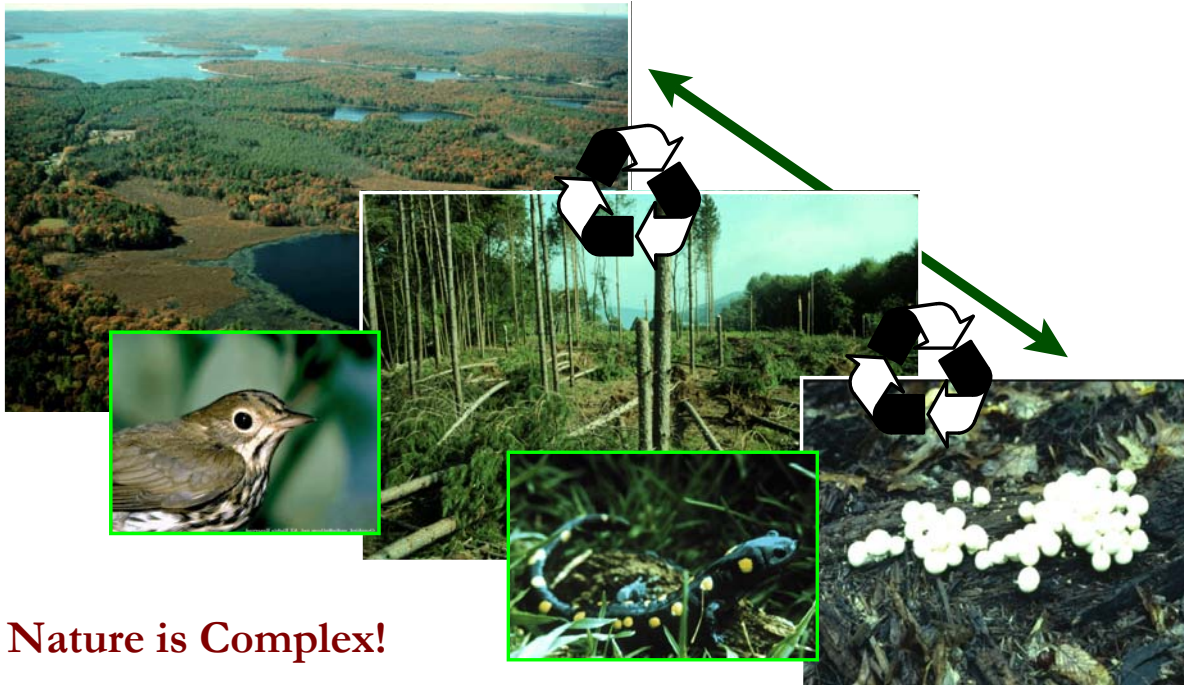


Multivariate Statistics: An Ecological Perspective



1

Advantages of Multivariate Statistics



- Reflect more accurately the true multidimensional, multivariate nature of natural systems.
- Provide a way to handle large data sets with large numbers of variables.
- Provide a way of summarizing redundancy in large data sets.
- Provide rules for combining variables in an "optimal" way.

2

Advantages of Multivariate Statistics



- Provide a solution to the multiple comparison problem by controlling experimentwise error rate.
- Provide a means of detecting and quantifying truly multivariate patterns that arise out of the correlational structure of the variable set.
- Provide a means of exploring complex data sets for patterns and relationships from which hypotheses can be generated and subsequently tested experimentally.

3

What is Multivariate Statistics?

Model

$$y = x_1 + x_2 + \dots + x_j \longrightarrow$$

Techniques

Regression
Analysis of Variance
Contingency Tables

$$y_1 + y_2 + \dots + y_i = x \longrightarrow$$

Multivariate ANOVA
Discriminant Analysis
CART, MRPP, MANTEL

$$y_1 + y_2 + \dots + y_i = x_1 + x_2 + \dots + x_j \longrightarrow$$

Canonical Corr. Analysis
Redundancy Analysis
Can. Correspond. Analysis

$$y_1 + y_2 + \dots + y_i \longrightarrow$$

Ordination
Cluster Analysis

Multivariate Statistics

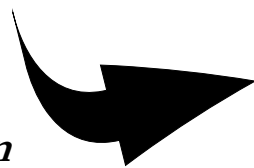
4

Example 1-Environmental Gradient

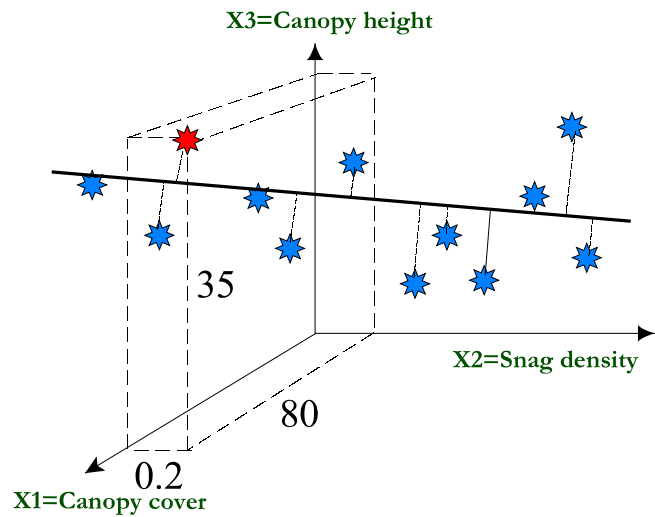
Data Matrix

Obs	Canopy Cover	Snag Density	Canopy Height
1	80	0.2	35
2	75	0.5	32
3	72	0.8	28
·	·	·	·
·	·	·	·
12	25	0.6	15

Ordination



3-Dimensional Data Space



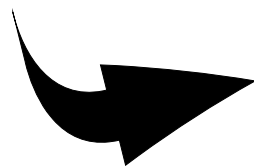
5

Example 1-Environmental Gradient

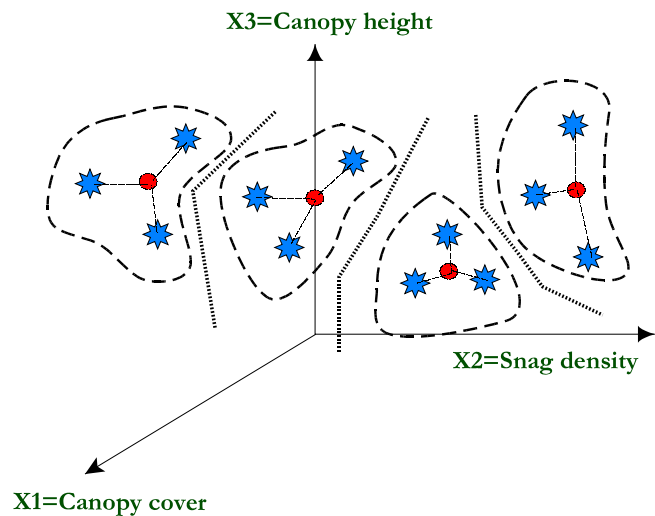
Data Matrix

Obs	Canopy Cover	Snag Density	Canopy Height
1	80	0.2	35
2	75	0.5	32
3	72	0.8	28
·	·	·	·
·	·	·	·
12	25	0.6	15

Cluster Analysis



3-Dimensional Data Space



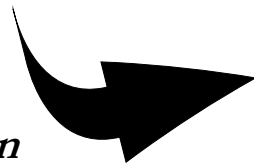
6

Example 2-Community Structure

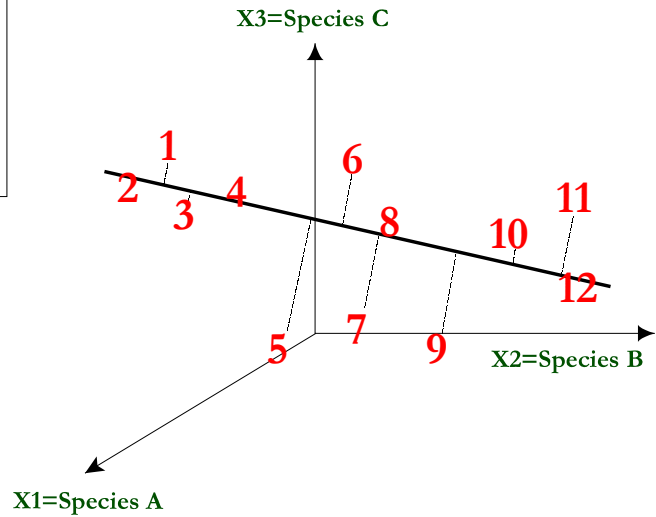
Data Matrix

Sample	Species A	Species B	Species C
1	80	1.2	35
2	75	0.5	32
3	72	0.8	28
⋮	⋮	⋮	⋮
12	25	0.6	15

Ordination



3-Dimensional Species Space



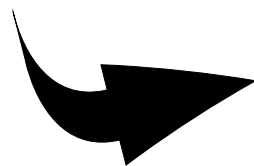
7

Example 2-Community Structure

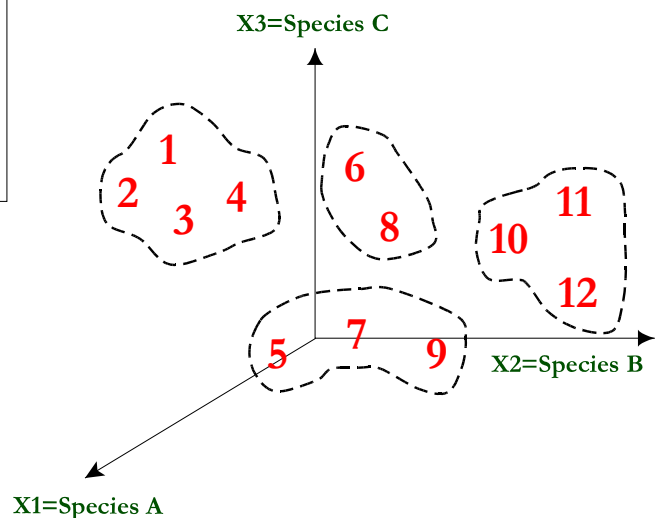
Data Matrix

Sample	Species A	Species B	Species C
1	80	1.2	35
2	75	0.5	32
3	72	0.8	28
⋮	⋮	⋮	⋮
12	25	0.6	15

Cluster Analysis



3-Dimensional Species Space



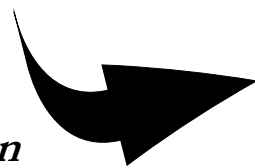
8

Example 2-Community Structure

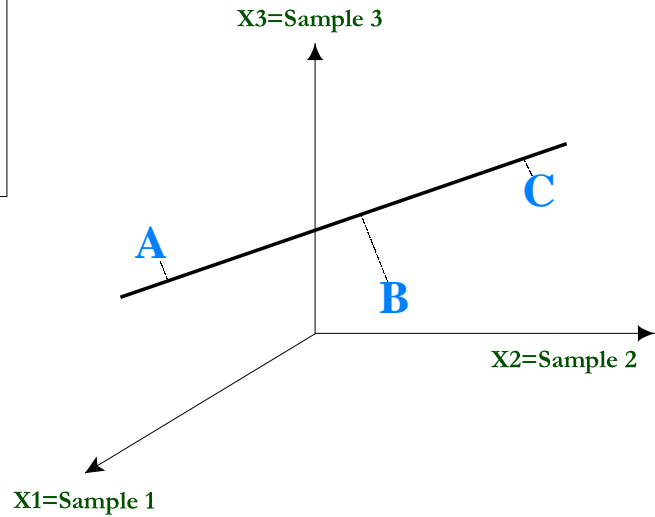
Data Matrix

Sample	Species A	Species B	Species C
1	80	1.2	35
2	75	0.5	32
3	72	0.8	28
·	·	·	·
·	·	·	·
12	25	0.6	15

Ordination



3-Dimensional Sample Space



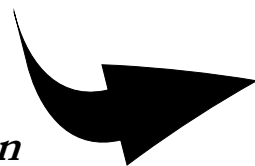
9

Example 2-Community Structure

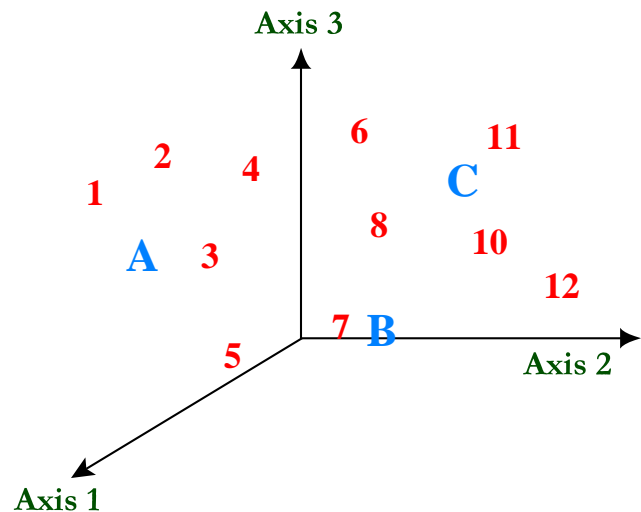
Data Matrix

Sample	Species A	Species B	Species C
1	80	1.2	35
2	75	0.5	32
3	72	0.8	28
·	·	·	·
·	·	·	·
12	25	0.6	15

Ordination



3-Dimensional Ordination Space

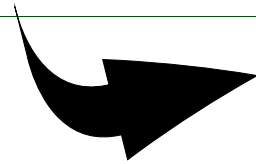


10

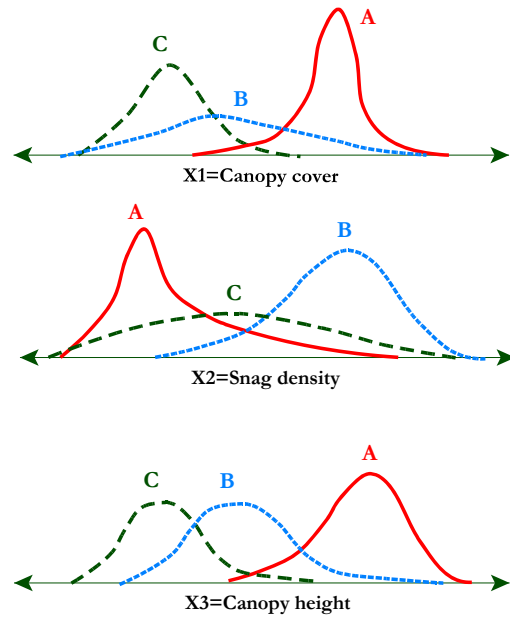
Example 3-Niche Separation

Data Matrix

Ind.	Species	Canopy Cover	Snag Density	Canopy Height
1	A	80	1.2	35
2	A	75	0.5	32
3	A	72	0.8	28
.
31	B	35	3.3	15
32	B	75	4.1	25
60	B	15	5.0	3
.
61	C	5	2.1	5
62	C	8	3.4	2
90	C	25	0.6	15



1-Dimensional Data Space

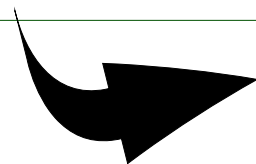


11

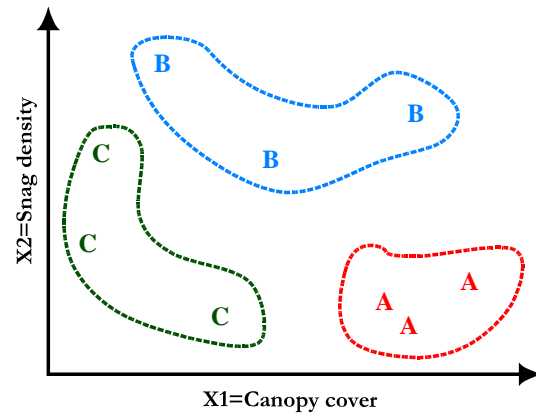
Example 3-Niche Separation

Data Matrix

Ind.	Species	Canopy Cover	Snag Density	Canopy Height
1	A	80	1.2	35
2	A	75	0.5	32
3	A	72	0.8	28
.
31	B	35	3.3	15
32	B	75	4.1	25
60	B	15	5.0	3
.
61	C	5	2.1	5
62	C	8	3.4	2
90	C	25	0.6	15



2-Dimensional Data Space

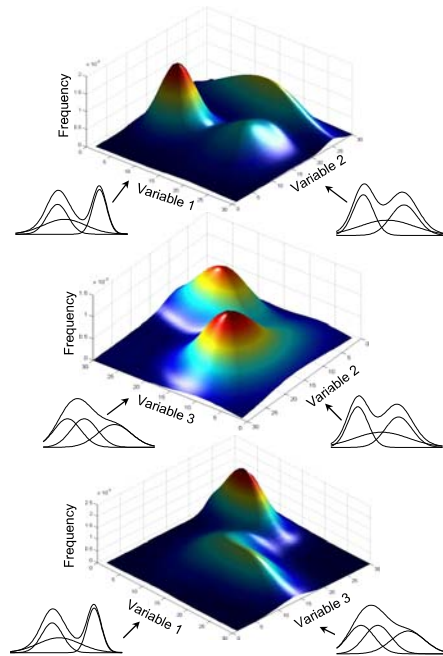
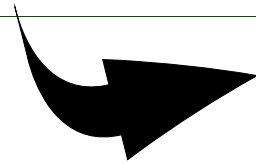


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Example 3-Niche Separation

Data Matrix

Ind.	Species	Canopy Cover	Snag Density	Canopy Height
1	A	80	1.2	35
2	A	75	0.5	32
3	A	72	0.8	28
.
31	B	35	3.3	15
32	B	75	4.1	25
60	B	15	5.0	3
.
61	C	5	2.1	5
62	C	8	3.4	2
90	C	25	0.6	15

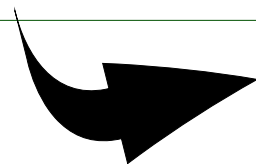


13

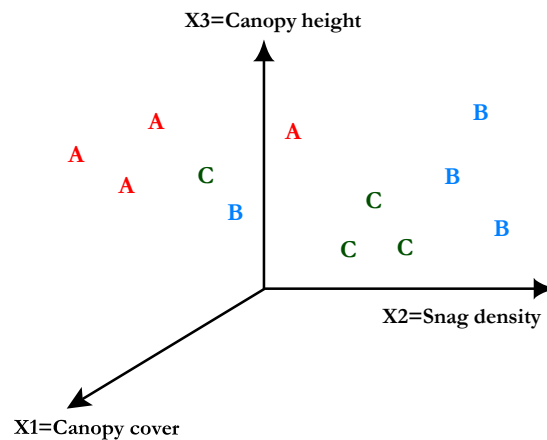
Example 3-Niche Separation

Data Matrix

Ind.	Species	Canopy Cover	Snag Density	Canopy Height
1	A	80	1.2	35
2	A	75	0.5	32
3	A	72	0.8	28
.
31	B	35	3.3	15
32	B	75	4.1	25
60	B	15	5.0	3
.
61	C	5	2.1	5
62	C	8	3.4	2
90	C	25	0.6	15



3-Dimensional Data Space

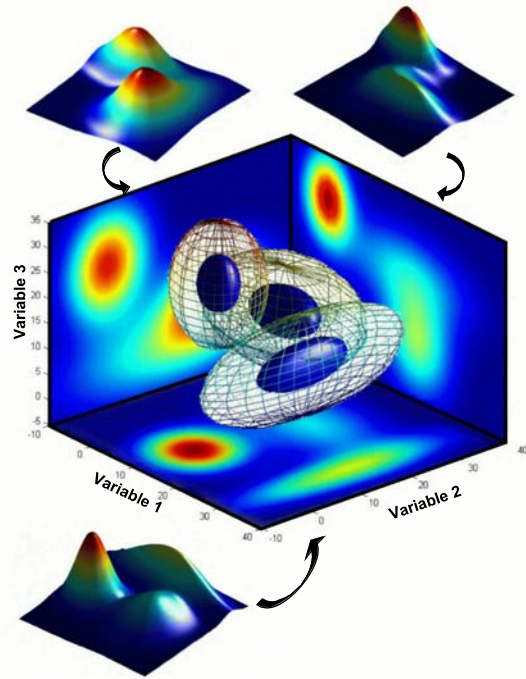


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Example 3-Niche Separation

Data Matrix

Ind.	Species	Canopy Cover	Snag Density	Canopy Height
1	A	80	1.2	35
2	A	75	0.5	32
3	A	72	0.8	28
.
31	B	35	3.3	15
32	B	75	4.1	25
60	B	15	5.0	3
.
61	C	5	2.1	5
62	C	8	3.4	2
90	C	25	0.6	15



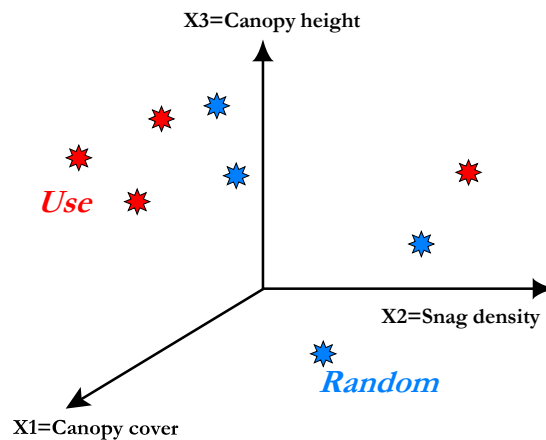
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Example 4-Habitat Use

Data Matrix

Obs	Group	Canopy Cover	Snag Density	Canopy Height
1	Use	80	1.2	35
2	Use	75	0.5	32
3	Use	72 <td 0.8	28	
4	Use	35	3.3	15
.
31	Random	5	2.1	5
32	Random	68	3.4	2
33	Random	25	0.6	15
34	Random	70	1.3	33
.

3-Dimensional Data Space



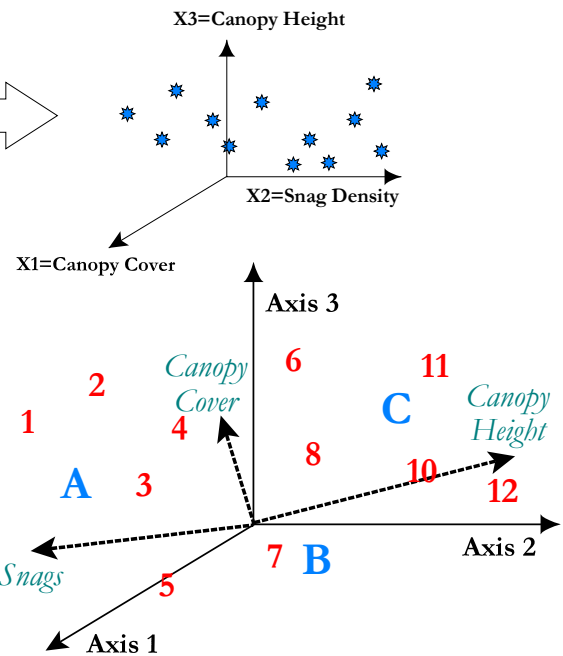
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Example 5-Constrained Ordination

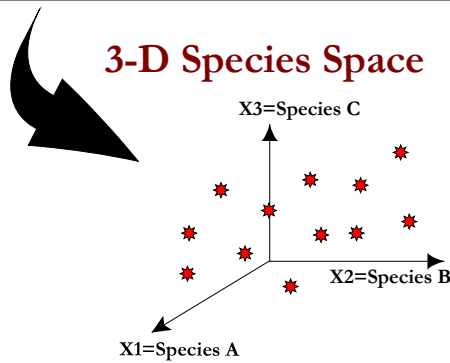
Data Matrix

Obs	Spec A	Spec B	Spec C	C-cov	Snags	C-hgt
1	10	1	3	80	1.2	35
2	8	0	1	75	0.5	32
3	12	0	5	72	0.8	28
4	0	7	3	35	3.3	15
5	1	9	1	75	4.1	25
6	0	12	5	15	5.0	3
7	2	1	6	5	2.1	5
8	0	3	9	8	3.4	2
9	4	4	8	25	0.6	15
.

3-D Environment Space



3-D Species Space



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Multivariate Statistics

Key Points

- Multivariate statistics involves cases involving multiple “dependent” variables, or a single set of variables presumed to be dependent on some underlying (latent) but unknown factors.
- All multivariate problems can be represented as a two-way data matrix in which rows represent sampling entities and columns represent variables; the internal structure of the matrix with respect to groups of sampling entities or dependence relationships among variables distinguishes among the various multivariate techniques.
- All multivariate problems can be conceptualized geometrically as a data cloud in a P -dimensional data space, where the dimensions (or axes) are defined by the variables of interest; it is the shape, clumping, and dispersion of this cloud that multivariate techniques seek to describe.

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Multivariate Description versus Inference

On the Descriptive Side:

- Provide rules for combining the variables in an optimal way. What is meant by 'optimal' may vary from one technique to the next.

On the Inferential Side:

- Provide explicit control over the experimentwise error rate. Many situations in which multivariate techniques are applied could be analyzed through a series of univariate significance tests

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Multivariate Confusion



- Which technique to use?
 - ▶ Ordination or cluster analysis?
 - ▶ Unconstrained or constrained ordination?
 - ▶ Polar ordination, principal components analysis, principal coordinates analysis, correspondence analysis, nonmetric multidimensional scaling?

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Multivariate Confusion



- **Alternative Terminology for Techniques**
 - ▶ Indirect Gradient Analysis or Unconstrained Ordination?
 - ▶ Reciprocal Averaging or Correspondence Analysis?
 - ▶ Canonical Ordination or Constrained Ordination?
 - ▶ Discriminant Analysis or Canonical Variates Analysis?

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Multivariate Confusion



- **Terminology for Variable Labels Based on Data Type and Measurement Scale**
 - **Categorical Variable**
 - ▶ Dichotomous
 - ▶ Polytomous
 - Ordinal Scale
 - Nominal Scale
 - **Continuous Variable**
 - Ratio Scale (true zero)
 - Interval Scale (arbitrary zero)
 - **Count Variable**

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Multivariate Confusion



■ Terminology for Variable Labels based on the Relationship with other Variables

● Independent Variable

- ▶ Variable presumed to be a cause of any change in a dependent variable; often regarded as fixed, either as in experimentation or because the context of the data suggests they play a causal role in the situation under study.

● Dependent Variable

- ▶ Variable presumed to be responding to a change in an independent variable; variables free to vary in response to controlled conditions.

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Multivariate Techniques

Technique

Unconstrained Ordination
(PCA, PO, CA, DCA, NMDS)

Cluster Analysis
(Family of techniques)

Discrimination
(MRPP, MANTEL, DA, CART, ...)

Constrained Ordination
(RDA, CCA, CAPS, CanCorr)

Objective

Extract gradients of maximum variation

Establish groups of similar entities

Test for or describe differences among groups of entities or predict group membership

Extract gradients of variation in dependent variables explainable by independent variables

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Multivariate Techniques

<u>Technique</u>	<u>Dependence Type</u>
Unconstrained Ordination (PCA, PO, CA, DCA, NMDS)	Interdependence
Cluster Analysis (Family of techniques)	Interdependence
Discrimination (MRPP, MANTEL, DA, CART, ...)	Dependence
Constrained Ordination (RDA, CCA, CAPS, CanCorr)	Dependence

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Multivariate Techniques

<u>Technique</u>	<u>Data Structure</u>
Unconstrained Ordination (PCA, PO, CA, DCA, NMDS)	One set; >>2 variables
Cluster Analysis (Family of techniques)	One set; >>2 variables
Discrimination (MRPP, MANTEL, DA, CART, ...)	Two sets; 1 grouping variable, >>2 discriminating variables
Constrained Ordination (RDA, CCA, CAPS, CanCorr)	Two sets; >>2 depend variables, >>2 independent variables

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Multivariate Techniques

Obs	Group	X-set				Y-set			
1	A	a_{11}	a_{12}	a_{13}	... a_{1p}	b_{11}	b_{12}	b_{13}	... b_{1m}
2	A	a_{21}	a_{22}	a_{23}	... a_{2p}	b_{21}	b_{22}	b_{23}	... b_{2m}
3	A	a_{31}	a_{32}	a_{33}	... a_{3p}	b_{31}	b_{32}	b_{33}	... b_{3m}
·	·	·	·	·	... ·	·	·	·	... ·
·	·	·	·	·	... ·	·	·	·	... ·
n	A	a_{n1}	a_{n2}	a_{n3}	... a_{np}	b_{n1}	b_{n2}	b_{n3}	... b_{nm}
n+1	C	c_{11}	c_{12}	c_{13}	... c_{1p}				
n+2	C	c_{21}	c_{22}	c_{23}	... c_{2p}				
n+3	C	c_{31}	c_{32}	c_{33}	... c_{3p}				
·	·	·	·	·	... ·				
·	·	·	·	·	... ·				
N	C	c_{n1}	c_{n2}	c_{n3}	... c_{np}				

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Multivariate Techniques

Technique

Sample Characteristics

Unconstrained Ordination
(PCA, PO, CA, DCA, NMDS)

N (from known or unknown #
pop's)

Cluster Analysis
(Family of techniques)

N (from known or unknown #
pop's)

Discrimination
(MRPP, MANTEL, DA, CART, ...)

N (from known # pop's) or
N1, N2, .. (from separate pop's)

Constrained Ordination
(RDA, CCA, CAPS, CanCorr)

N (from one pop)

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Multivariate Statistics

Key Points

- Multivariate statistics involves both descriptive and inferential statistics, although most applications are exploratory and descriptive in nature.
- Multivariate statistics includes a broad array of techniques and confusing and inconsistent use of terminology – *sorry, no way around this.*
- Research questions often warrant the use of more than one technique as the same technique can often be used to answer different questions and the same question can often be answered with different techniques.