# Stochastic Stability of General Equilibrium

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June 1, 2012

## 1 Introduction

The stability of Walrasian general equilibrium was widely studied in the years immediately following the proof of existence of equilibrium (Arrow and Debreu 1954), assuming Walras' tâtonnement process. Out of equilibrium, according to this assumption, there is a system of public prices shared by all agents, the time rate of change of which is a function of excess demand (Arrow and Hurwicz, 1958, 1959, 1960; Arrow, Block and Hurwicz, 1959; Nikaido 1959; McKenzie, 1960; Morishima 1960; Nikaido and Uzawa 1960; Uzawa 1961, 1962). This effort proved largely unsuccessful (Fisher 1983).

Even were a well-behaved tâtonnement dynamic to exist, such a dynamic could not be modified in any known manner to provide dynamics for decentralized and competitive market exchange. The problem is generic because the tâtonnement dynamic assumes that all prices are public (shared by all agents), and hence no agent is capable of altering a price. It follows that the Walrasian auctioneer cannot be replaced by a decentralized decision process involving the economic agents themselves.

In fact, in a decentralized economy out of equilibrium, in the absence of an auctioneer, there is no system of public prices at all (Hayek 1945). A price and quantity dynamic based on the decisions of economic agents must, by force of logic alone, be based on the results of agents adjusting private reservation prices to maximize profits or utility while producing and consuming out of equilibrium. This logically precludes the existence of a system of public prices faced by all agents.

We use the notion of stochastic stability (Young 1993) to characterize the asymptotic properties of a Markov process version of generalized competitive exchange in which each agent has a personal vector of prices used to specify individual supply and demand, as well as to judge the acceptability of trades. We show that market clearing prices obtain in the only stochastically stable state of this model.

The paper is organized as follows. In Section 2, we introduce the Markov process version of Walrasian general equilibrium model developed in Gintis (2012), which is a model of evolution with noise in the sense of (Ellison 2000). In Section 3 we characterize out-of-equilibrium trading in this economy and give sufficient conditions on the price updating mechanism to ensure the stochastic stability of equilibrium.

## 2 Evolutionary Dynamics in Exchange Economies

Consider an exchange economy with goods $l = 1, \ldots, L$, agent types $i = 1, \ldots, N$, and agents $j = 1, \ldots, M$ of each type. All agents have $Q := \mathbb{R}_+^L$ as consumption set. Agents of type $i$ are characterized by an utility function $u_i : Q \to \mathbb{R}$ and a vector of initial endowments $\omega_i \in Q$. Moreover, agent $j$ of type $i$ is endowed with a normalized vector $p_{ij}$ of private prices chosen from a finite subset $P$ of the unit simplex of $\mathbb{R}_+^L$, $S := \{p \in \mathbb{R}_+^L \mid \sum_{l=1}^L p_l = 1\}$. The population of agents is then characterized by a vector $\pi \in \Pi = P^{M \times N}$.

For instance, we might have $L = M$, $\omega_i$ consisting of $k_i$ units of good $i$. Thus in each period, each agent $j$ of type $i$ is endowed with $k_i$ units of good $i$, which he values at $k_i p_{ij(i)}$. Agent $j$ set his demand for other goods in his utility function by maximizing $u_i$, assuming income $k_i p_{ij}$. This agent then trades with agents of other types, agreeing to supply a fraction of his endowment in exchange for an amount of a good $i'$ for which he has positive demand at an exchange ratio at least as favorable as $p_{ij}(i')/p_{ij}(i)$.

Let $\Xi$ be the set of feasible allocations of goods to agents, so

$$\Xi = \left\{ \xi \in Q \middle| \sum_{i=1}^{N} \sum_{j=1}^{M} \xi_{ij} = M \sum_i \omega_i \right\}.$$

We then represent the trading process with prices $\pi$ as a probability distribution $\mathcal{T}_\pi$ over $\Xi$, which we endow with the Borel $\sigma-$algebra.

Given prices $\pi$ and an allocation $\xi \in \Xi$, we consider the imitation process as a probability distribution $\mathcal{I}_{(\pi,\xi)}$ over $\Pi$, which represent the state of prices in the suceeding period. This then becomes a Markov process in which the transition probabilities are given by

$$\mathcal{F}_{\pi,\pi'} = \int_{\xi \in \Xi} \mathcal{I}_{(\pi,\xi)}(\pi') d\mathcal{T}_\pi(\xi). \tag{1}$$

We are then concerned with the dynamics of private prices generated by the sequential iteration of trading and imitation processes. That is the process in which initial endowments are reinitialized at the beginning of each step, agents trade according to their private prices and update these as a function of the utility gained. This corresponds to the Markovian dynamics on $\Pi$ defined by the transition matrix $\mathcal{F}$ such that:

$$\mathcal{F}_{\pi,\pi'} = \int_{\xi\in\Xi} \mathcal{I}_{(\pi,\xi)}(\pi')\mathrm{d}\mathcal{T}_{\pi}(\xi) \tag{2}$$

If agents then randomly and independently mutate (i.e randomly choose a new price in $P$) with probability $\epsilon > 0$, the dynamics are modified according to:

$$\mathcal{F}^{\epsilon}_{\pi,\pi'} = \int_{\rho\in\Pi} R^{\epsilon}_{\rho,\pi'}\mathrm{d}\mathcal{F}_{\pi,\rho} = \sum_{\rho\in\Pi} R^{\epsilon}_{\rho,\pi'}\mathcal{F}_{\pi,\rho} \tag{3}$$

where $R^{\epsilon}(\rho,\pi') = (1-\epsilon)^{MN-\delta(\rho,\pi')} \times \left(\frac{\epsilon}{|P|-1}\right)^{\delta(\rho,\pi')}$ and $\delta(\rho,\pi')$ denotes the number of mutations, that is the cardinal of the set $\{(i,j) \mid \rho_{ij} \neq \pi'_{ij}\}$.

The family $\{\mathcal{F}^{\epsilon})|\epsilon \geq 0\}$ then is a model of evolution in the sense of (Ellison 2000)), and hence satisfies the following conditions:

1. $\mathcal{F}^{\epsilon}$ is ergodic for each $\epsilon > 0$,

2. $\mathcal{F}^{\epsilon}$ is continuous in $\epsilon$ and $\mathcal{F}_0 = \mathcal{F}$,

3. there exists a function $c : \Pi \times \Pi \to N$ such that for all $\pi, \pi' \in \Pi$, $\lim_{\epsilon\to 0} \frac{\mathcal{F}^{\epsilon}_{(\pi,\pi')}}{\epsilon^{c(\pi,\pi')}}$ exists and is strictly positive.

Condition (1) implies in particular that for each $\epsilon > 0$, $\mathcal{F}^{\epsilon}$ has a unique invariant distribution $\psi^{\epsilon}$. A population $\pi \in \Pi$ is then called stochastically stable if $\lim_{\epsilon\to 0} \psi^{\epsilon}(\pi) > 0$.

This notion of stochastic stability can be used for the analysis of the stability of the equilibria of the underlying exchange economy thanks to the identification of an equilibrium price $\overline{p}$ with the population $\overline{\pi}$ such that for each agent $j$ of type $i$, $\overline{\pi}_j = \overline{p}$). The equilibrium associated with the price $\overline{p}$ can then be called stochastically stable if $\overline{\pi}$ is. The interesting case is this where $\overline{\pi}$ is the only stochastically stable population which implies that $\lim_{\epsilon\to 0} \psi^{\epsilon}(\pi) = 1$ and that for vanishingly small perturbations the process eventually settles in $\overline{\pi}$ independently of the initial conditions, in other words converges to equilibrium.

## 3 Stochastic Stability in the Scarf Economy

We now want to define the outcome of out-of-equilibrium trading for a population of agents using private prices. In Gintis' model, agents assess the value of trades using their private prices and perform those which have a non-negative value. However, tracking all such possible trades lead to excessive combinatorial complexity. In order to provide a parsimonious analysis, we rather consider the benchmark situation where agents only trade with peers using the same price. This restriction in fact discards "lucky" trades which would increase the value of one's stock. It is also standard in the non-ttonnement literature (Fisher 1983).

An alternative interpretation of this restriction of trading to peers using the same price is to consider that different market places or trading posts coexist, in each of which exchanges are performed according to one price, and that the private price of an agent is a marker of the market he is "affiliated" to. The choice of a private price by an agent can then be seen as a form of voting with one's feet for a set of exchange ratios.

Formally, we place ourselves in the framework of section 2, with $N = 3$, utilities and endowments being those of the Scarf economy and we assume $P$ contains the equilibrium price $\overline{p} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. We also let $\mu_i(\pi, p)$ denote the number of agents of type $i$ using price $p$ in the population $\pi$, $\mu(\pi, p) = \sum_{i=1}^{N} \mu_i(\pi, p)$ denotes the total number of agents using price $p$ in the population $\pi$, and $\underline{\mu}(\pi, p) := \min(\mu_1(\pi, p), \mu_2(\pi, p), \mu_3(\pi, p))$ denotes the minimal number of agents using price $p$ among the different types in the population $\pi$.

In this setting the assumptions that agents only trade with peers using the same price and that they strategically restrict their trades amount to consider that agents of type $i$ using price $p$ collectively offer $\mu_i(\pi, p)(\omega_i - \overline{x}_i(p))$ units of good $i$ to their peers of other types using price $p$. To reduce the combinatorial complexity of the analysis, we shall restrict attention to the simplest allocations consistent with these constraints, namely those where $\underline{\mu}(\pi, p)$ among the agents using price $p$ receive the allocation $\overline{x}(p)$, while the remaining retain their initial allocation and do not trade. The random component of the trading process is hence restricted to the designation of trading and non-trading agents. This amounts to assume that for all population $\pi \in \Pi$ and type of agent $i$[1], one has:

$$\mathcal{T}_\pi \left\{ \xi \in \Xi \mid \begin{array}{l} \mathrm{card}\{j \mid \pi_{ij} = p \wedge \xi_{ij} = \overline{x}_i(p)\} = \underline{\mu}(\pi, p) \\ \mathrm{card}\{j \mid \pi_{ij} = p \wedge \iota_{ij} = \overline{x}_i(p)\} + \\ \mathrm{card}\{j \mid \pi_{ij} = p \wedge \xi_{ij} = \omega_i\} = \mu_i(\pi, p) \end{array} \right\} = 1 \qquad \text{(I)}$$

We hence obtain a simplified representation of the trading process of ($Gintis$ 2010), by assuming that trading is efficient and takes place only among subgroups consisting of a similar number of agents of each type.

[1] card$(E)$ denotes the cardinal of the set $E$.

### 3.1 Imitation process

In order to characterize the imitation process, we have to specify the probability of the event that the $j$th agent of type $i$ adopt the price of the $j'$th agent of type $i'$. We denote this event by $\{(i, j) \to (i', j')\}$ and assume that for all pair of agents $(i, j)$ and $(i', j')$, one has:

$$\epsilon I_{(\pi,\xi)} \{(i, j) \to (i', j')\} > 0 \\ \Leftrightarrow \\ \begin{cases} (i)\ i = i' \text{ and } \xi_{i,j} \neq \omega_i \text{ and } u_i(\xi_{i,j}) \leq u_i(\xi_{i,j'}) \\ \text{or} \\ (ii)\ \mu(\pi, \pi_{i',j'}) = \max_{p \in P} \mu(\pi, p) \\ \text{or} \\ (iii)\ (i, j) = (i', j') \end{cases} \tag{II}$$

Interpreted in conjunction with assumption (I), condition $(i)$ states that agents which have traded have a positive probability to copy the prices of a peer who has been more successful in trading[2].

Condition $(ii)$ states that agents always have a positive probability to copy the most widely used price in the population. This prevents in particular that the system gets stuck in the extremely inefficient situation where each type of agent uses a different price.

Condition $(iii)$ states that an agent can conserve his price with some probability.

As far as the relationships between agents' imitation behavior is concerned, we shall assume there is independent inertia, that is the imitation probabilities are independent. Namely, for all agents $(i, j)$, $(i', j')$, $(i'', j'')$, $(i''', j''')$ with $(i, j) \neq (i'', j'')$, one has:

$$\mathcal{I}_{(\pi,\xi)} \{(i, j) \to (i', j') \wedge (i'', j'') \to (i''', j'''')\} = \\ \epsilon I_{(\pi,\xi)} \{(i, j) \to (i', j')\} \times \epsilon I_{(\pi,\xi)} \{(i'', j'') \to (i''', j'''')\} \tag{III}$$

### 3.2 Stochastic stability of equilibrium in the Scarf economy

Note that the population $\overline{\pi}$ where each agent use the equilibrium price $\overline{p}$ (i.e for all $(i, j)$, $\overline{\pi}_{ij} = \overline{p}$) is an invariant distribution of the process $\mathcal{F}$ under assumption (II) and that under assumption (I) it can naturally be identified with the equilibrium of the Scarf economy as the trading process $\epsilon T_\pi$ then allocates to each agent the

[2] We could as well assume that all agents can copy a peer's price but we should then assume that they compare the utility each price yeilded to agents which have actually traded.

corresponding equilibrium allocation with probability one. Conditions (I), (II) and (III) are in fact sufficient to prove our main result:

Theorem 1. *Under conditions (I), (II), and (III), the population $\overline{\pi}$ is the only stochastically stable state of the dynamics $(\mathcal{F}^{\epsilon})^{\epsilon \geq 0}$ in the Scarf economy*

The proof mainly builds on (Ellison 2000) radius-coradius theorem, which makes use of the following notions:

- A path from $\pi \in \Pi$ to $\pi' \in \Pi$ is a finite sequence of states, $\pi^1, \cdots, \pi^K \in \Pi$, such that $\pi^1 = \pi$ and $\pi^K = \pi'$. The set of paths from $\pi$ to $\pi'$ is denoted by $S(\pi, \pi')$. The cost of a path $(\pi^1, \cdots, \pi^K)$ is defined as:

$$c(\pi^1, \cdots, \pi^K) = \sum_{k=1}^{K-1} c(\pi^k, \pi^{k+1}) \tag{4}$$

  One can then remark that equation (3) implies that $c(\pi, \pi') = 0$ whenever $\mathcal{F}_{\pi,\pi'} > 0$, that is whenever there is a positive probability to reach $\pi'$ from $\pi$ via the unperturbed process, and that $c(\pi, \pi')$ is bounded above by the number of distinct prices between $\pi$ and $\pi'$, that is $c(\pi, \pi') \leq \text{card}\,\{(i, j) \mid \pi_{i,j} \neq \pi'_{i,j}\}$.

- The basin of attraction of the population $\bar{\pi}$ is the set of initial states from which the unperturbed Markov process (with transition probability $\mathcal{F}$) converges to $\bar{\pi}$ with probability one, that is:

$$D(\overline{\pi}) = \{\pi \in \Pi \mid \lim_{T \to +\infty} \mathcal{F}^T_{\pi,\overline{\pi}} = 1\} \tag{5}$$

- The radius $r(\overline{\pi})$ of the population $\bar{\pi}$ is then defined as the minimal cost of a path leaving $D(\overline{\pi})$. That is letting $S(\overline{\pi}, D(\overline{\pi})^c) := \cup_{\pi \in D(\overline{\pi})^c} S(\overline{\pi}, \pi)$ denote the set of paths out of $D(\overline{\pi})$, one has:

$$r(\overline{\pi}) = \min\nolimits_{s \in S(\overline{\pi}, D(\overline{\pi})^c)} c(s) \tag{6}$$

- Finally, the coradius of $\overline{\pi}$ is defined as the maximal cost of a transition to $\overline{\pi}$

$$cr(\overline{\pi}) = \max\nolimits_{\pi \neq \overline{\pi}} \min\nolimits_{s \in S(\pi, \overline{\pi})} c(s) \tag{7}$$

Following theorem 1 in (Ellison 2000), in order to prove that $\overline{\pi}$ is stochastically stable, it suffices to prove that its radius if greater than its coradius. We shall precisely prove that:

$$r(\overline{\pi}) > 3 \text{ and } cr(\overline{\pi}) \leq 3 \tag{8}$$

- Let us first prove that $cr(\overline{\pi}) \leq 3$. Part $(ii)$ of assumption (II) ensures that there always exists a zero cost path from any population to a uniform one where each agent uses the same price. So, without loss of generality, we can restrict attention to populations $\pi$ such that for all $(i, j)$, $\pi_{i,j} = p$. Moreover, we can without loss of generality restrict attention to the cases where $p$ is such that $p_2 > p_1 > p_3$ or $p_1 > p_2 > p_3$. In either cases, one has $u_1(\overline{x}_1(p)) < \frac{1}{2}$ and $u_3(\overline{x}_3(p)) < \frac{1}{2}$.

  Let then $\pi'$ be such that for all $i$, $\pi'_{i,1} = \overline{p}$ and and for all $j \neq 1$, $\pi'_{i,j} = \pi_{i,j} = p$. One clearly has
$$c(\pi, \pi') = 3. \tag{9}$$

  Moreover, according to assumption (I), any $\xi$ such that $\epsilon T_{\pi'}(\xi) > 0$ should satisfy $u_1(\xi_{1,1}) = u_1(\overline{x}_1(\pi'_{1,1})) = u_1(\overline{x}_1(\overline{p})) = \frac{1}{2}$ and for $j \neq 1$ $u_1(\xi_{1,j}) = u_1(\overline{x}_1(\pi'_{1,j})) = u_1(\overline{x}_1(p)) < \frac{1}{2}$, so that for all $j \neq 1, u_1(\xi_{1,j}) < u_1(\xi_{1,1})$. Similarly, one obtains for all $j \neq 1, u_3(\xi_{3,j}) < u_1(\xi_{3,1})$.

  Let then $\pi''$ be such that for all $j$, $\pi''_{1,j} = \pi'_{1,j} = \overline{p}$, $\pi''_{3,j} = \pi'_{3,j} = \overline{p}$ and $\pi''_{2,j} = \pi'_{2,j}$. Using part $(i)$ of assumption (II), one clearly has
$$c(\pi, \pi'') = 0 \tag{10}$$

  Finally, as it is clear that $\mu(\pi'', \overline{p}) = \max_{p \in P} \mu(\pi, p)$, one has using part $(ii)$ of assumption (II) that
$$c(\pi'', \overline{\pi}) = 0. \tag{11}$$

  Equations (9), to (11) eventually yield:
$$cr(\overline{\pi}) \leq 3. \tag{12}$$

  This ends the first part of the proof.

- Let us then prove that $r(\overline{\pi}) > 3$ :
  - Assumption (II) clearly implies that $\mathcal{F}_{\overline{\pi},\overline{\pi}} = 1$, so that for any $\pi \neq \overline{\pi}$, one has $c(\overline{\pi}, \pi) \neq 0$. Hence $r(\overline{\pi}) > 0$
  - Moreover equation (3) implies that for any $\pi$ such that $c(\overline{\pi}, \pi) \leq 2$, one has for any $p \neq \overline{p}$, $\underline{\mu}(\pi, p) = 0$ as well as $\sum_{p \neq \overline{p}} \mu(\pi, p) \leq 2$. Let us consider $(i, j) \in$ such that $\pi_{i,j} \neq \overline{p}$ and $\xi \in \Xi$ with $\epsilon T_{\pi}(\xi) > 0$. Assumption (I) implies that $\xi_{i,j} = \omega_i$. As moreover, one clearly hasn't $\mu(\pi, \pi_{i,j}) = \max_{p \in P} \mu(\pi, p)$, assumption (II) in fact implies that for any $(i', j') \neq (i, j)$, $\epsilon I_{(\pi,\xi)}\{(i, j) \to (i', j')\} = 0$. Hence,

any $\pi'$ such that $c(\pi, \pi') = 0$ must satisfy $\underline{\mu}(\pi', p) = 0$ as well as $\sum_{p \neq \overline{p}} \mu(\pi', p) \leq 2$. By recursion the same can be proven for any population $\pi''$ such that $c(s) = 0$ for some $s \in S(\pi, \pi'')$. Moreover, part $(ii)$ of assumption (II) ensures that any such $\pi''$ satisfies $\mathcal{F}_{\pi'', \overline{\pi}} > 0$. This clearly implies that any such $\pi''$ and in particular $\pi$ belong to $D(\overline{\pi})$. We have hence proven that $r(\overline{\pi}) > 2$.

- Finally, if $\pi$ is such that $c(\overline{\pi}, \pi) \leq 3$, either one has for any $p \neq \overline{p}$, $\underline{\mu}(\pi, p) = 0$ as well as $\sum_{p \neq \overline{p}} \mu(\pi, p) \leq 3$ and one can prove as in the preceding case that $\pi \in D(\overline{\pi})$, or there exists $p \neq \overline{p}$ such that $\underline{\mu}(\pi, p) = 1$. Without loss of generality, we can then assume that for all $i$ $\pi(i, 1) = p$ and for all $j \neq 1$, $\pi(i, j) = \overline{p}$. Also, we can restrict attention to the cases where $p$ is such that $p_2 > p_1 > p_3$ or $p_1 > p_2 > p_3$. In either cases, one has $u_1(\overline{x}_1(p)) < \frac{1}{2}$ and $u_3(\overline{x}_3(p)) < \frac{1}{2}$ whereas one has respectively $u_1(\overline{x}_1(\overline{p})) = \frac{1}{2}$ and $u_3(\overline{x}_3(\overline{p})) = \frac{1}{2}$. Assumption (II) then implies that for any $\xi \in \Xi$ with $\epsilon T_\pi(\xi) > 0$, one has $\mathcal{I}_{(\pi,\xi)} \{(1, j) \to (1, 1)\} = 0$ and $\mathcal{I}_{(\pi,\xi)} \{(3, j) \to (3, 1)\} = 0$, although one might have $\mathcal{I}_{(\pi,x)}\{(2, j) \to (2, 1)\} > 0$[3]. This implies that for any $\pi'$ such that $c(\pi, \pi') = 0$, one has $\mu_1(\pi', p) \leq 1$, $\mu_2(\pi', p) \leq M$, and $\mu_3(\pi', p) \leq 1$, as well as $\mu_1(\pi', \overline{p}) \geq M - 1$, and $\mu_3(\pi', \overline{p}) \geq M - 1$. An immediate recursion show the same is true for any $\pi''$ such that $c(s) = 0$ for some $s \in S(\pi, \pi'')$. Part $(ii)$ of assumption (II) ensures that any such $\pi''$ satisfies $\mathcal{F}_{\pi'', \overline{\pi}} > 0$. This clearly implies that any such $\pi''$ and in particular $\pi$ belong to $D(\overline{\pi})$. We have hence proven that $r(\overline{\pi}) > 3$.

This ends the proof.

Basic asymptotic properties of ergodic Markov chains yield a straightforward interpretation of theorem 1 : as the mutation rate tends towards zero, the frequency with which the system lies in equilibrium tends towards one.

## 4 Concluding Remarks

We have shown that in a framework where trading is governed by private prices and strategic behavior, agents who update their private prices by imitation and random mutation will eventually adopt the equilibrium price (but for vanishingly small perturbations) and obtain their equilibrium allocation in the Scarf economy. Hence, we provide some analytical support to the result obtained in series of simulations performed by Gintis: (Gintis 2010) and (Gintis 2012). One should however take

[3]That is agents of type 2 might adopt price $p$ but not agents of type 1 and 3.

note that this result has only been obtained at the expense of a considerable simplification of Gintis' modelsec of market exchange. It is also the case that our results rely crucially on the fact that mutations are drawn from an uniform distribution and hence the remark in (Fudenberg and Harris 1992) is particularly relevant in our setting: *"Intuitively, the likelihood that a Wiener process will be able to swim upstream k meters against a deterministic flow depends both on the distance k and on the strength of the flow, while the probability that a discrete-time system jumps k or more meters over the flow in a single period depends on k but not on the strength of the flow. This explains the differences in the generality of the models conclusions, and suggests that long-run behavior may depend on the precise form of the deterministic dynamics in any model with continuous sample paths."* As a matter of fact, we would conjecture that our results hold for weaker sets of assumption than those put forward here but our current results suggest that the problem becomes more and more complex from the combinatorial point to view as these assumptions are relaxed.

Further investigations are however necessary to determine whether the results presented here and in (Gintis 2010) and (Gintis 2012) can be turned into the conjecture that the general equilibrium of an economy always is stochastically stable for a certain class of evolutionary dynamics.

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