Who Should Bear the Administrative Costs of an Emissions Tax?

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Abstract: All environmental policies involve costs of implementation and management that are distinct from pollution sources’ abatement costs. In practice, regulators and sources usually share these administrative costs. We examine theoretically an optimal policy consisting of an emissions tax and the distribution of administrative costs between the government and regulated sources of pollution. Our focus is on the optimal distribution of administrative costs between polluters and the government and the optimal level of the emissions tax in relation to marginal pollution damage. We demonstrate how the policy variables affect aggregate equilibrium administrative costs and show that these effects are generally indeterminate, as is the effect of the distribution of administrative costs on aggregate emissions. Consequently, the optimal sharing of administrative costs and whether the optimal emissions tax is higher or lower than marginal damage depend on specific contexts.

Keywords: Emission taxes, Pigouvian taxes, Administrative costs, Pollution control

JEL Codes L51, Q58

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1. Introduction

Any environmental policy involves costs beyond what would normally be categorized as abatement costs. These costs include the costs of monitoring polluters for compliance and imposing sanctions when a violation is found. In addition, resources are expended for monitoring policy performance, conducting research, and record keeping and reporting by both regulated firms and regulators. For environmental taxes, these costs include the costs of setting up and maintaining a system for collecting tax revenue. Perhaps for lack of a better term, these costs may be referred to as administrative costs. These are the costs of implementing and managing environmental policies that extend beyond abatement costs. In practice, these costs are typically borne by both regulated firms and regulators.

Policy analysts have long known that administrative costs can affect the setting of optimal emissions taxes. Polinsky and Shavell (1982) show how administrative costs can affect the level of a uniform Pigouvian tax. Brock and Evans (1985) examine how the existence of administrative costs can justify setting a lower tax on smaller firms than on larger firms. Cremer and Gahvari (2002) and Stranlund, Chavez, and Villena (2009) show that enforcement costs can produce discriminatory emissions taxes. Researchers also understand that administrative costs can affect the relative efficiency of different kinds of environmental taxes. For example, Smulders and Vollebergh (2001) examine how administrative costs, particularly monitoring costs, determine the use of emissions and input taxes with specific reference to the choice between taxing CO2 emissions or taxing energy products. Schmutzler and Goulder (1997) provide another analysis of mixed environmental taxes that is motivated by differences in monitoring costs.\(^1\)

While it is well known that administrative costs can be important determinants of the design and performance of environmental policies, in the literature on emissions taxes the distribution of administrative costs between the public (via government regulators) and polluting firms is taken

\(^1\)Others have investigated how administrative costs can affect the relative efficiency of taxes and other policy instruments. For example, Kampas and White (2004) provide an empirical analysis of the relative efficiency of several different policies for the control of an agricultural nonpoint pollutant. They find that the presence of administrative costs favors an input tax even though this instrument does not minimize aggregate abatement costs.
Policymakers cannot sidestep the problem of distributing administrative costs and have confronted it in a variety of ways. Many emissions control policies require that regulated firms bear the costs of monitoring and reporting their own emissions while the government takes on the burden of checking emissions reports and sanctioning violations. (The SO2 Allowance Trading Program is a good example). Some programs collect administrative fees from pollution sources to help finance enforcement and other activities. For example, Title V of the 1990 amendments to the U.S. Clean Air Act requires most large sources and some smaller sources of air pollution to obtain operating permits, and to pay an annual fee per ton of each regulated pollutant to cover the costs of administering the permit program.

In this paper we examine theoretically the joint determination of an efficient emissions tax and the distribution of administrative costs between the public and regulated firms. We identify three factors that determine whether the distribution of administrative costs affects social welfare. 1) Firm-level emissions may increase administrative costs directly. For example, firms with higher emissions may require more expensive monitoring equipment. 2) Having firms bear a larger part of administrative costs can reduce the size of the industry, holding the emissions tax and the industry’s output price constant. 3) Public funds devoted to administering the emissions tax may be more expensive than private funds due to the deadweight costs of other taxes used to generate government revenue. When these factors are not present the distribution of administrative costs does not affect the social costs and benefits of controlling emissions with an emissions tax, so regulators need not be concerned with specifying an optimal distribution.

Even if the distribution of administrative costs does not affect social welfare, a distribution must be chosen (by some criterion unrelated to economic efficiency) and this choice helps determine

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2See Tietenberg (2003) and Schrank, Arnason, and Hannesson (2003) for examples of fees on fishing quota owners for recovering public management costs in several fisheries.
the optimal emissions tax. Whether the tax should be higher or lower than marginal damage depends on the distribution of administrative costs and how the tax affects equilibrium aggregate administrative costs. These results are relevant when there is not an efficiency consequence of choosing a particular distribution of administrative costs, or when these consequences exist but are not accounted for when specifying the distribution in the regulation.

Given that the distribution of administrative costs does impact social welfare, and hence should be specified in a truly optimal tax regulation, the optimal distribution and tax become fairly complex decisions requiring determination on a case-by-case basis. The reason is that, in general, the distribution of administrative costs has ambiguous impacts on equilibrium aggregate emissions and administrative costs. Consequently, in any particular situation the optimal distribution of administrative costs can range from having polluting firms bear all administrative costs, to the firms and government sharing these costs, to having the government take on the entire burden. Moreover, the optimal emissions tax can be less than, greater than, or equal to marginal damage.

While it is difficult to reach specific conclusions about the optimal distribution of administrative costs and the tax in the general case, we can obtain insight into the problem by examining special cases. We analyze two of these cases. The first is when the price of the industry’s output is fixed, and hence, is relevant for the emissions control of a domestic (or regional) industry that faces a perfectly elastic demand curve because it is a relatively small part of a much larger international (or national) market. In this case, we demonstrate that it is optimal for the firms to bear all administrative costs and the emissions tax to be below marginal damage if aggregate administrative costs are nondecreasing in the tax.

In our second special case, administrative costs do not depend directly on firm-level emissions and the distribution of these costs does not affect the size of an industry. This case is relevant when administrative costs are so “small” that they have a negligible impact on firms’ decisions. If public funds are more expensive than private funds, then it is optimal for firms to bear all administrative costs and the optimal tax is less than marginal damage. Our second special case also covers common assumptions in the theoretical literature on enforcing emissions taxes. These models typ-
ically involve a fixed number of firms, enforcement costs that do not depend on firms’ emissions (there are no other administrative costs) and these costs are borne entirely by the government. Examples of such models are found in Montero (2002), Cremer and Gahvari (2002), Macho-Stadler and Perez-Castrillo (2006), Stranlund, Chávez, and Villena (2009), and Evans, Gilpatric, and Liu (2009).\(^3\) We demonstrate that under the first two assumptions, having the government bear all enforcement costs never welfare-dominates having the polluters bear all enforcement costs. Thus the standard assumption that the government takes on all enforcement costs often imposes an inefficient policy design on a model.

The rest of the paper proceeds as follows. In the next section we lay out the elements of our model and determine how the policy parameters, an emissions tax and distribution of administrative costs, affect equilibrium market and emissions outcomes. Section 3 contains the main results of our analysis, those concerning the optimal distribution of administrative costs. We conclude in section 4.

2. A model of an emissions tax with administrative costs

In this section we present a model of the distribution of administrative costs under an emissions tax. We begin by laying out the basic elements of our model and then move on to specifying firm behavior and market equilibrium, given an emissions tax and a distribution of the costs of administering the tax. We then determine the effects of these policy choices on equilibrium aggregate administrative costs and aggregate emissions. This analysis yields new insights about how the costs of administering an emissions tax may be affected by how these costs are distributed between polluters and the government and by the level of the tax. In addition, we demonstrate how the distribution of administrative costs can affect the environmental performance of an emissions

\(^3\)These assumptions are also common in studies of emissions trading (Malik 1992, Stranlund 2007, and Caffera and Chávez 2011), emissions standards (Arguedas 2008), as well as in the broader literature on the economic theory of law enforcement (Polinsky and Shavell 2000).
2.1 Basic elements of the model

Throughout we consider competitive firms that produce a homogeneous output \( q \) at price \( p \). Firms generate emissions \( x \) as a byproduct of production. Emissions are uniformly mixed and cause damage at constant rate \( \delta \).\(^5\) A firm’s cost function is \( c(q, x, \lambda) \), which is strictly increasing in output, decreasing in emissions, and strictly convex in \( q \) and \( x \). Moreover, we make the common assumption that \( c_{qx} < 0 \) so that firms’ marginal production costs are decreasing in their emissions. \( \lambda \) is a parameter that is used to order the firms: we will refer to \( \lambda \) as a firm’s type. The firms’ emissions are controlled with a per unit tax \( t \). We assume throughout the paper that enforcement is sufficient to induce full compliance, which means that each operating firm submits a truthful report of its tax liability to the authorities.\(^6\)

As noted in the introduction, the administrative costs associated with maintaining any environmental policy are likely to come from several sources (e.g., monitoring and enforcement, research, record keeping, etc). Rather than model each of these sources, we aggregate all such costs and make assumptions about how they may vary. First, we assume that aggregate administrative costs are nondecreasing in the level of an emissions tax, given a fixed number of regulated firms. This assumption is justified largely by the monitoring and enforcement literature, which suggests that firms have a stronger incentive to evade a higher emissions tax causing regulators to expend greater enforcement resources to maintain compliance. It is also possible that administrative costs

\(^4\)Our model borrows from and extends the work of Polinsky and Shavell (1982) who examine how the structure and distribution of administrative costs affects the determination of an optimal emissions tax. There are several important differences between our effort and theirs, but the most important is that we look for the optimal sharing of administrative costs between regulated firms and the government, while they assume that the distribution is fixed and either the firms bear all administrative costs or the government does. We also allow for a difference in the costs of public funds and private funds and we incorporate an output market into the analysis, both of which turn out to have important impacts on the optimal distribution of administrative costs and the level of the emissions tax. Polinsky and Shavell (1982) do not include these features.

\(^5\)We assume constant marginal damage to simplify the analysis. This assumption does not change our results.

\(^6\)Recent work by Stranlund, Chavez, and Villena (2009) suggests that there is a very limited set of circumstances under which it may be efficient to implement emissions taxes that are not fully enforced. Moreover, designing an enforcement strategy that induces full compliance is very simple, because it only requires that firms face an expected marginal penalty that exceeds the tax.
are affected directly by firms’ emissions. Perhaps higher firm-level emissions are associated with
greater expenditures on monitoring. For example, suppose that firms must monitor each point of
discharge and that higher emissions are associated with more discharge points. In this case the
costs of installing and operating the monitoring technology at the firm level may increase with
higher emissions. Hence, we assume that aggregate administrative costs are non-decreasing in
firm-level emissions. There may also be administrative costs that do not change with the string-
genency of the regulation or firm-level emissions, but may vary from firm to firm and, in aggregate,
increase with the number of regulated firms. Certain kinds of permitting and record keeping may
involve such costs.

As a device to simplify the analysis, we assume that all administrative costs are associated with
individual firms that operate under an emissions tax. This does not mean that the firms will bear
the costs associated with them; it does, however, allow us to describe aggregate administrative
costs as an aggregation of these costs over regulated firms. Let $m(t, x, \lambda)$ denote the administrative
cost associated with a $\lambda$-type firm that faces a tax $t$ and emits $x$. Given our assumptions above,
$m(t, x, \lambda) > 0$, $m_t(t, x, \lambda) \geq 0$, and $m_x(t, x, \lambda) \geq 0$ for $t \geq 0$. We also assume $m_{xt}(t, x, \lambda) \geq 0$, which
simply helps guarantee that a firm reduces its emissions as the tax increases.

We model the distribution of administrative costs by letting $\alpha$ be the share of these costs asso-
ciated with a particular firm that is borne by the government; $(1 - \alpha)$ is the share that is borne by
the firm. Assume that this share is constant across firms. The determination of $\alpha$ along with an
efficient tax is the main question addressed in this paper.

There is one final component of administrative costs. Typically, governments must raise rev-
ue with distortionary taxes, which implies that a dollar of government expenditure costs more
than one dollar. Consequently, let the marginal cost of public funds be the constant $\mu \geq 1$, so
the cost of one dollar of funds to finance administrative costs is $1 - \alpha + \alpha \mu$. Clearly this term is
increasing in the marginal cost of public funds.\footnote{The marginal cost of public funds is equal to one plus the marginal excess burden of taxation, where the latter is the efficiency loss of a higher tax per dollar of increased revenue. In their simulation study of the costs and distributional consequences of alternative policies to reduce U.S. CO$_2$ emissions, Parry and Williams (2010) set marginal excess burden equal to 25 cents. One may wonder why we model the difference between public and private costs when the...}
Given $\alpha$ and $t$, the administrative costs borne by an operating $\lambda$-type firm are $(1 - \alpha)m(t, x, \lambda)$ while the government takes on $\alpha \mu m(t, x, \lambda)$. In practice, implementation of this distribution can be done by distributing responsibilities for the necessary services between the firms and regulators and with fees and subsidies, although we do not model implementation explicitly. To illustrate, imagine a situation in which the only administrative costs are monitoring and enforcement costs. If it is optimal for the firms and public to share administrative costs, then sharing responsibilities may be a good approximation to the optimal sharing rule. For example, suppose as in actual settings that the firms are responsible for installing and operating an emissions monitoring technology and submitting emissions reports to the government, while the government bears the costs of monitoring the firm’s reports and data related to the operation of the monitoring device, as well as the services necessary for sanctioning noncompliant firms. However, if it is optimal for the firms to bear a larger part of monitoring and enforcement costs, then the policy may include a fee charged by the government to recover a portion of the costs of its activities. If it is optimal for the government to bear a larger part of monitoring and enforcement costs, the policy may include a subsidy to firms to help finance their purchase and installation of the monitoring equipment, for example. The distribution of administrative responsibilities and fees and subsidies can be structured to implement any sharing requirement.

For easy reference, Table 1 provides a list of the most important variables and functions used in this paper.

### 2.2. Firms’ choices

The timing of events in our model is standard: the regulator commits to an emissions tax and distribution of the costs of administering the tax and communicates these to the regulated firms before they make their choices. Given $t$, $\alpha$, and enforcement stringent enough to motivate each government could use emissions tax receipts to finance its share of administrative costs. We do so because using the emissions tax revenue in this way would still involve an opportunity cost if that revenue could be used to help finance other government activities that would instead have been financed with other distortionary taxes.
Table 1: Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q, Q )</td>
<td>A firm’s output and aggregate output; ( \hat{q} ) and ( \hat{Q} ) are the equilibrium values.</td>
</tr>
<tr>
<td>( p )</td>
<td>Output price; ( \hat{p} ) is the equilibrium output price.</td>
</tr>
<tr>
<td>( x, X )</td>
<td>A firm’s emissions and aggregate emissions; ( \hat{x} ) and ( \hat{X} ) are the equilibrium values.</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Constant marginal damage.</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Firm type, used to order firms.</td>
</tr>
<tr>
<td>( \lambda^m )</td>
<td>Type of the least profitable firm; ( \hat{\lambda}^m ) is the equilibrium value.</td>
</tr>
<tr>
<td>( \hat{q}^m, \hat{x}^m )</td>
<td>Output and emissions of the ( \hat{\lambda}^m )-type firm.</td>
</tr>
<tr>
<td>( c(q,x,\lambda) )</td>
<td>Cost function of a ( \lambda )-type firm.</td>
</tr>
<tr>
<td>( t )</td>
<td>Emissions tax.</td>
</tr>
<tr>
<td>( m(t,x,\lambda) )</td>
<td>Administrative costs associated with a ( \lambda )-type firm.</td>
</tr>
<tr>
<td>( \hat{M} )</td>
<td>Equilibrium aggregate administrative costs.</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Proportion of administrative costs borne by the government.</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Constant marginal cost of public funds.</td>
</tr>
<tr>
<td>( \pi )</td>
<td>A firm’s profit.</td>
</tr>
</tbody>
</table>

firm’s full compliance, an operating \( \lambda \)-type firm chooses output and emissions to maximize profit:

\[
\pi = pq - c(q,x,\lambda) - tx - (1 - \alpha)m(t,x,\lambda).
\]

(1)

We restrict our attention to policies that induce interior choices of output and emissions for all operating firms. The first order conditions for a firm’s output and emissions are:

\[
p - c_q(q,x,\lambda) = 0 \quad \text{and} \quad -c_x(q,x,\lambda) - t - (1 - \alpha)m_x(t,x,\lambda) = 0,
\]

(2)

respectively. As is standard, competitive firms choose their output to equate marginal production costs to the output price. The conventional result about emissions taxes is that firms choose their emissions to equate the tax with their marginal costs of abatement. This is an important result because it implies the minimization of aggregate abatement costs, which has always been one of
the main motivations for implementing price-based pollution policies. However, (2) indicates that firms will not choose their emissions to equate the tax and marginal abatement costs if they have to bear a part of the costs of administering the tax and these costs increase directly with their emissions. Under these circumstances, the usual cost-effectiveness result about emissions taxes does not hold.

The firm’s first-order conditions (2) implicitly define its optimal production and emissions as:

\[ q = q(t, \alpha, p, \lambda) \text{ and } x = x(t, \alpha, p, \lambda). \] (3)

We show in the Appendix that a firm’s output and emissions are strictly decreasing in the emissions tax and strictly increasing in the output price. A firm’s emissions and output are strictly increasing as the government takes on more of the burden of administrative costs as long as \( m_x > 0 \). If this effect is zero, then firms’ emissions and output choices are independent of the distribution of administrative costs. These and all such results are collected for ease of reference in Table 2.

Now let us turn to the entry and exit condition that determines the size of the industry. Using (1) and (3), define the maximum profit for a \( \lambda \)-type firm as:

\[
\pi(t, \alpha, p, \lambda) = pq(t, \alpha, p, \lambda) - c(q(t, \alpha, p, \lambda), x(t, \alpha, p, \lambda), \lambda) - tx(t, \alpha, p, \lambda) - (1 - \alpha)m(t, x(t, \alpha, p, \lambda), \lambda).
\] (4)

Assume that firms are continuously ordered so that operating firms with higher \( \lambda \)'s are more profitable; that is, \( \pi_\lambda = -c_\lambda - (1 - \alpha)m_\lambda > 0 \) for all \( t \) and for all \( \alpha \in [0, 1] \). Suppose that \( \lambda \) is then distributed over \([0, \bar{\lambda}]\), with \( \pi(t, \alpha, p, \lambda = 0) < 0 \) and \( \pi(t, \alpha, p, \bar{\lambda}) > 0 \). These imply, for given policy parameters \( t \) and \( \alpha \), that there is a unique cut-off value of \( \lambda > 0 \) below which firms earn negative profit and thus will not operate. Firms identified by \( \lambda \)'s greater than or equal to the cut-off \( \lambda \) continue to operate. The cut-off value of \( \lambda \) is determined as the implicit solution to \( \pi(t, \alpha, p, \lambda) = 0 \).

Let this value be:

\[
\lambda^m = \lambda^m(t, \alpha, p).
\] (5)
Table 2: Comparative statics for the determination of the effects of the policy variables on aggregate emissions and administrative costs.

<table>
<thead>
<tr>
<th>Tax</th>
<th>Administrative Cost Burden</th>
<th>Output Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_t &lt; 0$</td>
<td>$q_\alpha \geq 0$; $q_\alpha = 0$ iff $m_x = 0$</td>
<td>$q_p &gt; 0$</td>
</tr>
<tr>
<td>$x_t &lt; 0$</td>
<td>$x_\alpha \geq 0$; $x_\alpha = 0$ iff $m_x = 0$</td>
<td>$x_p &gt; 0$</td>
</tr>
<tr>
<td>$\lambda^m_t &gt; 0$</td>
<td>$\lambda^m_\alpha \leq 0$</td>
<td>$\lambda^m_p &lt; 0$</td>
</tr>
<tr>
<td>$\hat{p}_t &gt; 0$</td>
<td>$\hat{p}<em>\alpha \leq 0$; $\hat{p}</em>\alpha = 0$ iff $m_x = \lambda^m_\alpha = 0$</td>
<td></td>
</tr>
<tr>
<td>$\hat{x}_t &lt; 0$, by assumption</td>
<td>$\hat{x}_\alpha \leq 0$ in general; however,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\hat{x}<em>\alpha = 0$ if $m_x = \lambda^m</em>\alpha = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\hat{x}<em>\alpha &lt; 0$ if $m_x = 0$ and $\lambda^m</em>\alpha &lt; 0$</td>
<td></td>
</tr>
<tr>
<td>$\hat{\lambda}^m_t &gt; 0$, by assumption</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>$\hat{\lambda}^m_\alpha &lt; 0$ if $m_x = 0$ and $\lambda^m_\alpha &lt; 0$</td>
<td></td>
</tr>
</tbody>
</table>

Note that a higher value of $\lambda^m$ indicates that the industry is smaller. We show in the Appendix (and repeat in Table 2) that the regulation reduces the number of active firms as the tax increases ($\lambda^m_t > 0$) or if more of the administrative cost burden is placed on the firms ($\lambda^m_\alpha < 0$), holding the output price constant. However, the number of active firms increases with a higher output price ($\lambda^m_p < 0$). We recognize that some of these effects may be zero empirically; in particular, the firms’ administrative cost burden may have a negligible effect on the number of firms if these costs are a very small part of their total costs. Hence, we allow $\lambda^m_\alpha = 0$.

2.3 Market equilibrium

We now specify the output market equilibrium in terms of the policy parameters, ultimately reaching a specification about how equilibrium aggregate emissions and administrative costs depend on
$t$ and $\alpha$. Begin with the industry supply function:

$$Q^S(t, \alpha, p) = \int_{\lambda_m}^{\lambda} q(t, \alpha, p, \lambda) d\lambda. \quad (6)$$

The effects of the output price, the emissions tax, and the distribution of administrative costs on the industry supply function consist of the effect for a fixed number of firms and an industry-size effect. As expected the industry supply function is increasing in the price of output ($Q^S_p > 0$), because individual firms increase their output and more firms enter the industry. The supply function is decreasing in the emissions tax because each operating firm’s output and the size of the industry are both decreasing in the tax. The supply function is non-decreasing in the share of administrative costs borne by the government. Note from Table 2 that each firm’s output increases with $\alpha$ as long as $m_x > 0$. Moreover, an increase in $\alpha$ increases the size of the industry as long as $\lambda^m_{\alpha} < 0$. Both effects work to increase industry supply. However, the industry supply function is independent of the distribution of administrative costs if $m_x = \lambda^m_{\alpha} = 0$.

Let $Q^D(p)$ be the demand function for the industry’s output and assume that it is downward sloping. The equilibrium price is the solution to $Q^S(t, \alpha, p) = Q^D(p)$. Denote this price as $\hat{p}$ and note that it is a function of the policy parameters $t$ and $\alpha$. That is:

$$\hat{p} = \hat{p}(t, \alpha). \quad (7)$$

All equilibrium values from here on will be indicated with a “hat.” We show in the Appendix that the equilibrium output price is increasing in the emissions tax ($\hat{p}_t > 0$) and non-increasing in the share of administrative costs borne by the public ($\hat{p}_\alpha \leq 0$). If there is no direct effect of firm-level emissions on administrative costs and if the distribution of administrative costs does not affect the size of the industry, then $\hat{p}_\alpha = 0$ if $m_x = \lambda^m_{\alpha} = 0$, because industry supply is then independent of the distribution of administrative costs. However, $\hat{p}_\alpha < 0$ if either $m_x > 0$, $\lambda^m_{\alpha} < 0$, or both.

We will also consider the optimal distribution of administrative costs when the industry faces an exogenous output price. This case is important because it is relevant for the control of firms that
are a relatively small part of a larger market; for example, a domestic industry that is a small part of a larger international market. In this case, we simply set \( \hat{\rho}_t = \hat{\rho}_\alpha = 0 \).

With the equilibrium output price we can use (3) and (5) to define the equilibrium values for a firm’s output and emissions, as well as the equilibrium cut-off firm type:

\[
\hat{q} = q(t, \alpha, \hat{\rho}, \lambda), \quad \hat{x} = x(t, \alpha, \hat{\rho}, \lambda), \quad \text{and} \quad \hat{\lambda}_m = \lambda^m(t, \alpha, \hat{\rho}).
\] (8)

We will see that the effects of the policy parameters on firms’ equilibrium levels of output have no bearing on the optimal policy; the effects of the policy parameters on their choices of emissions and the size of the industry are what matter. The effects of the tax are indeterminate at this level of generality because they depend on direct effects and countervailing indirect price effects. For example, the effect of the tax on individual emissions is \( \hat{x}_t = x_t + x_p \hat{\rho}_t \). The direct effect of the emissions tax is to reduce an operating firm’s emissions \( (x_t < 0) \), but an increase in the tax can increase the output price, which has a positive indirect effect on a firm’s emissions \( x_p \hat{\rho}_t > 0 \). Despite this indeterminacy, we will assume that equilibrium firm-level emissions are strictly decreasing in the tax; that is:

\[
\hat{x}_t = x_t + x_p \hat{\rho}_t < 0.
\] (9)

The effect of the tax on industry size is similarly indeterminate. However, we assume:

\[
\hat{\lambda}_m = \lambda^m + \lambda^m \hat{\rho}_t \geq 0,
\] (10)

because it seems unlikely that increasing firms’ costs by taxing emissions can increase their profitability so that more firms enter the industry.

The effect of the public’s share of administrative costs on individual firms’ emissions is also ambiguous. Note that:

\[
\hat{x}_\alpha = x_\alpha + x_p \hat{\rho}_\alpha.
\] (11)

The direct effect on a firm’s emissions of having the public bear a higher portion of administrative
costs is to weakly increase emissions \( (x_\alpha \geq 0) \), but this may decrease the equilibrium output price leading to reduced emissions \( (x_p \hat{p}_\alpha \leq 0) \). We are not comfortable asserting a sign for (11). However, we do know that \( \hat{x}_\alpha = 0 \) when \( m_x = \lambda_\alpha^m = 0 \), because in this case \( x_\alpha = \hat{p}_\alpha = 0 \). We also know that \( \hat{x}_\alpha < 0 \) when \( m_x = 0 \) and \( \lambda_\alpha^m < 0 \), because in this case \( x_\alpha = 0 \) and \( x_p \hat{p}_\alpha < 0 \).

Not surprisingly the effect of \( \alpha \) on the equilibrium size of the industry is also ambiguous. Note first that:

\[
\hat{\lambda}_\alpha^m = \lambda_\alpha^m + \lambda_p^m \hat{p}_\alpha. \tag{12}
\]

Increasing the government’s share of administrative costs can increase the size of the industry \( (\lambda_\alpha \leq 0) \), but it can also decrease the output price leading to a reduction in industry size \( (\lambda_p \hat{p}_\alpha \geq 0) \). Despite being unable to sign \( \hat{\lambda}_\alpha^m \) in general, we know its sign in special cases. For example, \( \hat{\lambda}_\alpha^m = 0 \) when \( m_x = \lambda_\alpha^m = 0 \). However, if \( \lambda_\alpha^m = 0 \) but \( m_x > 0 \), then \( \hat{\lambda}_\alpha^m > 0 \). The opposite sign is obtained in the case of \( \lambda_\alpha^m < 0 \) and \( m_x = 0 \).

### 2.4 Aggregate emissions and administrative costs

We are now ready to specify equilibrium aggregate emissions and administrative costs. (Again we will see that the impacts of the policy variables on equilibrium output do not affect the optimal policy). They are:

\[
\hat{X} = \int_{\lambda_m}^{\lambda} \hat{x} d\lambda; \tag{13}
\]

\[
\hat{M} = (1 - \alpha + \alpha \mu) \int_{\lambda_m}^{\lambda} m(t, \hat{x}, \lambda) d\lambda. \tag{14}
\]

The effects of the policy variables on aggregate emissions are:

\[
\hat{X}_t = \int_{\lambda_m}^{\lambda} \hat{x}_t d\lambda - \hat{\lambda}_t^m \hat{x}_m; \tag{15}
\]

\[
\hat{X}_\alpha = \int_{\lambda_m}^{\lambda} \hat{x}_\alpha d\lambda - \hat{\lambda}_\alpha^m \hat{x}_m. \tag{16}
\]
where \( \hat{x}^m = x(t, \alpha, \hat{\rho}, \hat{\lambda}^m) \) is the equilibrium emissions of the cut-off firm in the industry. While it is not possible at this level of generality to give \( \hat{X}_t \) a definite sign we insist that it is negative since it makes no sense to impose a tax that increases emissions.

\( \hat{X}_\alpha \) is also indeterminate in general, but we do not have a reason for asserting that it has a particular sign. Two effects determine the overall impact of the distribution of administrative costs on aggregate emissions. (1) A change in the distribution of administrative costs can change the aggregate emissions of a fixed number of firms. The direction of this effect depends on the sign of \( \hat{x}_\alpha \), which recall is generally indeterminate. (2) A change in the distribution of administrative costs can affect the size of the industry, which in turn affects aggregate emissions. This effect depends on the sign of \( \hat{\lambda}^m_\alpha \), which recall is also generally indeterminate. Although we cannot sign \( \hat{X}_\alpha \) in general, we can in specific cases. The cases that we use later to gain insight into the optimal regulation are specified in the following lemma.

**Lemma 1:** The effect of the distribution of administrative costs on equilibrium aggregate emissions is indeterminate in general. However:

(a) \( \hat{X}_\alpha = 0 \) for all \( \alpha \) if \( m_x = \lambda^m_\alpha = 0 \).

(b) \( \hat{X}_\alpha \geq 0 \) for all \( \alpha \) if \( \hat{\rho} \) is fixed. The inequality is strict if \( m_x > 0 \) or \( \lambda^m_\alpha < 0 \).

**Proof:** For part (a), note from Table 2 that \( m_x = \lambda^m_\alpha = 0 \) implies \( \hat{x}_\alpha = 0 \) and \( \hat{\lambda}^m_\alpha = 0 \). Using (16), \( \hat{x}_\alpha = \hat{\lambda}^m_\alpha = 0 \) implies \( \hat{X}_\alpha = 0 \) for all \( \alpha \). For part (b), if \( \hat{\rho} \) is fixed, then using (11) and (12), (16) becomes:

\[
\hat{X}_\alpha = \int_{\hat{\lambda}}^{\lambda} x_\alpha d\lambda - \lambda^m_\alpha \hat{x}^m \geq 0,
\]

the sign of which follows from \( x_\alpha \geq 0 \) and \( \lambda^m_\alpha \leq 0 \) (consult Table 2). The inequality is strict if \( x_\alpha > 0 \), which requires \( m_x > 0 \) or \( \lambda^m_\alpha < 0 \). □

Part (a) of Lemma 1 reveals that aggregate emissions are unaffected by the distribution of administrative costs if these costs do not depend directly on firm-level emissions and if the size
of the industry is unaffected by the distribution of administrative costs. If either of these effects are present, then the distribution of administrative costs does impact aggregate emissions, but the direction of the impact is indeterminate in general. Part (b) of the lemma reveals that this indeterminacy stems from the variable output price. When the industry’s output price is fixed, equilibrium aggregate emissions are non-decreasing as the government takes on a larger part of the administrative cost burden.

The effects of the policy variables on aggregate administrative costs are also critically important for determining the optimal policy. The effect of the emissions tax on equilibrium aggregate administrative costs is:

\[
\hat{M}_t = (1 - \alpha + \alpha \mu) \left\{ \int_{\hat{\lambda}_m}^{\lambda} m_t d\hat{\lambda} + \int_{\hat{\lambda}_m}^{\lambda} \hat{\lambda}_m m_t d\hat{\lambda} - \hat{\lambda}_m m(t, \hat{x}_m, \hat{\lambda}_m) \right\}. \tag{17}
\]

Although we cannot sign \(\hat{M}_t\) in general, we can do so in specific cases. These are included in the following lemma.

**Lemma 2:** The effect of the tax on equilibrium aggregate administrative costs is indeterminate in general. However:

(a) \(\hat{M}_t = 0\) for all \(\alpha \in [0, 1]\) if \(m_t = m_x = \hat{\lambda}_m = 0\).

(b) \(\hat{M}_t < 0\) if \(m_t = 0\) and either \(m_x > 0\) or \(\hat{\lambda}_m > 0\).

(c) \(\hat{M}_t > 0\) only if \(m_t > 0\).

**Proof:** These results follow directly from (17). □

Three effects determine the overall effect of the emissions tax on aggregate administrative costs. (1) Increasing the tax can increase administrative costs directly if these costs increase directly with the stringency of the regulation \((m_t > 0)\). (2) Increasing the tax can decrease administrative costs indirectly through a decrease in emissions if administrative costs increase in firm-level
emissions ($\hat{x}_t m_x < 0$). (3) Increasing the tax can also decrease administrative cost by reducing the number of firms in the industry ($\hat{\lambda}_m^\alpha \geq 0$). When each of these effects is absent, equilibrium administrative costs are independent of the level of the emissions tax. However, if only the direct tax effect is absent ($m_t = 0$), then equilibrium administrative costs decline as the tax is increased. Consequently, equilibrium administrative costs increase as the tax is increased only if the tax has a direct effect on these costs.

Now let us turn to the effect of the distribution of administrative costs on equilibrium administrative costs:

\[
\hat{M}_\alpha = (1 - \alpha + \alpha \mu) \left\{ \int_{\hat{\lambda}_m}^{\hat{\lambda}_m^\alpha} \hat{x}_\alpha m_x d\lambda - \hat{\lambda}_m^m m(t, \hat{x}_m, \hat{\lambda}_m^m) \right\} + (\mu - 1) \int_{\hat{\lambda}_m}^{\hat{\lambda}_m^m} m(t, \hat{x}_m, \hat{\lambda}_m^m) d\lambda.
\] (18)

Three effects determine the overall effect of the distribution of administrative costs on aggregate administrative costs. (1) A change in the distribution of administrative costs can change aggregate emissions, which in turn changes aggregate administrative costs if these costs increase with firm-level emissions ($\hat{x}_\alpha m_x$). The direction of this effect depends on the sign of $\hat{x}_\alpha$, which recall is generally indeterminate. (2) A change in the distribution of administrative costs can affect the size of the industry, which in turn affects aggregate administrative costs. This effect depends on the sign of $\hat{\lambda}_m^m$, which recall is also generally indeterminate. (3) If public funds are more expensive than private funds, then having the government take on a greater burden of administrative costs will increase these costs. Although we cannot sign $\hat{M}_\alpha$ in the general case, we can do so in special cases that are informative. These are included in Lemma 3.

**Lemma 3:** The effect of the distribution of administrative costs on equilibrium administrative costs is indeterminate in general. However:

(a) $\hat{M}_\alpha = 0$ for all $\alpha \in [0, 1]$ if and only if $m_x = \hat{\lambda}_m^m = \mu - 1 = 0$.
(b) $\hat{M}_\alpha > 0$ if $m_x = 0$ and either $\hat{\lambda}_m^m < 0$, $\mu - 1 > 0$, or both.
(c) \( \hat{M}_\alpha < 0 \) only if \( m_x > 0 \).

(d) \( \hat{M}_\alpha \geq 0 \) if \( \hat{p} \) is fixed, and the inequality is strict if \( m_x > 0, \lambda^m_\alpha < 0 \), or \( \mu - 1 > 0 \).

**Proof:** For part (a) of the lemma, note from Table 2 that \( m_x = \lambda^m_\alpha = 0 \) implies \( \hat{\lambda}^m_\alpha = 0 \). Plug \( m_x = \hat{\lambda}^m_\alpha = \mu - 1 = 0 \) into (18) to show that \( \hat{M}_\alpha = 0 \) independently of \( \alpha \). However, if \( m_x > 0, \lambda^m_\alpha < 0 \), or \( \mu - 1 > 0 \), then \( \hat{M}_\alpha \) is not independent of the level of \( \alpha \). For part (b) of the lemma, note from Table 2 that \( m_x = 0 \) and \( \lambda^m_\alpha \leq 0 \) imply \( \hat{\lambda}^m_\alpha \leq 0 \), and the inequality is strict if \( \lambda^m_\alpha < 0 \). Set \( m_x = 0 \) in (18) to show that \( \hat{M}_\alpha > 0 \) if either \( \hat{\lambda}^m_\alpha < 0 \) (which requires \( \lambda^m_\alpha < 0 \), \( \mu - 1 > 0 \), or both. Part (c) of the lemma follows as a consequence of part (b). For part (d) of the lemma: i) From (11) and Table 2, if \( \hat{p} \) is fixed, then \( \hat{x}_\alpha = x_\alpha \geq 0 \) and the inequality is strict if \( m_x > 0 \). Therefore, fixed \( \hat{p} \) implies \( \hat{x}_\alpha m_x \geq 0 \) and the inequality is strict if \( m_x > 0 \). ii) Moreover, from (12) and Table 2, if \( \hat{p} \) is fixed, then \( \hat{\lambda}^m_\alpha = \lambda^m_\alpha \geq 0 \). Part (d) of the lemma follows from (18), i), ii) and \( \mu - 1 \geq 0 \). □

Part (a) of Lemma 3 reveals the circumstances under which the level of aggregate administrative costs is affected by the distribution of these costs. If administrative costs are not affected directly by firm-level emissions, the distribution of administrative costs does not affect the size of the industry, and public funds are not more expensive than private funds, then aggregate administrative costs are independent of their distribution. Part (b) of the lemma reveals that the latter two effects (\( \lambda^m_\alpha < 0 \) or \( \mu > 1 \)) tend to make administrative costs increase in the government’s share of these costs, while part (c) reveals that the first effect (\( m_x > 0 \)) tends to make administrative costs decrease in the government’s share. Like the impact of the cost distribution on aggregate emissions, it is the variable output price that makes the sign of \( \hat{M}_\alpha \) indeterminate in the general case. Part (d) of the lemma reveals that when this price is fixed, equilibrium administrative costs are nondecreasing in the government’s share of these costs.
3. Optimal emissions tax and distribution of administrative costs

Having specified firm and market reactions to an emissions tax and distribution of administrative costs, as well as the effects of the policy variables on aggregate emissions and administrative costs, we are now ready to characterize the optimal policy. This policy will maximize consumer plus producer surplus, less emissions damage and aggregate administrative costs, subject to the equilibrium output and emissions induced by the policy. The social welfare function is:

\[ W(t, \alpha) = \int_0^{\hat{Q}} p(Q)dQ - \int_{\hat{\lambda}_m}^{\hat{\lambda}} c(\hat{q}, \hat{x}, \lambda) d\lambda - \delta \int_{\hat{\lambda}_m}^{\hat{\lambda}} \hat{x} d\lambda - (1 - \alpha + \alpha \mu) \int_{\hat{\lambda}_m}^{\hat{\lambda}} m(t, \hat{x}, \lambda) d\lambda, \]  

where \( p(Q) \) is the inverse demand function for the industry’s output. The optimal policy is \( (t, \alpha) \) that maximizes \( W(t, \alpha) \) subject to \( \alpha \in [0, 1] \) and \( t > 0 \). Assume that the first-order conditions for this problem are necessary and sufficient to identify an optimal policy. In the Appendix we show that these first-order conditions can be written as:

\[ W_t(t, \alpha) = (t - \delta) \hat{X}_t - \alpha \mu \left\{ \int_{\hat{\lambda}_m}^{\hat{\lambda}} (m_t + m_t \hat{x}_t) d\lambda - \hat{\lambda}_m m(t, \hat{x}_m, \hat{\lambda}_m) \right\} - (1 - \alpha) \int_{\hat{\lambda}_m}^{\hat{\lambda}} m_t d\lambda = 0; \]  

\[ W_\alpha(t, \alpha) = (t - \delta) \hat{X}_\alpha - \alpha \mu \left\{ \int_{\hat{\lambda}_m}^{\hat{\lambda}} \hat{x}_\alpha m_t d\lambda - \hat{\lambda}_m m(t, \hat{x}_m, \hat{\lambda}_m) \right\} - (\mu - 1) \int_{\hat{\lambda}_m}^{\hat{\lambda}} m(t, \hat{x}, \lambda) d\lambda \begin{cases} 
\leq 0, & \text{if } \mu < 0, \text{ then } \alpha = 0 \\
\geq 0, & \text{if } \mu > 0, \text{ then } \alpha = 1.
\end{cases} \]  

The first term of each first order condition captures how the policy variable changes the difference between pollution damage and the firms’ aggregate abatement costs and their share of administrative costs. The signs of these effects depend on the relationship between the tax and
marginal damage and how the policy variable affects equilibrium aggregate emissions. Recall that aggregate emissions strictly decrease with the tax, but the effect of the distribution of administrative costs on emissions is ambiguous. The second and third terms of each first order condition are not equal to the effects of the policy variables on aggregate administrative costs, but their signs are determined by the same factors; hence, the absence of administrative costs results in the standard prescription to set the emissions tax equal to marginal damage.⁸

We begin the analysis of optimal tax policies by specifying the factors that determine whether the distribution of administrative costs has efficiency consequences.

**Proposition 1:** Social welfare is unaffected by the distribution of administrative costs if and only if
\[ m_x = \lambda_\alpha^m = \mu - 1 = 0. \]

**Proof:** Note first from Lemma 1 that \( m_x = \lambda_\alpha^m = 0 \) imply \( \hat{X}_\alpha = 0 \) independently of the level of \( \alpha \). Moreover, \( m_x = \lambda_\alpha^m = \mu - 1 = 0 \) imply that the remaining terms of (21) are equal to zero independently of \( \alpha \). Consequently, \( W_\alpha(t, \alpha) = 0 \) regardless of the value of \( \alpha \), which indicates that social welfare is unaffected by the distribution of administrative costs if \( m_x = \lambda_\alpha^m = \mu - 1 = 0. \) However, if \( m_x > 0, \lambda_\alpha^m < 0, \) or \( \mu > 1, \) then \( W_\alpha(t, \alpha) \) does depend on the level of \( \alpha \). Hence, social welfare is independent of the distribution of administrative costs only if \( m_x = \lambda_\alpha^m = \mu - 1 = 0. \)

Proposition 1 reveals that there are three fundamental factors that determine whether the distribution of administrative costs affects the efficiency of an emissions tax: (1) Holding the tax fixed, administrative costs increase directly with firm-level emissions; (2) holding the tax and the industry’s output fixed, industry size increases as the government takes on more of the burden of administrative costs, or (3) the cost of public funds for administration is greater than the cost of private funds. Each of these effects can make social welfare dependent on the choice of the distribution of administrative costs, but regulators do not have to worry about setting the correct

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⁸Note that the effects of the policy variables on industry output do not play a role in the determination of the optimal policy. This is due to the assumption of perfect competition in the output market, because firms choose efficient levels of output given the policy choice. In the conclusion we suggest that considering the optimal tax and distribution of administrative costs under imperfect competition is a worthwhile extension of this work.
distribution of administrative costs if they are all absent. The reason is that both equilibrium aggregate emissions and administrative costs are independent of the distribution of administrative costs when these factors are absent (see Lemmas 1 and 3, respectively).

Although it is possible that social welfare is unaffected by the distribution of administrative costs, that does not imply that we can ignore the distribution of these costs in setting the optimal tax when this occurs. A distribution of administrative costs must be chosen and this choice helps determine the level of the optimal tax. The following proposition shows the relationship between the optimal emissions tax and marginal damage.

**Proposition 2:** The qualitative relationship between the optimal emissions tax and marginal damage is given by:

\[
\text{sign}(t - \delta) = -\text{sign}\left(\frac{\alpha \mu \hat{M}_t}{1 - \alpha + \alpha \mu} + (1 - \alpha) \int_{\lambda_m}^{\hat{\lambda}} m_t d\lambda\right).
\]  

(22)

**Proof:** Using (17) we can rewrite (20) as:

\[
W_t(t, \alpha) = (t - \delta)\hat{X}_t - \frac{\alpha \mu \hat{M}_t}{1 - \alpha + \alpha \mu} - (1 - \alpha) \int_{\lambda_m}^{\hat{\lambda}} m_t d\lambda = 0.
\]

(22) follows because \(\hat{X}_t < 0\). □

The following corollary follows directly from (22). It reveals how the optimal emissions tax is affected by the distribution of administrative costs and on how the tax affects aggregate administrative costs.

**Corollary 1:** The emissions tax is weakly less than marginal damage if the industry bears all administrative costs, or if aggregate equilibrium administrative costs are nondecreasing in the tax. The emissions tax is strictly less than marginal damage if aggregate equilibrium administrative costs are strictly increasing in the tax. The optimal tax exceeds marginal damage only if the government bears a part of administrative costs and these costs are strictly decreasing in the emissions
tax, although these conditions are not sufficient.

**Proof:** Using (22), note that $(1 - \alpha) \int_{\lambda_0}^{\lambda_t} m_t d\lambda \geq 0$. Then, (22) reveals that $t \leq \delta$ if $\alpha = 0$. If $\alpha > 0$ and $\dot{M}_t \geq 0$, then the right side of (22) is nonpositive implying $t \leq \delta$. Now assume that $\dot{M}_t > 0$ and recall from Lemma 2 that this can only occur if $m_t > 0$. Together, $\dot{M}_t > 0$ and $m_t > 0$ imply the right side of (22) is strictly negative regardless of the level of $\alpha$. Hence, $t < \delta$ if $\dot{M}_t > 0$. Finally, note that $t > \delta$ only if $\alpha \mu \dot{M}_t / (1 - \alpha + \alpha \mu) < 0$, which requires both $\alpha > 0$ and $\dot{M}_t < 0$.

□

It is intuitive that when aggregate administrative costs are strictly increasing with a higher emissions tax, it is optimal to pursue less stringent emissions control via a lower emissions tax to conserve administrative costs. However, we should note that while $\dot{M}_t > 0$ is sufficient to produce a lower tax, it is not necessary. Suppose for example that $\dot{M}_t < 0$, but that firms bear all administrative costs and $m_t > 0$. In this case, the tax is strictly lower than marginal damage despite the fact that aggregate administrative costs decline with the tax. In fact, regardless of the sign of $\dot{M}_t$, the tax cannot be above marginal damage if the firms bear all administrative costs. The only way that the optimal tax is above marginal damage is if the public bears a part of administrative costs and aggregate equilibrium administrative costs are strictly decreasing in the emissions tax. While these conditions are necessary to produce a higher tax, they are not sufficient because it is easy to come up with combinations of $\alpha > 0$, $\dot{M}_t < 0$, and $m_t > 0$ that make (22) negative.

In Proposition 1 we determined when the distribution of administrative costs affects social welfare, and hence, needs to be specified in a truly optimal tax policy. However, we have not yet said anything about how to determine the optimal distribution when it matters beyond stating its first order condition. In the general case it is difficult to reach specific conclusions about the optimal distribution of administrative costs. The main reason is that the distribution of these costs has ambiguous effects on aggregate emissions and aggregate administrative costs. As a consequence, in any particular situation in which specifying the distribution of administrative costs affects social welfare, the optimal distribution can range from having firms bear all administrative costs, to the
firms and the government sharing administrative costs, to having the government take on the entire burden of these costs.

While it is difficult to reach specific conclusions about the jointly optimal distribution of administrative costs and the emissions tax in the general case, we can obtain some insight into the problem in special cases. The remainder of our analysis looks at two such cases. The distribution of administrative costs affects social welfare in both cases, so the optimal distribution of these costs is a relevant question.

It is clear that the effects of the tax and distribution of administrative costs on the industry’s output price can have a major impact on the optimal policy, in part because the variable output price makes several critical effects indeterminate, in particular the effects of the distribution of administrative costs on aggregate emissions and aggregate administrative costs (recall Lemmas 1 and 3, respectively). The following proposition and corollary suggest optimal policies when the industry’s output price is fixed. This case is relevant when the regulated industry is a domestic (regional) one operating in a competitive international (national) market.

**Proposition 3:** Suppose that the industry’s output price is fixed and that the distribution of administrative costs affects social welfare. If the optimal tax is not greater than marginal damage, then the optimal policy has the industry bearing all administrative costs. The industry and the government share administrative costs only if the optimal tax is strictly greater than marginal damage and aggregate emissions are strictly increasing in the government’s share of administrative costs. A sufficient condition for it to be optimal for the public to bear a portion of administrative costs is

\[
W_\alpha(t, \alpha = 0) = (t - \delta)\hat{X}_\alpha - (\mu - 1)\int_{\hat{\lambda}}^{\hat{\lambda}_m} m(t, \hat{x}, \lambda) d\lambda > 0. \quad (23)
\]

**Proof:** If the output price is fixed, then (21) becomes:

\[
W_\alpha(t, \alpha) = (t - \delta)\hat{X}_\alpha - \alpha\mu \left\{ \int_{\hat{\lambda}_m}^{\hat{\lambda}} x_\alpha m d\lambda - \lambda_\alpha m(t, \hat{x}, \lambda) \right\}
\]

23
Recall from Lemma 1 that $\hat{X}_α ≥ 0$ when $\hat{p}$ is fixed. Hence, given $t ≤ δ$, $(t − δ)\hat{X}_α ≤ 0$. Moreover, from Table 2, $x_α ≥ 0$ and the inequality is strict if $m_x > 0$. Therefore,

$$\int_{λ_m}^{λ} x_α m_x dλ - λ_m m(t, x, λ) ≥ 0,$$

and the inequality is strict if $m_x > 0$ or $λ_m < 0$. Finally,

$$(μ − 1) \int_{λ_m}^{λ} m(t, x, λ) dλ ≥ 0$$

and the inequality is strict if $μ > 1$. Recall from Proposition 1 that the distribution of administrative costs affects social welfare if $m_x > 0$, $λ_m < 0$, or $μ > 1$. Hence, the first term of the right side of (24) is weakly negative while the sum of the second and third terms is strictly negative. This implies, $W_α(t, α) < 0$ and $α = 0$. Hence, it is optimal for the industry to bear all administrative costs if the optimal tax is not greater than marginal damage. The only way for it to be optimal for the industry and government to share the administrative cost burden is if $(t − δ)\hat{X}_α > 0$, which given $\hat{X}_α ≥ 0$, only occurs if $t > δ$ and $\hat{X}_α > 0$. Finally, (23) follows directly from (24). It says that a sufficient condition for it to be optimal that the public take on a part of administrative costs is if social welfare is increasing in the public’s share of administrative costs when that share is zero. □

Fixing the output price makes aggregate emissions nondecreasing and aggregate administrative costs strictly increasing in the distribution of administrative costs. The latter effect is a strong motivation to push more of the burden of administrative costs onto the firms, while the role of the former effect depends on whether the emissions tax is greater than, equal to, or less than marginal damage. Fixing the output price does not help us determine the sign of $(t − δ)$ under the optimal policy, so the results of Corollary 1 apply. Combining Proposition 3 and Corollary 1 gives us the following corollary.
Corollary 2: Suppose that the industry’s output price is fixed and that the distribution of administrative costs affects social welfare. Then, it is optimal for the industry to bear all administrative costs if these costs are nondecreasing in the emissions tax. Consequently, the government shares a part of administrative costs only if these costs are strictly decreasing in the tax, although this is not a sufficient condition.

If the optimal tax is less than or equal to marginal damage, social welfare is strictly decreasing as the government takes on a greater burden of administrative costs. Hence, it is optimal to have the industry bear all administrative costs and, from (20), to set the tax according to

\[ W_t(t, \alpha) = (t - \delta) \hat{X}_t - \int_{\lambda_m}^{\hat{\lambda}} m_t d\lambda = 0. \]  

(25)

In this case the optimal tax is equal to marginal damage if \( m_t = 0 \), but is strictly less than marginal damage if \( m_t > 0 \).

However, recall from Proposition 2 that it may be optimal to set the tax above marginal damage, but only if the government bears a part of administrative costs and these costs are strictly decreasing in the emissions tax. With the additional condition that aggregate emissions are strictly decreasing in the administrative cost share of the government, it is possible that the optimal policy involves the government bearing at least a part of administrative costs and the optimal tax is strictly above marginal damage.

In our second special case, administrative costs do not depend directly on firm-level emissions and the distribution of these costs does not affect the size of an industry. This case is relevant when administrative costs are so “small” that they have a negligible impact on firms’ decisions. This case also covers common assumptions in the theoretical literature on enforcing emissions taxes and other environmental policies.

Proposition 4: Suppose that administrative costs do not depend directly on firm-level emissions and that the distribution of these costs does not affect the size of an industry (\( m_x = \lambda_{\alpha}^m = 0 \)). If
public funds are more costly than private funds ($\mu - 1 > 0$), then it is optimal for the firms to bear all administrative costs and the optimal tax is weakly less than marginal damage.

**Proof:** The conditions $m_x = \lambda^{m_m}_x = \lambda^{m}_x = 0$ imply $\hat{x}_x = \hat{\lambda^{m}_x} = 0$ from Table 2, and $\hat{X}_x = 0$ from Lemma 1. Given, $\hat{X}_x = \hat{x}_x = \hat{\lambda^{m}_x} = 0$, (21) reduces to

$$W_\alpha(t, \alpha) = -(\mu - 1) \int_{\hat{\lambda}_m} \lambda m(t, \hat{x}, \lambda) d\lambda \leq 0, \text{ if } \alpha < 0, \text{ then } \alpha = 0. \quad (26)$$

Since $\mu - 1 > 0$, $W_\alpha(t, \alpha) < 0$, indicating that optimality requires that the firms bear all administrative costs. From Corollary 1, the optimal emissions tax is weakly less than marginal damage when the firms bear all administrative costs. □

The reasons that the firms bear all administrative costs in this situation are because the distribution of these costs does not affect aggregate emissions (as noted in the proof) and because aggregate equilibrium administrative costs are strictly increasing as the government takes on more of these costs (see Lemma 3 part (b)). The sole reason for this latter effect is that public funds are more expensive than private funds; hence, it is optimal for firms to bear all administrative costs to eliminate the deadweight costs of using public funds to administer the tax.\(^9\)

Recall from Table 2 that $m_x = 0$ implies $q_\alpha = x_\alpha = 0$. Thus, under the conditions of Proposition 4, $q_\alpha = x_\alpha = \hat{\lambda^{m}_x} = 0$. This can occur if administrative costs are so small relative to firms’ production and abatement costs that they do not affect their decisions about production, emissions, and market participation.

**Corollary 3:** If the distribution of administrative costs does not affect firms’ production, emissions, and market participation choices, and if public funds are more expensive than private funds, then it is optimal to have the firms bear all administrative costs, and the optimal tax is weakly less than marginal damage.

\(^9\)If public funds and private funds are equally costly ($\mu = 1$) then we are in the situation specified in Proposition 1 and the distribution of administrative costs does not affect social welfare.
Proposition 4 is also relevant for existing theoretical models of enforcing emissions taxes and other environmental policies. Typically, in these models administrative costs consist of only enforcement costs which are borne entirely by the government. Moreover, enforcement costs do not depend directly on firm-level emissions and there are a fixed number of firms. That is, \( \alpha = 1 \) while \( m_x = \lambda^m_{\alpha} = 0 \). Models with these features include those of Montero (2002), Cremer and Gahvari (2002), Macho-Stadler and Perez-Castrillo (2006), Stranlund, Chávez, and Villena (2009), and Evans, Gilpatric, and Liu (2009). While these works simply assume that the government bears all enforcement costs, Proposition 4 reveals that when \( m_x = \lambda^m_{\alpha} = 0 \) and public funds are more expensive than private funds, it is optimal for firms to bear all administrative costs. Therefore, Proposition 4 implies the following corollary:

**Corollary 4:** The following assumptions are common in the literature on enforcing environmental taxes and other environmental policies: Enforcement costs are the only administrative costs, enforcement costs do not depend directly on firm-level emissions, and the number of firms is fixed. Under these conditions and if public funds are more expensive than private funds, then it is optimal to have the firms bear all administrative costs and the optimal tax is weakly less than marginal damage.

Under the usual assumptions in the literature on enforcing emissions taxes, having the government bear all enforcement costs cannot welfare-dominate having the firms bear all of these costs. In fact, taking into account the costs of public funds devoted to enforcement makes it optimal for firms to bear all enforcement costs. Hence, economic models of enforcing emissions taxes that assume that the government takes on all enforcement costs are not only unrealistic, because in practice these costs are shared between regulators and firms, they also involve suboptimal policies.
4. Conclusion

We have examined the efficient distribution of the costs of administering an emissions tax between regulated firms and the general public via government regulators. Our results suggest three ways in which the distribution of administrative costs can matter. Firm-level emissions may have a direct impact on administrative costs, the distribution of these costs may affect the size of the regulated industry, and public funds for administration may be more expensive than private funds due to the deadweight costs of generating public revenue. If all three of these effects are absent, then the distribution of administrative costs has no impact on social welfare, although the chosen distribution does help determine the optimal tax.

When the distribution of administrative costs does have welfare consequences, the optimal sharing of administrative costs and the level of the tax relative to marginal damage is largely determined by the effects of these policy variables on aggregate emissions and administrative costs. Except for the effect of an emissions tax on aggregate emissions, the other effects are indeterminate in the most general case, implying that the optimal level of a tax and the distribution of the costs of administering the tax have to be determined on a case-by-case basis.

Our ambiguous results imply that implementation of the optimal distribution of administrative costs can be difficult, because there are no simple rules concerning the distribution of administrative costs to use in designing an optimal emission tax. In fact, implementing the optimal distribution of administrative costs in a particular setting appears to require careful estimates of the impact of the distribution of these costs on firm and market behavior. Unfortunately, we are not aware of any such estimates. Hence, one general conclusion about implementing emissions taxes in consideration of administrative costs is the need for empirical research about the effects of these costs on firms’ choices and market outcomes.

Without the information necessary to implement an optimal sharing rule in general, considering the special cases of our model can be helpful. For example, using our Corollary 3, it is optimal for firms to bear all administrative costs in cases in which it is unlikely that the distribution of these costs affects firms’ choices. This also suggests that careful estimates of the effects of the
distribution of administrative costs are most important when regulators believe that this distribution can cause firms’ to alter their production, emissions, and market participation choices. As another example, our Proposition 3 and Corollary 2 suggest for domestic industries that operate in a competitive world market that regulators need only consider bearing part of the burden of administrative costs if they believe that aggregate equilibrium administrative costs are strictly decreasing in the level of the emissions tax; otherwise, it is optimal for the industry to bear all administrative costs.

Of course, equity or political factors may trump efficiency in implementing administrative cost sharing rules. The opposition to emissions taxes can be intense, and making firms bear all the costs of administering a tax can only enhance that opposition. Policy designers may be motivated to relieve some of the administrative cost burden on regulated firms to make an emission tax more palatable to industry and to reduce the risk of forcing some firms out of business. This may come at a cost, but it also may be an efficiency-enhancing move if the alternative is a policy that is less efficient than an emissions tax. In cases in which the distribution of administrative costs is not chosen optimally, it is important to remember that the chosen distribution does affect the optimal emissions tax.

This work can be extended in many ways, but let us mention just two. It would be useful to examine the design of an emissions trading scheme considering the distribution of administrative costs. We doubt that our results about distributing the administrative costs of an emissions tax can be applied directly to emissions trading. The analysis is likely to be more complex because the distribution of administrative costs can affect two markets—the output and permit markets—simultaneously.

Another potentially fruitful area of research would be to consider the distribution of the costs of administering an emissions tax when the industry is characterized by imperfect competition. Examining the effects of imperfect competition on optimal environmental policies has been an active area of research for several decades. See Requate (2007) for a thorough review of this literature. A policy of an emissions tax and the distribution of the costs of administering the tax would confront
the inefficiency in the product market, as well as deal with the emissions control. Katsoulacos and Xepapadeas (1995) demonstrated three welfare effects of a tax on an industry consisting of identical Cournot firms. The first is the positive welfare effect due to reduced emissions. The second is a negative welfare effect that is due to the tax reducing output, which is already inefficiently low in the absence of the tax. The third welfare effect of an emissions tax is a positive one that is due to the tax reducing the number of firms in an industry, which is higher than the optimal number of firms in the absence of the tax. The distribution of administrative costs would no doubt affect each of these factors as well. It is particularly intriguing that reducing the number of Cournot firms in an industry might be an important reason for pushing a larger part of the costs of administering an emissions tax on the industry.
Appendix

Derivations of the results in Table 2. Use (2) and (3) to obtain:

\[
\begin{bmatrix}
q_t \\
x_t
\end{bmatrix}
=\begin{bmatrix}0 \\
1 + (1 - \alpha)m_{xt}
\end{bmatrix},
\begin{bmatrix}q_\alpha \\
x_\alpha
\end{bmatrix}
=\begin{bmatrix}0 \\
-m_{xt}
\end{bmatrix},
\begin{bmatrix}q_p \\
x_p
\end{bmatrix}
=\begin{bmatrix}-1 \\
0
\end{bmatrix},
\]

(27)

where

\[
S = \begin{bmatrix}-c_{qq} & -c_{qx} \\
-c_{sx} & -c_{xx} - (1 - \alpha)m_{xx}
\end{bmatrix}.
\]

Let \(|S| = c_{qq}(c_{xx} + (1 - \alpha)m_{xx}) - (c_{qx})^2\) denote the determinant of the Hessian matrix of the firm’s total cost function (production cost plus the fraction of the administrative costs it bears). \(|S| > 0\) is required for concavity of the firm’s profit function. In the usual manner find the following comparative statics:

\[
q_t = c_{qx}(c_{xx} + (1 - \alpha)m_{xx})/|S| < 0, q_\alpha = -c_{qxm}/|S| \geq 0, \text{ and } q_p = (c_{xx} + (1 - \alpha)m_{xx})/|S| > 0;
\]

(28)

\[
x_t = -c_{qq}(1 + (1 - \alpha)m_{xt})/|S| < 0, x_\alpha = c_{qxm}/|S| \geq 0, \text{ and } x_p = c_{xx}/|S| > 0;
\]

(29)

To determine the direct effects of \(t\), \(\alpha\), and \(p\) on the size of the industry substitute (5) into (4) to obtain \(\pi(t, \alpha, p, \lambda^m) \equiv 0\) and:

\[
\lambda_t^m = (x + (1 - \alpha)m_t)/\pi\lambda > 0, \lambda_\alpha^m = -m/\pi\lambda \leq 0, \text{ and } \lambda_p^m = -q/\pi\lambda < 0.
\]

(30)

To determine the effects of \(t\), \(\alpha\), and \(p\) on the supply function of the regulated industry, first define:

\[
q^m = q(t, \alpha, p, \lambda^m) \text{ and } x^m = x(t, \alpha, p, \lambda^m)
\]

(31)
as the output and emissions of the cut-off firm in the industry. Then:

\[
Q^S_p(t, \alpha, p) = \int_{\lambda^m}^{\lambda} q_p d\lambda - \hat{\lambda}^m_p q^m > 0; \quad (32)
\]

\[
Q^S_t(t, \alpha, p) = \int_{\lambda^m}^{\tilde{\lambda}} q_t d\lambda - \hat{\lambda}^m_t q^m < 0; \quad (33)
\]

\[
Q^S_\alpha(t, \alpha, p) = \int_{\lambda^m}^{\lambda} q_\alpha d\lambda - \hat{\lambda}^m_\alpha q^m \geq 0; \quad (34)
\]

From the market-clearing condition \(Q^S(t, \alpha, p) = Q^D(p)\) obtain:

\[
\hat{p}_t = -\frac{Q^S_t}{Q^S_p - Q^D_p} > 0 \quad \text{and} \quad \hat{p}_\alpha = -\frac{Q^S_\alpha}{Q^S_p - Q^D_p} \leq 0. \quad (35)
\]

The sign of \(\hat{p}_t\) follows from \(Q^S_p - Q^D_p > 0\) and (33). The sign of \(\hat{p}_\alpha\) follows from \(Q^S_p - Q^D_p > 0\) and (34), noting the sign of \(q_\alpha\) from (28).

The signs of \(\hat{x}_t\) and \(\hat{\lambda}_m^m\) in Table 2 are by assumption as noted in section 2.3. The sign of \(\hat{x}_\alpha\) is demonstrated in the main text.

To show the results for \(\hat{\lambda}_m^m\), recall from (12) that \(\hat{\lambda}_m^m = \lambda^m_\alpha + \lambda^m_p \hat{p}_\alpha\). From Table 2, \(\lambda^m_\alpha = m_x = 0\) imply \(\hat{p}_\alpha = 0\), and \(\hat{\lambda}_m^m = 0\). Moreover, \(m_x > 0\) and \(\lambda^m_\alpha = 0\) imply \(\hat{p}_\alpha < 0\). Then, since \(\lambda^m_p \hat{p}_\alpha > 0\), \(m_x > 0\) and \(\lambda^m_\alpha = 0\) imply \(\hat{\lambda}_m^m > 0\). To show \(\hat{\lambda}_m^m < 0\) when \(\lambda^m_\alpha < 0\) and \(m_x = 0\) use (35) and (34) to rewrite \(\hat{\lambda}_m^m = \lambda^m_\alpha + \lambda^m_p \hat{p}_\alpha\) as:

\[
\hat{\lambda}_m^m = \lambda^m_\alpha - \lambda^m_p \left\{ \int_{\lambda^m}^{\lambda} q_\alpha d\lambda - \frac{\hat{\lambda}^m_\alpha q^m}{Q^S_p - Q^D_p} \right\}. 
\]

Since \(m_x = 0\) implies \(q_\alpha = 0\) (see (28)):

\[
\hat{\lambda}_m^m = \lambda^m_\alpha \left\{ \frac{Q^S_p - Q^D_p + \lambda^m_p q^m}{Q^S_p - Q^D_p} \right\}. 
\]
Substitute in for $Q_p^S$ using (32) to obtain:

$$
\hat{\lambda}^m_{\alpha} = \lambda^m_{\alpha} \left\{ \frac{\int_{\lambda^m_{\alpha}}^{\hat{\lambda}} q_p d\lambda - Q^D_{p}}{Q_p^S - Q^D_p} \right\},
$$

which is strictly negative when $\lambda^m_{\alpha} < 0$, because $q_p > 0$, $Q^D_{p} < 0$, and $Q_p^S - Q^D_p > 0$.

**Derivation of first order conditions (20) and (21).** From (19) the first order condition for a strictly positive tax can be written as:

$$
W_t(t, \alpha) = p(\hat{Q}) \left( \int_{\hat{\lambda}_{m}}^{\hat{\lambda}} \hat{q}_t d\lambda - \hat{\lambda}^m_{t} q^m \right)
- \left( \int_{\hat{\lambda}_{m}}^{\hat{\lambda}} (c_q \hat{q}_t + c_x \hat{x}_t) d\lambda - \hat{\lambda}^m_{t} c(q^m, \hat{x}^m, \hat{\lambda}^m) \right)
- \delta \left( \int_{\hat{\lambda}_{m}}^{\hat{\lambda}} \hat{x}_t d\lambda - \hat{\lambda}^m_{t} \hat{x}^m \right)
- (1 - \alpha + \alpha \mu) \left( \int_{\hat{\lambda}_{m}}^{\hat{\lambda}} (m_t + m_x \hat{x}_t) d\lambda - \hat{\lambda}^m_{t} m(t, \hat{x}, \hat{\lambda}^m) \right) = 0,
$$

where $\hat{q}^m = q(t, \alpha, \hat{\rho}, \hat{\lambda}^m)$ is the equilibrium output of the cut-off firm in the industry. From a firm’s first order conditions for its optimal choices of output and emissions (2), substitute $p(\hat{Q}) = c_q$ and $t = -c_x - (1 - \alpha)m_x$ into (36) and rearrange terms to obtain:

$$
W_t(t, \alpha) = \int_{\hat{\lambda}_{m}}^{\hat{\lambda}} ((t - \delta) \hat{x}_t - \alpha \mu m_x \hat{x}_t - (1 - \alpha + \alpha \mu)m_t) d\lambda
- \hat{\lambda}^m_{t} \left( p(\hat{Q}) \hat{q}^m - c(q^m, \hat{x}^m, \hat{\lambda}^m) \right)
- \delta \hat{x}^m - (1 - \alpha + \alpha \mu)m(t, \hat{x}^m, \hat{\lambda}^m) = 0.
$$

Substitute equilibrium values of the cut-off firm into (4) to obtain:

$$
\pi(t, \alpha, \hat{\lambda}^m) = p \hat{q}^m - c(q^m, \hat{x}^m, \hat{\lambda}^m) - t \hat{x}^m - (1 - \alpha)m(t, \hat{x}^m, \hat{\lambda}^m) \equiv 0.
$$

33
Subtract $\delta \dot{x}^m$ and $\alpha \mu m(t, \dot{x}^m)$ from both sides of (38) and rearrange terms to obtain:

$$p\dot{q}^m - c(q^m, \dot{x}^m, \dot{\lambda}^m) - \delta \dot{x}^m - (1 - \alpha + \alpha \mu)m(t, \dot{x}^m, \dot{\lambda}^m) = (t - \delta)\dot{x}^m - \alpha \mu m(t, \dot{x}^m, \dot{\lambda}^m).$$

(39)

Substitute (39) into (37) and rearrange terms to obtain:

$$W_t(t, \alpha) = (t - \delta) \left\{ \int_{\dot{\lambda}_m}^{\dot{\lambda}} \dot{\lambda} \dot{x} d\lambda - \dot{\lambda}_m^m \dot{x}^m \right\} - \alpha \mu \left\{ \int_{\dot{\lambda}_m}^{\dot{\lambda}} (m_t + m_x \dot{x}_t) d\lambda - \dot{\lambda}_m^m m(t, \dot{x}^m, \dot{\lambda}^m) \right\} - (1 - \alpha) \int_{\dot{\lambda}_m}^{\dot{\lambda}} m_t d\lambda = 0.$$

(40)

Substitute (15) into (40) to obtain the desired result (20).

Now turn to the first order condition for $\alpha$, which from (19) is:

$$W_{\alpha}(t, \alpha) = p(\dot{Q}) \left( \int_{\dot{\lambda}_m}^{\dot{\lambda}} \dot{q}_\alpha d\lambda - \dot{\lambda}_m^m \dot{q}^m \right) - \left( \int_{\dot{\lambda}_m}^{\dot{\lambda}} (c_q \dot{q}_t + c_x \dot{x}_t) d\lambda - \dot{\lambda}_m^m c(q^m, \dot{x}^m, \dot{\lambda}^m) \right) - \delta \left( \int_{\dot{\lambda}_m}^{\dot{\lambda}} \dot{\lambda}_m^m \dot{x}^m \right) - (1 - \alpha + \alpha \mu) \left\{ \int_{\dot{\lambda}_m}^{\dot{\lambda}} m_x \dot{x}_\alpha d\lambda - \dot{\lambda}_m^m m(t, \dot{x}^m, \dot{\lambda}^m) \right\} - (\mu - 1) \int_{\dot{\lambda}_m}^{\dot{\lambda}} m(t, \dot{x}, \dot{\lambda}) d\lambda \left\{ \begin{array}{ll} 
\leq 0, & \text{if } < 0, \text{ then } \alpha = 0 \\
\geq 0, & \text{if } > 0, \text{ then } \alpha = 1. 
\end{array} \right.$$ 

(41)

Substitute $p(\dot{Q}) = c_q$ and $t = -c_x - (1 - \alpha)m_x$ into (41) and rearrange terms to obtain:

$$W_{\alpha}(t, \alpha) = \int_{\dot{\lambda}_m}^{\dot{\lambda}} ((t - \delta)\dot{x}_\alpha - \alpha \mu m_x \dot{x}_\alpha) d\lambda - \dot{\lambda}_m^m (p(\dot{Q})q^m - c(q^m, \dot{x}^m, \dot{\lambda}^m) - \delta \dot{x}^m - (1 - \alpha + \alpha \mu)m(t, \dot{x}^m, \dot{\lambda}^m))$$

34
\[-(\mu - 1) \int_{\hat{\lambda}^m}^{\lambda} m(t, \hat{x}, \lambda) d\lambda \begin{cases} \\ \leq 0, & \text{if } < 0, \text{ then } \alpha = 0 \\ \geq 0, & \text{if } > 0, \text{ then } \alpha = 1. \end{cases}\] (42)

Substitute (39) into (42) and rearrange terms to obtain:

\[W_\alpha(t, \alpha) = (t - \delta) \left\{ \int_{\hat{\lambda}^m}^{\lambda} \hat{x}_\alpha d\lambda - \hat{\lambda}_\alpha^m \hat{x}_m^m \right\} - \alpha \mu \left\{ \int_{\hat{\lambda}^m}^{\lambda} m(\hat{x}_\alpha) d\lambda - \hat{\lambda}_\alpha^m m(t, \hat{x}_m, \hat{\lambda}_m^m) \right\} - (\mu - 1) \int_{\hat{\lambda}^m}^{\lambda} m(t, \hat{x}, \lambda) d\lambda \begin{cases} \\ \leq 0, & \text{if } < 0, \text{ then } \alpha = 0 \\ \geq 0, & \text{if } > 0, \text{ then } \alpha = 1. \end{cases}\] (43)

Substitute (16) into (43) to obtain (21). □
References


