Preface

> Was sich sagen läßt, läßt sich klar sagen, und wovon man nicht sprechen kann, darüber muß man schweigen.
>
> Ludwig Wittgenstein

This book is a problem-centered introduction to classical and evolutionary game theory. For most topics, I provide just enough in the way of definitions, concepts, theorems, and examples to begin solving problems. Learning and insight come from grappling with and solving problems. I provide extensive answers to some problems, sketchy and suggestive answers to most others. Students should consult the answers in the back of the book only to check their work. If a problem seems too difficult to solve, the student should come back to it a day, a week, or a month later, rather than peeking at the solution.

Game theory is logically demanding, but on a practical level, it requires surprisingly few mathematical techniques. Algebra, calculus, and basic probability theory suffice. However, game theory frequently requires considerable notation to express ideas clearly. The reader should commit to memory the precise definition of every term, and the precise statement of most theorems.

Clarity and precision do not imply rigor. I take my inspiration from physics, where sophisticated mathematics is common, but mathematical rigor is considered an impediment to creative theorizing. I stand by the truth and mathematical cogency of the arguments presented in this book, but not by their rigor. Indeed, the stress placed on game-theoretic rigor in recent years is misplaced. Theorists could worry more about the empirical relevance of their models and take less solace in mathematical elegance.

For instance, if a proposition is proved for a model with a finite number of agents, it is completely irrelevant whether it is true for an infinite number of agents. There are, after all, only a finite number of people, or even bacteria. Similarly, if something is true in games in which payoffs are finitely divisible (e.g., there is a minimum monetary unit), it does not matter whether it is true when payoffs are infinitely divisible. There are no payoffs in the universe, as far as we know, that are infinitely divisible. Even time, which is continuous in principle, can be measured only by devices with a finite number of quantum states. Of course, models based on the real and complex numbers can be hugely useful, but they are just approximations, because there are only a finite number of particles in the universe, and we can construct only a finite set of numbers, even in principle. There is thus no intrinsic value of a theorem that is true for a continuum of agents on a Banach space, if it is also true for a finite number of agents on a finite choice space.

Evolutionary game theory is about the emergence, transformation, diffusion, and stabilization of forms of behavior. Traditionally, game theory has been seen as a theory of how rational actors behave. Ironically, game theory, which for so long was predicated upon high-level rationality, has shown us, by example, the limited capacity of the concept of rationality alone to predict human behavior. I explore this issue in depth in *Bounds of Reason* (Princeton, 2009), which develops themes from epistemic game theory to fill in where classical game theory leaves off. Evolutionary game theory deploys the Darwinian notion that good strategies diffuse across populations of players rather than being learned by rational agents.

The treatment of rationality as preference consistency, a theme that we develop in chapter 2, allows us to assume that agents choose best responses, and otherwise behave as good citizens of game theory society. But they may be pigs, dung beetles, birds, spiders, or even wild things like Trogs and Klingons. How do they accomplish these feats with their small minds and alien mentalities? The answer is that the *agent* is displaced by the *strategy* as the dynamic game-theoretic unit.

This displacement is supported in three ways. First, we show that many static optimization models are stable equilibria of dynamic systems in which agents do not optimize, and we reject models that do not have attractive stability properties. To this end, after a short treatment of evolutionary stability, we develop dynamical systems theory (chapter 11) in sufficient depth to allow students to solve dynamic games with replicator dynamics (chapter 12). Second we provide animal as well as human models. Third, we provide agent-based computer simulations of games, showing that really stupid critters can evolve toward the solution of games previously thought to require "rationality" and high-level information processing capacity.

The Wittgenstein quote at the head of the preface means "What can be said, can be said clearly, and what you cannot say, you should shut up about." This adage is beautifully reflected in the methodology of game theory, especially epistemic game theory, which I develop in *The Bounds of Reason* (2009),and which gives us a language and a set of analytical tools for modeling an aspect of social reality with perfect clarity. Before game theory, we had no means of speaking clearly about social reality, so the great men and women who created the behavioral sciences from the dawn of the Enlightenment to the mid-twentieth century must be excused for the raging ideological battles that inevitably accompanied their attempt to talk about what could not be said clearly. If we take Wittgenstein seriously, it may be that those days are behind us.

Game Theory Evolving, Second Edition does not say much about how game theory applies to fields outside economics and biology. Nor does this volume evaluate the empirical validity of game theory, or suggest why rational agents might play Nash equilibria in the absence of an evolutionary dynamic with an asymptotically stable critical point. The student interested in these issues should turn to the companion volume, *The Bounds of Reason.*

Game Theory Evolving, Second Edition was composed on a word processor that I wrote in Borland Pascal, and the figures and tables were produced by a program that I wrote in Borland Delphi. The simulations are in Borland Delphi and C++Builder, and the results are displayed using SigmaPlot. I used NormalSolver, which I wrote in Delphi, to check solutions to many of the normal and extensive form games analyze herein. *Game Theory Evolving, Second Edition* was produced by LaTeX.

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