DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
MASTER'S OPTION EXAM-APPLIED MATHEMATICS
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Do five of the following problems. All problems carry equal weight.
Passing level: 60% with at least two substantially correct.

1. Consider the ordinary differential equation

$$x' = (x-1)(x+1)$$

   (a) What are the constant solutions?

   (b) Sketch the solutions of the ODE with initial data $x(0) = -2, 0, 2$ and find $\lim_{t\to\pm\infty} x(t)$ for each solution. Justify your results. Do not solve the equation!

2. (a) Give a physical interpretation of the equation

$$u_t + xu_x = 0$$

   (b) Draw the characteristics and solve the above equation with initial data $u(x,0) = x2$.

3. Express the solution of the initial boundary value problem

$$u_{tt} = u_{xx}, \quad 0 < x < 2, \quad -\infty < t < \infty$$

$$u(0,t) = 0, \;\; u_x(2,t) = 0, \;\; u(x,0) = 0, \quad u_t(x,0) = x \quad 0 < x < 2,$$

   as a Fourier series, and provide the details of the calculations used in in obtaining thes and their coefficients in your answer.

4. Solve the boundary value problem, providing a full explanation and/or calculation for various expressions in your answer.

$$u_{xx} + u_{yy} = xy, \;\; 0 < x < \pi, \quad 0 < y < 1$$

$$u(x,0) = u(x,1) = 0, \;\; 0 < x < \pi, \;\; u(0,y) = u(\pi,y) = 0, \;\; 0 < y < 1,$$

5. Find all rest or fixed points of the following system of differential equations, and determine the behavior of all solutions of the system in a small neighborhood of each rest point.

$$\frac{dx}{dt} = 2x - xy$$

$$\frac{dy}{dt} = y - xy,$$

6. Solve the boundary problem $u'' = 0$ and $0 < x < 1$ with $u'(0) + ku(0) = 0$ and $u'(1) + ku(1) = 0$. Do the + and - cases separately. What is special about the case $k = 2$?

7. Prove that the(total) energy

$$E = \frac{1}{2} \int_0 1(u_t)2 + (u_x)2dx$$

is conserved for the wave equation

$$u_{tt} = u_{xx}, \quad 0 < x < 1.$$

where the boundary conditions are $u_x(0,t) = 1$ and $u_x(1,0) = 1$(Hint: Use integration by parts to simplify).