)^{\text{st}}$ sequence a^{N+1} is zero (the reason is really that $\delta^{N+1}m^N = 0$). The table of these sequences is reminiscent of Pascal triangles and this can be used to reconstruct the zero$^{\text{th}}$ row a^0 by going up from the N^{th} row. This will again manifest the powers m^N in the zero$^{\text{th}}$ sequence as a combination of binomial functions with the coefficients which are manifestly integers.

Problem 6. *Through the center of a cube (of side length one) draw a straight line in such a way that the sum of the squares of its distances from the vertices is (a) minimal, (b) maximal.*

To get a feel for the situation, we calculate the sum of the squared distances for special lines anticipating that we will find candidates for the solution:

- A line connecting the centers of two opposing faces with sum of squared distances to the vertices: $8(\frac{\sqrt{2}}{2})^2 = 4$.

- A diagonal of the cube with sum of squared distances to the vertices: $6(1 - (\frac{\sqrt{3}}{3})^2) = 4$.

- A line connecting the midpoints of opposite edges through the center of the cube with sum of squared distance to the vertices: $4(\frac{1}{2})^2 + 4(\frac{\sqrt{3}}{2})^2 = 4$.

At this point one wonders what is going on and whether the question is purposely misleading. In fact, as one can check by some basic vector algebra, the sum of squared distances to the vertices from any line through the center of the cube is 4. The Problem Master is wondering* whether this could be true for the tetrahedron and perhaps for any Platonic solid?

*The Omniscient Editor has a proof, applying Schur's Lemma to the moment of inertia matrix L for the set V of vertices, with the center of mass 0: if the symmetry group of V acts irreducibly on W=span(V), as true for a Platonic solid, then L is a fixed multiple of projection to W, and so this sum is the same for any line though 0.

We always like hearing from you. Please send solutions to Professor Franz Pedit <pedit@math.umass.edu> with the subject line "Challenge Problems 2018."

UMASS HOSTS MATH EDUCATORS, FOSTERING CLASSROOM COLLABORATION

About 70 math educators from across the region convened at UMass Amherst late June for the one-day institute organized by the Western Massachusetts Math Partnership (WMMP) with the hopes of solving not only math problems, but some classroom challenges as well. Over the past seven years, several hundred math educators from the region have participated in WMMP activities, including one- and multi-day institutes.

Professor **George Avrunin** helped organize the event, which aims to elevate math curricula that will propel students into higher performance levels and give elementary, intermediate and high school teachers a place to exchange ideas and enhance their teaching repertoire. Avrunin hopes educators who attended this year's seminar now see math as an “active, sense-making endeavor” and better “grasp how mathematical topics fit together.”

“Without a firm understanding of math, many doors remain closed to young people—especially those from underserved populations,” says Avrunin. He hopes that his work with WMMP and K-12 teachers will have a positive impact on children throughout Western Massachusetts.

FOUNDATIONS OF DATA SCIENCE: A NEW COURSE FOR NON-MAJORS

In 2011, The McKinsey Global Institute, a research arm of the consulting firm, predicted that the demand for analytical talent could be 50–60% greater than supply by 2021. A 2017 IBM study showed an increase of 364,000 data science positions over the year before, and predicted there would be 2,720,000 by 2020. Demand for data scientists was so much greater than supply that their 2016 median salary was $105,000. The problem seems clear: we need more people with education in data science! But what is data science?

The National Academy of Sciences convened a committee of statisticians, mathematicians, engineers, and computer scientists from industry, government, and academia to prepare a report titled *Frontiers in Massive Data Analysis* to advise policy-makers on what data science is, what the promises of data science are, and how to address the needs presented by massive data sets. The committee concluded in their report that “…the analysis of massive data goes well beyond the province of a single discipline, and … [there is] the need for a thoroughgoing interdisciplinarity in approaching problems of massive data.” The report further concludes that computer scientists, statisticians, mathematicians, and domain scientists all have essential roles in the design of data analysis systems. Modern data science is interdisciplinary.

Many organizations now promise training in data science, often without the rigors of mathematics, statistics, or computer science. The result is confusion surrounding the use of the phrase “data scientist.” When confronted with an applicant for a data science position, organizations wonder: is this someone who has a deep education in computational and inferential thinking that will allow them to adapt to new problems throughout their career or is this someone who has a superficial understanding of mathematics and computation? To the National Academies Committee on the Analysis of Massive Data Analysis, a data scientist should have “experience with massive data and with computational infrastructure that permits the real problems associated with massive data to be revealed”. A data scientist is someone who combines deep computational and inferential thinking.

How did we develop our solution to the “data science gap”?

In 2017, Assistant Professor **Patrick Flaherty** partnered with Benjamin Marlin of the College of Information & Computer Sciences to propose a new cross-listed course Foundations of Data Science to meet the demand for data science education at UMass Amherst. Based based on a course by the same name taught by John DeNero at UC Berkeley, its initial offering at UMass in Fall 2018 was open to 200 undergraduate students — and the number is expected to grow. It aims to provide a foundation in computational and inferential thinking primarily for students across the university who are not majors in Math or Stat or Computer Science, who can develop a deep understanding of quantitative reasoning with large data sets in their own academic disciplines.

As data science disrupts traditional jobs and careers, we hope our partnership will place UMass Amherst at the forefront of providing our students with the kind of educational opportunities that lay the foundation for lifelong growth. Foundations of Data Science is an initial step towards this long-term vision, and we expect to report new data science developments in coming years.

NEW FACES IN THE DEPARTMENT

After an initial year-long leave, Owen Gwilliam joined the department as an assistant professor in September 2018. His work revolves around quantum field theory, focusing on foundational aspects as well as applications to pure mathematics itself. With Kevin Costello, he has written a book that explores how to interpret formalisms from quantum gauge theories in terms of contemporary higher algebra (e.g., operads). He has also studied examples of field theories that encode important topological invariants, like the Todd and Witten genera.

Owen received his PhD from Northwestern University in 2012 under the guidance of Costello, and then spent two years at the University of California, Berkeley, as an NSF postdoctoral fellow with Nicolai Reshetikhin as mentor. The last four years he enjoyed a postdoc at the Max Planck Institute for Mathematics, in Bonn, Germany, while his wife Sophie worked as a doctor for the US Army. Owen and Sophie look forward to settling in Amherst after fourteen years on the move. They have two young sons, Laszlo (5) and Hadrian (1).

Vince Lyzinski joined the Department as an assistant professor in September 2017. His research has focused on both developing classical statistical methodology – such as hypothesis testing, classification, and clustering – for network data, and constructing and analyzing network-specific data mining tasks and algorithms—such as graph matching, data de-anonymization, and vertex nomination. Network data is becoming increasingly common across the social and natural sciences, and Vince's work has been motivated by the need for scalable, principled statistical methodologies to analyze these nonstandard data types. For example, in social sciences, graph matching can be used to identify users across different social network platforms. Vince and his coauthors have developed robust procedures that can estimate these correspondences in very large networks, and their current work investigates the theoretical limits of graph matchability; for instance, determining critical regimes in which the identities cannot be recovered and the graphs can be effectively anonymized.

Vince received his PhD degree in applied mathematics and statistics from John Hopkins University in 2013 under the supervision of Professor James Allen Fill. He was a postdoctoral fellow in the Applied Mathematics and Statistics Department at Johns Hopkins University under the supervision of Professor Carey Priebe in 2013–2014; then, over the next three years, he served as both a Senior Research Scientist at the Human Language Technology Center of Excellence, and also as an Assistant Research Professor in the Applied Mathematics and Statistics Department, both at Johns Hopkins.

Alejandro Morales joined the department as an assistant professor in September 2017. Alejandro's research is in enumerative and algebraic combinatorics. In the past years, he has been part of a project studying and classifying new formulas from geometry to count fundamental objects in algebraic combinatorics called standard Young tableaux of skew shape. The number of such objects gives the dimension of irreducible representations of affine Hecke algebras and there is no simple product formula to count them. The new formulas from geometry are related to statistical mechanics and tilings; they have led to new asymptotic results for the number of tableaux and a link to the simple inequality that π is less than twice the golden ratio. Alejandro has also worked on the study of polytopes coming from flows on graphs related to Kostant's vector partition function; he found a link between these polytopes and the space of diagonal harmonics, and he has studied formulas for their volumes and number of lattice points.

Originally from Colombia, Alejandro earned a PhD at MIT under the supervision of Alex Postnikov. Before coming to UMass, he was a CRM postdoctoral fellow at the Université du Québec à Montréal for two years and at UCLA for three years. Alejandro is part of the "comunidad colombiana de combinatorial" group, which organizes yearly summer schools on discrete math in Colombia.

Annie Raymond joined the department as an assistant professor in January 2018. With her collaborators, she established an unexpected direct connection between the celebrated flag algebra methods of Alexander Razborov and the classical theory of sums of squares polynomials in optimization and real algebraic geometry. The key insight was to identify the

symmetries in the flag method and connect them to standard symmetry reduction techniques for sums of squares polynomials and semidefinite optimization. This work opened the door for a systematic approach to an array of problems in computer science, combinatorics and optimization, and has already received considerable attention from these communities. These days, she is applying these ideas to graph profiles, a concept that provides a way of "understanding" very large graphs which are ubiquitous in modern-day applications.

Annie received her PhD in 2014 from the Technical University in Berlin under the supervision of Martin Groetschel. Her thesis work was centered around problems in polyhedral combinatorics and combinatorial optimization, spanning both theory and applications. She was an Acting Assistant Professor at the University of Washington in Seattle from 2014–17. In the fall of 2017, Annie spent a semester as the Gamelin Endowed Postdoctoral Fellow at the Mathematical Sciences Research Institute in their Geometric and Topological Combinatorics program.

Annie gives herself tirelessly to math outreach and education. She has taught weekly college math classes at the Monroe Correctional Complex and at the San Quentin State Prison, so that inmates can obtain their associates degrees. She also runs the Instagram feed "_forall" that aims to increase the representation of women and other minorities in mathematics and computer science highlighted in our cover feature.

Naitian Wang started his new job as the Business Manager in the Department of Mathematics & Statistics this summer; he's excited to learn everything new to support the professional business of the department head, faculty, students, and staff.

He's been working in the finance field for many years, most recently in the Dean's Office in the College of Education as the Associate Director of Financial Services. Naitian is also an alumnus of UMass Amherst, and he's happy to share an interesting feature of his given name: "NAITIAN" is a palindrome!

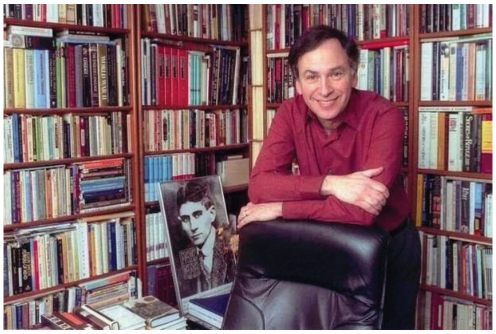
PUTTING A NUMBER ON TRUST continued from cover

Katsoulakis says the work is immediately applicable to computational chemistry and materials science problems where there is a need to piece together computational models with data from different scales, from quantum to mesoscale to macroscopic or "real life" scale. Key mathematical tools to provide performance guarantees include uncertainty quantification, information theory, robust optimization, extreme events and approximate inference.

Specifically, for this grant the researchers will investigate the effect of model and data uncertainties in materials science problems with multiple spatio-temporal scales, such as understanding how uncertainties at the quantum level propagate to meso- and macro-scales, and the impact of rare and extreme events on predictions. Further, they intend to provide reliable computational strategies for design and optimization under uncertainty focused on energy research problems such as designing better batteries, thermoelectric devices and fuel cells.

An important feature of their effort to build more reliable computational models, they plan to address the predictive accuracy of machine learning algorithms, using similar mathematical, statistical and computational tools. Katsoulakis says, "A notable and important example across application domains of machine learning methods that we plan to address is providing prediction guarantees for classes of approximate inference algorithms such as variational inference and expectation propagation methods."

Ricci flow, an important new tool in geometry and low-dimensional topology, is still under development and getting its "bugs" worked out (note the little bug on the first C).



Photograph from *Amherst Bulletin* by Carol Lollis

Professor Richard S. Ellis, long-time colleague and co-editor of this Newsletter, passed away 2 July 2018.

He was born 15 May 1947 in Brookline and grew up in Boston. He attended Boston Latin School and then Harvard, where he majored in mathematics and German literature and graduated in 1969. He earned a Ph.D. at the Courant Institute of Mathematical Sciences at New York University in 1972 and began his research and teaching career at Northwestern University that same year.

He joined the UMass Amherst Department of Mathematics and Statistics in 1975 and became a full professor in 1981. He also taught classes about Judaism and the Torah as an adjunct professor of Judaic and Near Eastern Studies at UMass and elsewhere.

He was a pioneer in the area of probability theory known as large deviations and proved an important theorem in this field, the Gärtner-Ellis Theorem. Professionally, he was most proud of having inspired and mentored many undergraduate and graduate students across his 45-year teaching career as a professor. Always trying to help others, he was proud of having brought meditation to his classroom. He published two research-level mathematics books, numerous research papers and a book about meditation.

Professor Emeritus Samuel S. Holland, Jr. passed away at his Eastham home on 13 October 2018. He was a veteran of the Korean War, serving as part of Operation Teapot, the US Army's atmospheric nuclear test program.

Sam received his BS degree from MIT, his Master's in Mathematics from the University of Chicago, and his PhD in Mathematics from Harvard. Before joining the Department in 1967, Sam worked as an industrial mathematician with Technical Operations and taught for six years at Boston College.

While at UMassAmherst, Sam received the Distinguished Teaching Award and authored numerous scholarly journal articles and the groundbreaking book, Applied Analysis by the Hilbert Space Method. This reflected his passion for improving mathematics education: the book aimed to convey traditionally advanced mathematics concepts to undergraduate students. After Sam's retirement in 1997, this same passion led him to author an extensive NSF proposal on how to attract more PhD mathematicians into public high school teaching.

REMEMBRANCE OF WALTER ROSENKRANTZ

Walter Rosenkrantz came to UMass in 1971, two years after I did and one year after Donald Geman. Don and I were newly minted Ph.D.s when we arrived, whereas Walter had held a named instructorship at Dartmouth, a postdoc at Courant Institute, and an associate professorship at NYU before coming here. The three of us constituted a nascent probability group, to which Walter, being a little older, added an aura of maturity.

Walter graduated from the University of Chicago at age 19 and obtained his Ph.D. in 1963 from the University of Illinois, under the supervision of J.L. Doob, one of the luminaries of the theory of stochastic processes. His dissertation (*Trans. Amer. Math. Soc.*, Aug. 1965) is a blend of probabilistic potential theory and classical Fourier analysis. Probabilistic potential theory refers to the interplay (in fact in some sense almost an isomorphism) between classical and "modern" potential theory, Markov processes, martingales (about which more below), and associated analysis (e.g., semigroups of operators). Doob was one of the pioneers of the field, as well as of the theory of continuous-time stochastic processes, which involves very complicated measure theory. A summary of Doob's work that also gives the flavor of probabilistic potential theory is given by a subluminary, R.K. Getoor, *Ann. Prob.* 37, 2009. It was one of two main areas in stochastic process research that were active roughly between the 1950s and 2000. The second was invariance principles or functional central limit theorems (fCLTs) for stochastic processes; these are results that generalize the classical central limit theorem(s) to convergence of probability measures on infinite-dimensional spaces, especially function spaces such as C[0,1]. These two streams, probabilistic potential theory (especially Markov processes and martingales) and fCLTs, flow through almost all of Walter's work, sometimes separately, sometimes intertwined.

Aside from his dissertation, Walter's oeuvre consists of some forty papers, several technical reports, lecture notes, and a textbook (1997) on probability and statistics "for science and engineering," at

about the level of STAT 515-516. The second edition (2008) of the book added "and finance" to the title. Late in the 20th century many probabilists, including Walter and (true confession) myself, became interested in mathematical finance because of its connection with Brownian motion, stochastic differential equations, and the like. Many mathematicians (and some physicists) went to Wall Street and became "quants." Except for some serious browsing I did not pursue the subject, but Walter went so far as to develop an undergraduate course in mathematics of finance. I don't know whether the mathematicians contributed to the global financial crisis of 2008.

Several of Walter's earliest papers deal with *rates of convergence* for invariance principles in theoretical statistics. An example (*Ann. Math. Stat*. 43, 1972) is the rate of convergence of the distribution of linear combinations of order statistics to a normal distribution, as the sample size tends to infinity. Convergence to normality was already known; Walter showed how fast it takes place by embedding the problem into an artfully chosen sequence of stochastic processes related to Brownian motion, which made explicit probability calculations possible. (This 3-line summary does not do justice to the difficulty of the problem.) Other papers from this period deal with fCLTs for solutions of stochastic differential equations as time and space are scaled appropriately, and with diffusion approximations to processes discrete in time and/or space, e.g., transport processes (*Indiana Univ. Math. J.* 26, 1977, with Richard Ellis).

Starting around 1980, Walter's research took a turn toward applied probability, resulting in a series of papers on applications of martingales to queueing systems and communications protocols. A *martingale* is a real-valued stochastic process that can be regarded as a model for a fair game; similarly super- (respectively, submartingales) are models for unfair (resp., advantageous) games. The terminology is reminiscent of that of harmonic (and super- and subharmonic) functions, and, indeed, there are deep connections between the two subjects. Martingales have good convergence properties and behavior at certain random times, and they lurk nearby to Markov processes and some queueing processes.

Walter was a master at pulling martingales related to the systems he was studying out of a hat (so it seems) to prove new and/or better results than previous ones, often with simpler and more powerful proofs. For example, using martingales he derives new results for the M|G|1 queue (*ZW* 58, 1981) and shows that a particular communications protocol "explodes" in the sense that the number of blocked messages tends to infinity over time *(IEEE Proc. Automatic Control*, 28, 1983, with Don Towsley), contrary to what had been assumed in the literature. Walter and I collaborated on one paper (*Methods Appls. of Anal.*, 9, 2002) in which we analyzed a relatively simple model for internet traffic that exhibits long range dependence and self-similarity, two properties that had been observed empirically but had not been explained theoretically; this paper used queueing theory and fCLTs, but no martingales.

Two late papers (*Amer. Statistician* 54, 2000; *Biometrical Journal* 55, 2013) of Walter's are contributions to the goodness-of-fit problem when parameters need to be estimated, and may seem like outliers from the main corpus of his work, but in fact borrow ideas from his results on rates of convergence in statistics, alluded to above.

Looking at the totality of Walter's research, I find it interesting that he is the sole author of about half of the papers in his CV, which seems quaint nowadays, and that relatively few references are cited in each paper, just some standard treatises, a few fundamental papers, and Walter's own previous work. The fundamental results on Markov processes and martingales (many due to Doob) and invariance principles that Walter was using were relatively new and still under development at the time, and he used them sometimes close to the limits of their applicability. Nevertheless, his papers are readable, meant "for human consumption," and, all in all, constitute an impressive body of work.

Walter's scholarly proclivities were reflected in his teaching, but were not always appreciated by his students. One semester he presented a derivation of Kepler's laws of planetary motion from Newton's law of gravitation, a nontrivial but accessible exercise – the kind of example I think students would (should) find very interesting. They hated it, he told me later. Another time Walter told of a teaching evaluation he had received: "Dear Professor Rosenkrantz: You talk too much. You explain too much. Just shut up and give us the formula!" He told me that with a great deal of mirth.

In 2004 Walter retired from UMass and moved to Washington, D.C., to be near his children. During the intervening years he taught at George Washington University, finished the second edition of his book, and wrote two papers, one on Kolmogorov-Smirnov mentioned above, the other (*Stat. & Prob. Letters* 83, 2013) with colleagues from GW. He died 19 September 2017, after a brief battle with cancer.

Walter was a fount of jokes and stories, many about famous mathematicians (Doob, for example), often laced with mimicry of the characters he was describing. Several times, while I was preparing this piece, questions arose in my mind about some mathematical or historical point or personage. The best person to answer them would have been Walter himself, of course, and I have no doubt that he would have answered the questions and included an amusing story about the principals involved. I would have liked that. We both would have.

–Joseph Horowitz
Professor Emeritus

ELEANOR KILLAM REMEMBRANCE

March 24, 2018
Eleanor Killam's Memorial Service
The Union Congregational Church, Holyoke, MA

Good Morning. My name is Sue Hrobar and I am the baby sister of Eleanor Killam. I am 16 years young than Eleanor and 11 years younger than our brother Everett, who passed away in 2016. My nephew Jason, from Montana and I from Wisconsin along with the rest of our family, who were unable to attend, thank you for coming to this service to honor my sister.

My stories and memories to share with you about Eleanor, begin while Eleanor was in college and then while starting her career teaching math at UMass. I hope you enjoy hearing these various snippets and will learn some new things about her life.

One of Eleanor's personal traits included using her words sparingly in her conversations unless she was talking about baseball, music, mathematics, or playing board and card games. Many people don't realize that underneath her sometimes austere outer appearance, she was a "good sport". This showed through when I was young as Eleanor would submit to her little sister styling her hair in creative ways and on rare occasions, allowed me to apply make-up on her face. Our mother thought that little girls should always wear bows in their hair. Eleanor wore her hair in the same style with the bows until she was 16. I, on the other hand, balked at wearing bows when I was in second grade. I would also remove my bows and hide them in the lilac bushes, and my mother would crawl around to find them.

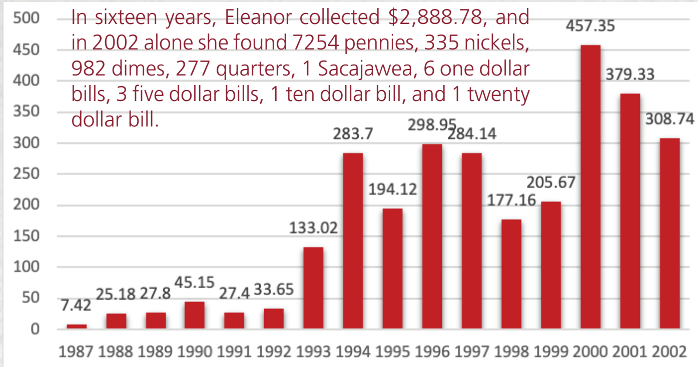
Baseball was one of Eleanor's passions, and even though she disliked driving a car, Eleanor would occasionally take me to Boston for a Red Sox game at Fenway Park. One year in the spring, Eleanor came to our parents' home in NH to get me, and with my parents permission, I was allowed to miss a Friday school day to attend a UMass championship baseball game. In her mind, baseball would be one of the few legitimate reasons for skipping classes. When Eleanor visited us in Wisconsin, when possible, we would take her to see Milwaukee Brewers games. Eleanor also enjoyed watching our daughters, Jessica and Julie, play on the summer softball teams.

During the summers, while I was still in junior and senior high school, Eleanor would invite me to spend a couple of weeks at her house. In 1964 we took a charter bus to see the New York World's Fair. During our day at the fair, we encountered a pouring rain storm, and we had no umbrellas or raincoats. We "made do" by going into exhibit buildings to get out of the rain and try to dry out. After paying for the bus ride and the fair entrance fees, I don't remember that we spent any other money that day. We had brought peanut butter sandwiches with us and ate those on the bus.

Eleanor was a very private person, a very frugal person, and seldom spoke about her accomplishments in the world of university mathematics. In 1955 Eleanor was the first woman to graduate as Valedictorian from the College of Technology at the University of New Hampshire. There are several newspaper articles touting her accomplishment. The following year she received her Master's Degree from UNH, and then was awarded a four-year full scholarship to Yale for her Ph.D. Eleanor was so careful in using her scholarship funds, that when she left Yale, she had enough money left over to pay cash for her first car and furniture for her apartment at UMass. When my parents and I came to visit for weekends, I often brought a friend with me. We would actually get up on a Saturday morning to attend the 8:00am math class with Eleanor. We would sit in the back row and try to make her laugh during her lecture. We did accomplish our goal at least once. I always wondered what Eleanor's students thought about their professor suddenly laughing in class.

After a couple of years living in an apartment, Eleanor bought a lot in Amherst and had a house built — a big priority for her was to have the house close enough to the university to walk to and not need a car to get there or to pay the parking fees. To pay for the new house, she went to a bank and wanted to take out a 10 year mortgage loan. The bank balked about having such a short term mortgage, and a compromise was reached with a 15 year mortgage. I don't think the bank was happy to lose several years worth of interest when Eleanor paid off the mortgage in 5 years.

Eleanor's "eagle eyesight" came in handy for finding money on sidewalks and parking lots, as she took her daily 5 mile walk around the campus area. When she was 81 she cut her walking distance to 2 1/2 miles. One year she found over $400.00. Winter did not deter Eleanor from finding money on her walks. She carried a screwdriver in her pocket to dig coins out of the snow and ice. After John and I were married and moved to Wisconsin, Eleanor would come for a yearly visit. Until John & I retired, Eleanor would be by herself during the work week. She kept busy during the day walking around our neighborhood and reading books. During the semesters, she did not feel that she had time to read, and she made up for it when she came to visit. Before she arrived, I would go to the library and pick out 10-12 books for her two week stay. She often finished those, and I would go back to the library to get more, or she would read books that were in our bookcases. In order not to repeat books, I kept a list of the books she had read.



Eleanor was willing to trust John and agree to try new adventures that she never would have done on her own. She loved going fishing with John on our pontoon boat during the Wisconsin "free fishing" weekend in June. She was most happy when the results culminated in a "fish fry" dinner. She did not understand the concept of going fishing, releasing the catch, and not bringing home fish for dinner. Along with our two daughters, Jessica and Julie, we took some canoe trips, and Eleanor helped to paddle one of the canoes. We made a day trip to Chicago by train, just for the adventure of it. We took her to Michigan's Upper Peninsula, where John spent a lot time for work, and went to museums, had a glass bottom boat ride, and toured a mine shaft. We also enticed her to roll up her pant legs and wade in Lake Superior. On another excursion along the Mississippi River, John coaxed Eleanor to pick up and hold a good-sized Mississippi River mud turtle that we saw in a park. We have the photo to prove it.

Besides Eleanor's book collection and reading hobbies, she sewed all her clothes, tatted, knitted, crocheted, quilted, made tote bags, embroidered, did needlepoint, and made many, many stuffed animals. One of her favorite things to make were small mice for faculty members and friends that had a new baby. She once told me that she saved small fabric scraps if they were big enough to make a mouse's ear. I still have about 50 handkerchiefs with tatted edges that she made. You can see some of the handkerchiefs on one of the photo boards. At one time, Eleanor would watch a Red Sox game on TV while tatting or knitting and also have an open book on her lap to keep up with her reading.

I debated about telling this story, but after seeing Steve Bennett today and hearing him speak, I decided that he would not be embarrassed to hear this. Steve was a student of Eleanor's and she began asking him to do some house repairs for her. Part of the payment for Steve was that Eleanor would cook a dinner for him. When the pie for dessert came to the table after his first dinner with her, Eleanor looked at him, and very seriously said, "Do you want a quarter or a half?" Poor Steve had no idea what she meant, he had never been asked that question. As he gaped at her, Eleanor, calmly said, "Well, you can start with a quarter and come back for more." She, then promptly put the other half of the pie onto her plate. To this day one of our family's running jokes, "Do you want a quarter or a half?"

I hope these thoughts and memories help to give you a more rounded picture of Eleanor.

In conclusion, I would like to thank all of you for being a part of Eleanor's life. Most of all, a huge thank you to Roy Perdue for his long time friendship with Eleanor, and for the care he provided my sister in having her come to live with him in her time of need.

Alumnus Roy Perdue arranged Eleanor Killam's memorial service and performed several piano pieces enjoyed by those in attendance (including alumni John Baillieul and Steve Bennett, professors George Avrunin and Rob Kusner, and department head Nate Whitaker). The annual Jacob-Cohen-Killam Mathematics Competition, which Eleanor helped create, is named in her honor.

The following alumni and friends have made generous contributions to the Department of Mathematics and Statistics between January 2017 and June 2018. Your gifts help us improve our programs and enrich the educational experiences of our students. We deeply appreciate your continuing support.

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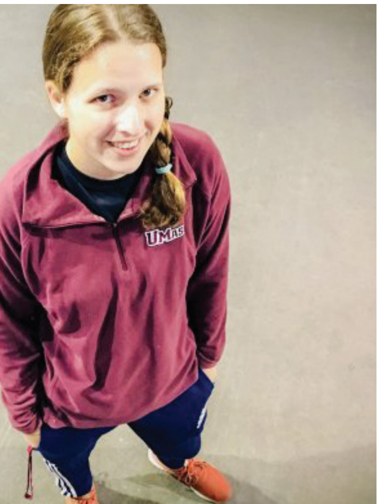

RISING RESEARCHER AWARD

Commonwealth Honors College student **Shelby Cox** '18 is a Mathematics & Statistics and Linguistics double major who has a track record of winning awards that reflect her superb academic performance and leadership abilities in the field of mathematics and statistics. Along with establishing and serving as President of the UMass chapter of the Association for Women in Mathematics, Cox has received two Outstanding Academic Achievement Awards from the Department of Mathematics and Statistics as well as the 2017 William F. Field Alumni Scholar Award, which recognizes and honors third-year students for their academic achievements. Shelby won a prestigious National Science Foundation graduate research fellowship to pursue a PhD in pure mathematics this fall at the University of Michigan.

Cox's research accomplishments began when she participated in a summer 2016 National Science Foundation Research Experience for Undergraduates (NSF/REU) at the University of Maryland. The project concerned calculating the Euler characteristic of geometric objects, known as Hilbert schemes, which are mathematical structures in algebraic geometry that occur under symmetry. The Euler characteristic is a rough measure of the topology, or shape, of an object. The heart of Cox's achievement was to reduce these calculations to previous known calculations that are more mathematically manageable. Cox and her collaborator gave a talk and also presented a poster on their work in January 2017 at the Joint Mathematics Meetings – the largest gathering of mathematicians in the United States and the largest annual meeting of mathematicians in the world.

According to Associate Professor Eric Sommers, her advisor and teacher, "Shelby's superb performance in research and departmental coursework, as well as her role in establishing and leading the UMass chapter of the Association for Women in Mathematics, makes her deserving of the Rising Researcher award."

Cox acknowledged just how much what it means to be a scientist has changed—but that the principles of science have remained the same:

> *As we move on to new adventures as educators, actuaries, data scientists, software engineers and researchers, we probably won't have to remember the pigeonhole principle or the first 10 digits of π, but we will continue to solve problems.*

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