1. A topological space $X$ is normal if points are closed, and, for any pair of disjoint closed sets $A, B \subset X$, there are disjoint open sets $U, V \subset X$ with $A \subset U$ and $B \subset V$. Let $A$ be a closed subspace of a normal space $X$. Show that $A$ and the quotient $X/A$ are normal.

2. (a) Say what it means for a space $X$ to be locally path-connected.
(b) If $X$ is locally path-connected and connected, show that $X$ is path-connected.
(c) If $X$ is locally path-connected and compact, show that $X$ has finitely many path components.

3. (a) Give a connected 2-dimensional CW complex $X$ which has fundamental group isomorphic to the cyclic group of order 4. You should describe what the cells are, and what are the attaching maps.
(b) The space you have constructed has a connected 2-sheeted covering space $X'$. Describe it as a CW complex, so that the covering map is cellular.
(c) Calculate the integral homology groups of $X'$.

4. Let $X = S^2/\{p,q\}$ be the two-sphere with two points identified.
   (a) Compute the local homology groups $H_*(X, X \setminus \{x\})$
      (1) when $x = [p]$ is the image of $p$ or $q$ in $X$, and
      (2) when $x$ is any other point.
   (b) Explain why this implies that a homeomorphism of $X$ with itself must take $[p]$ to itself.

5. Let $X = S^6 \vee \mathbb{CP}^2$ and $Y = S^2 \vee (\mathbb{CP}^3/\mathbb{CP}^1)$.
   (a) Show that $X$ and $Y$ have isomorphic cohomology groups, with any coefficients.
   (b) Show that $X$ and $Y$ are not homotopy equivalent.
   (c) Show that there is a map $Y \to X$ which induces an isomorphism on $H_2$, but that any map $X \to Y$ induces the zero map on $H_2$.

   (a) Prove that its Euler characteristic $\chi(Y) = \sum_i (-1)^i \mathrm{rank}(H_i(Y))$ is zero.
   (b) Show that if $Y$ is non-orientable, then it has infinite fundamental group.
      (Hint: part (a) will be useful.)