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## Evolutionarily Stable Strategies

There is but a step between the sublime and the ridiculous.

Leo Tolstoy

In 1973 the biologist John Maynard Smith and the mathematician G. R. Price wrote an article in *Nature* showing how game theory applies to the behavior of animals (Maynard Smith and Price 1973). Maynard Smith went on to write a book on the subject (Maynard Smith 1982), which has become a classic. The idea of applying game theory to animals, and not just the higher primates, but fish, dung beetles, fireflies, and pond scum as well, seemed strange at the time, because game theory had always been the preserve of hyperrationality. Animals hardly fit the bill. Maynard Smith made three critical shifts from traditional game theory. The first is in the concept of a strategy, the second in the concept of equilibrium, and a third in the nature of agent interactions.

**Strategy.** In classical game theory, *players* have strategy sets from which they choose particular strategies. In biology, *species* have strategy sets (genotypic variants), of which *individuals* inherit one or another variant, perhaps mutated, that they then play in their strategic interactions. This extends nicely to the treatment of culture in human society, in which case we say that *society* has the strategy set (the set of alternative cultural forms) and *individuals* inherit or choose among them.

**Equilibrium.** In place of the Nash equilibrium, Maynard Smith and Price used the *evolutionarily stable strategy* (ESS) concept. A strategy is an ESS if a whole population using that strategy cannot be invaded by a small group with a mutant genotype. Similarly, a cultural form is an ESS if, upon being adopted by all members of a society (firm, family, etc.), no small group of individuals using an alternative cultural form can invade. We thus move from explaining the actions of individuals to modeling the diffusion of forms of behavior (“strategies”) in society.

**Player interactions.** In place of the one-shot and repeated games of classical game theory, Maynard Smith introduced the notion of the *repeated*,

*random pairing of agents* who play strategies based on their genome but not on the previous history of play.

The ESS concept is particularly useful because it says something about the dynamic properties of a system without being committed to any particular dynamic model. As we shall see, however, an evolutionary system with a symmetrical two-player stage game can be dynamically stable without being an ESS (§12.9).

### 10.1 Evolutionarily Stable Strategies: Definition

Consider a two-player normal form game in which both players have the set  $S = \{s_1, \dots, s_n\}$  of pure strategies, and the payoffs to an agent playing  $s_i \in S$  and another agent playing  $s_j \in S$  are  $\pi_{ij}^1$  for the first and  $\pi_{ij}^2 = \pi_{ji}^1$  for the second. We call such a game *symmetric in payoffs*. In addition, we assume the agents cannot condition their play on whether they are “player 1” or “player 2.” We call such a game *symmetric in strategies*. If a game is symmetric in both payoffs and strategies, we simply call the game *symmetric*. We call  $A = (\pi_{ij}^1)$  the *matrix* of the symmetric game. Note that  $A$  represents only the payoffs for the row player, because the payoffs to the column player are just the transpose of  $A$ .

Let  $\mathcal{G}$  be a symmetric game with matrix  $A$  (we’ll call it the *stage game*) and large population of agents. In each period  $t = 1, 2, \dots$ , agents are randomly paired and they play the stage game  $\mathcal{G}$  once. Each agent is of type  $i$  for some  $s_i \in S$ , meaning that the agent uses strategy  $s_i$  in the stage game. If the proportion of agents of type  $j$  is  $p_j$  at a particular time, we say the *state* of the population is  $\sigma = p_1 s_1 + \dots + p_n s_n$ . Note that we must have  $p_1, \dots, p_n \geq 0$  and  $\sum_i p_i = 1$ . The payoff at that time to a player of type  $i$  when the state of the population is  $\sigma$  is defined by

$$\pi_{i\sigma} = \sum_{j=1}^n \pi_{ij} p_j, \quad (10.1)$$

which is the player’s expected payoff before being assigned a particular partner. These conditions define a new game, called the *evolutionary game* corresponding to the stage game  $\mathcal{G}$ .

Suppose the state of the population is  $\sigma$ , and some small subpopulation plays a “mutant” strategy  $\tau = q_1 s_1 + \dots + q_n s_n$ , in the sense that  $q_i$  is the frequency of pure strategy  $s_i$  in this subpopulation. We say the mutant is of

“type  $\tau$ ,” and has payoff

$$\pi_{\tau\sigma} = \sum_{i,j=1}^n q_i \pi_{ij} p_j,$$

when a random member of its population meets a random member of the population  $\sigma$ .

Suppose the state of the population is  $\sigma = p_1 s_1 + \dots + p_n s_n$ . The expected payoff to a randomly chosen member of the population is thus just  $\pi_{\sigma\sigma}$ . If we replace a fraction  $\epsilon > 0$  of the population with a “mutant” of type  $\tau$ , the new state of the population is

$$\mu = (1 - \epsilon)\sigma + \epsilon\tau,$$

so the payoff to a randomly chosen nonmutant is

$$\pi_{\sigma\mu} = (1 - \epsilon)\pi_{\sigma\sigma} + \epsilon\pi_{\sigma\tau},$$

and the expected payoff to a mutant is

$$\pi_{\tau\mu} = (1 - \epsilon)\pi_{\tau\sigma} + \epsilon\pi_{\tau\tau}.$$

We say the mutant type can *invade* the population if  $\sigma \neq \mu$  and for all sufficiently small  $\epsilon > 0$ ,

$$\pi_{\tau\mu} \geq \pi_{\sigma\mu},$$

which says that, on average, a mutant does at least as well against the new population as does a nonmutant. We say  $\sigma$  is an *evolutionarily stable strategy* (ESS) if it cannot be invaded by any mutant type, in a sense defined precisely below.

We assume that mutants can employ mixed strategies in applying the ESS criterion, because as we shall see later (§12.7), with this assumption evolutionarily stable strategies have powerful dynamic properties. A Nash equilibrium in an evolutionary game can consist of a *monomorphic* population of agents, each playing the same mixed strategy, or a *polymorphic* population, a fraction of the population playing each of the underlying pure strategies in proportion to its contribution to the mixed Nash strategy. The two interpretations are interchangeable under many conditions, and we shall not commit ourselves exclusively to either interpretation. Because the stage

game is a one-shot, it is rarely plausible to hold that an individual will play a strictly mixed strategy. Thus, in general, the heterogeneous population interpretation is superior. The heterogeneous mutant  $\tau$  must then possess some internal mechanism for maintaining the constant frequency distribution  $q_1, \dots, q_n$  from period to period. We relax this assumption when we treat evolutionary games as dynamical systems in chapter 12.

## 10.2 Properties of Evolutionarily Stable Strategies

Prove the following properties of evolutionarily stable strategies:

- a. Strategy  $\sigma \in \Delta S$  is an ESS if, for every mutant type  $\tau \in \Delta S$ , there is an  $\epsilon_\tau > 0$  such that for all  $\epsilon \in (0, \epsilon_\tau)$  and defining  $\mu = (1 - \epsilon)\sigma + \epsilon\tau$ , we have

$$\pi_{\sigma\mu} > \pi_{\tau\mu}. \quad (10.2)$$

- b. We say that  $\sigma \in \Delta S$  has a *uniform invasion barrier* if there is some  $\epsilon_o \in (0, 1)$  such that (10.2) holds for all  $\tau \neq \sigma$  and all  $\epsilon \in (0, \epsilon_o)$ . Strategy  $\sigma$  is an ESS if and only if it has a uniform invasion barrier.
- c. Strategy  $\sigma \in \Delta S$  is an ESS if and only if, for any mutant type  $\tau \in \Delta S$ , we have

$$\pi_{\sigma\sigma} \geq \pi_{\tau\sigma},$$

and if  $\pi_{\sigma\sigma} = \pi_{\tau\sigma}$ , then

$$\pi_{\sigma\tau} > \pi_{\tau\tau}.$$

This says that  $\sigma \in \Delta S$  is an ESS if and only if a mutant cannot do better against an incumbent than an incumbent can do against another incumbent, and if a mutant does as well as an incumbent against another incumbent, then an incumbent must do better against a mutant than a mutant does against another mutant. Note here that we are assuming mutants can use mixed strategies.

- d. An evolutionarily stable strategy is a Nash equilibrium that is isolated in the set of symmetric Nash equilibria (that is, it is a strictly positive distance from any other symmetric Nash equilibrium).
- e. Every strict Nash equilibrium in an evolutionary game is an ESS.

### 10.3 Characterizing Evolutionarily Stable Strategies

**THEOREM 10.1** *Suppose symmetric two-player game  $\mathcal{G}$  has two pure strategies. Then, if  $\pi_{11} \neq \pi_{21}$  and  $\pi_{12} \neq \pi_{22}$ ,  $\mathcal{G}$  has an evolutionarily stable strategy.*

**PROOF:** Suppose  $\pi_{11} > \pi_{21}$ . Then, pure strategy 1 is a strict Nash equilibrium, so it is an evolutionarily stable strategy. The same is true if  $\pi_{22} > \pi_{12}$ . So suppose  $\pi_{11} < \pi_{21}$  and  $\pi_{22} < \pi_{12}$ . Then, we can show that the game has a unique completely mixed symmetric equilibrium  $p$ , where each player uses strategy 1 with probability  $\alpha_p \in (0, 1)$ . The payoff to strategy 1 against the mixed strategy  $(\alpha_p, 1 - \alpha_p)$  is then  $\alpha_p \pi_{11} + (1 - \alpha_p) \pi_{12}$ , and the payoff to strategy 2 against this mixed strategy is  $\alpha_p \pi_{21} + (1 - \alpha_p) \pi_{22}$ . Because these must be equal, we find that  $\alpha_p = (\pi_{22} - \pi_{12}) / \Delta$ , where  $\Delta = \pi_{11} - \pi_{21} + \pi_{22} - \pi_{12} < 0$ . Note that under our assumptions,  $0 < \alpha_p < 1$ , so there is a unique completely mixed Nash equilibrium  $(\alpha_p, 1 - \alpha_p)$ .

Now let  $\alpha_q$  be the probability a mutant player uses pure strategy 1. Because each pure strategy is a best response to  $\alpha_p$ ,  $\alpha_q$  must also be a best response to  $\alpha_p$ , so clearly,  $\pi_{qp} = \pi_{pp}$ . To show that  $p$  is an ESS, we must show that  $\pi_{pq} > \pi_{qq}$ . We have

$$\pi_{pq} = \alpha_p [\pi_{11} \alpha_q + \pi_{12} (1 - \alpha_q)] + (1 - \alpha_p) [\pi_{21} \alpha_q + \pi_{22} (1 - \alpha_q)]$$

and

$$\pi_{qq} = \alpha_q [\pi_{11} \alpha_q + \pi_{12} (1 - \alpha_q)] + (1 - \alpha_q) [\pi_{21} \alpha_q + \pi_{22} (1 - \alpha_q)].$$

Subtracting and simplifying, we get

$$\pi_{pq} - \pi_{qq} = -(\alpha_p - \alpha_q)^2 \Delta > 0,$$

which proves we have an ESS. ■

**THEOREM 10.2** *Using the same notation, the stage game has a strictly mixed Nash equilibrium if and only if  $\pi_{11} > \pi_{21}$  and  $\pi_{22} > \pi_{12}$ , or  $\pi_{11} < \pi_{21}$  and  $\pi_{22} < \pi_{12}$ . The equilibrium is an ESS only if the second set of inequalities holds.*

PROOF: It is easy to check that if there is a mixed strategy equilibrium, the frequency  $\alpha$  of pure strategy 1 must satisfy

$$\alpha = \frac{\pi_{22} - \pi_{12}}{\Delta}, \quad \text{where } \Delta = \pi_{11} - \pi_{21} + \pi_{22} - \pi_{12}.$$

Suppose  $\Delta > 0$ . Then  $0 < \alpha < 1$  if and only if  $0 < \pi_{22} - \pi_{12} < \pi_{11} - \pi_{21} + \pi_{22} - \pi_{12}$ , which is true if and only if  $\pi_{11} > \pi_{21}$  and  $\pi_{22} > \pi_{12}$ . If  $\Delta < 0$ , a similar argument shows that  $0 < \alpha < 1$  if and only if the other pair of inequalities holds.

Suppose there is a “mutant” that uses pure strategy 1 with probability  $\beta$ . Thus, in general,

$$\begin{aligned} \pi_{\gamma\delta} &= \gamma\delta\pi_{11} + \gamma(1-\delta)\pi_{12} + (1-\gamma)\delta\pi_{21} + (1-\gamma)(1-\delta)\pi_{22} \\ &= \gamma\delta\Delta + \delta(\pi_{21} - \pi_{22}) + \gamma(\pi_{12} - \pi_{22}) + \pi_{22}. \end{aligned}$$

It follows that

$$\pi_{\alpha\alpha} - \pi_{\beta\alpha} = (\alpha - \beta)[\alpha\Delta - (\pi_{22} - a_{12})] = 0,$$

so the equilibrium is an ESS if and only if  $\pi_{\alpha\beta} > \pi_{\beta\beta}$ . But

$$\begin{aligned} \pi_{\alpha\beta} - \pi_{\beta\beta} &= \alpha\beta\Delta + \beta(a_{21} - a_{22}) + \alpha(a_{12} - a_{22}) + a_{22} \\ &\quad - \beta^2\Delta - \beta(a_{21} - a_{22}) - \beta(a_{12} - a_{22}) - a_{22} \\ &= \beta(\alpha - \beta)\Delta + (\alpha - \beta)(a_{12} - a_{22}) \\ &= (\alpha - \beta)(\beta\Delta + a_{12} - a_{22}) \\ &= (\alpha - \beta)(\beta\Delta - \alpha\Delta) \\ &= -(\alpha - \beta)^2\Delta. \end{aligned}$$

Thus, the equilibrium is an ESS if and only if  $\Delta < 0$ , which is equivalent to  $a_{11} < a_{21}$  and  $a_{22} < a_{12}$ . This proves the assertion. ■

**THEOREM 10.3** *Suppose  $\sigma = \alpha_1 s_1 + \dots + \alpha_n s_n \in \Delta S$  is an ESS, where  $s_i$  is a pure strategy and  $\alpha_i > 0$  for  $i = 1, \dots, n$ . Suppose  $\tau = \beta_1 s_1 + \dots + \beta_n s_n \in \Delta S$  is also an ESS. Then,  $\beta_i = \alpha_i$  for  $i = 1, \dots, n$ . In other words, the support of an ESS cannot strictly contain the support of another ESS.*

**THEOREM 10.4** *If  $\sigma \in \Delta S$  is weakly dominated, then  $\sigma$  is not an ESS.*

**THEOREM 10.5** *An evolutionary game whose stage game has a finite number of pure strategies can have only a finite number of evolutionarily stable strategies.*

**PROOF:** Suppose there are an infinite number of distinct evolutionarily stable strategies. Then there must be two, say  $\sigma$  and  $\tau$ , that use exactly the same pure strategies. Now  $\tau$  is a best response to  $\sigma$ , so  $\sigma$  must do better against  $\tau$  than  $\tau$  does against itself. But  $\sigma$  does equally well against  $\tau$  as  $\tau$  does against  $\tau$ . Thus,  $\sigma$  is not an ESS and similarly for  $\tau$ . ■

By the *distance* between two strategies  $\sigma = \sum_i p_i s_i$  and  $\tau = \sum_i q_i s_i$ , we mean the distance in  $\mathbf{R}^n$  between the points  $(p_1, \dots, p_n)$  and  $(q_1, \dots, q_n)$ . The following is proved in Hofbauer and Sigmund (1998). Note that the theorem implies that an evolutionarily stable strategy  $\sigma$  is an *isolated Nash equilibrium*, in the sense that there is an  $\epsilon > 0$  such that no strategy  $\tau \neq \sigma$  within distance  $\epsilon$  of  $\sigma$  is a Nash equilibrium.

**THEOREM 10.6** *Strategy  $\sigma \in \Delta S$  is an ESS if and only if there is some  $\epsilon > 0$  such that  $\pi_{\sigma\tau} > \pi_{\tau\tau}$  for all  $\tau \in \Delta S$  within distance  $\epsilon$  of  $\sigma$ .*

**PROOF:** Suppose  $\sigma$  is an ESS, so for any  $\tau \neq \sigma$ , there is an  $\tilde{\epsilon}(\tau)$  such that

$$\pi_{\tau, (1-\epsilon)\sigma + \epsilon\tau} < \pi_{\sigma, (1-\epsilon)\sigma + \epsilon\tau} \quad \text{for all } \epsilon \in (0, \tilde{\epsilon}(\tau)). \quad (10.3)$$

In fact, we can choose  $\tilde{\epsilon}(\tau)$  as follows. If (10.3) holds for all  $\epsilon \in (0, 1)$ , then let  $\tilde{\epsilon}(\tau) = 1$ . Otherwise, let  $\tilde{\epsilon}$  be the smallest  $\epsilon > 0$  such that (10.3) is violated and define

$$\tilde{\epsilon}(\tau) = \frac{\pi_{\sigma\sigma} - \pi_{\tau\sigma}}{\pi_{\tau\tau} - \pi_{\tau\sigma} - \pi_{\sigma\tau} + \pi_{\sigma\sigma}}.$$

It is easy to check that  $\tilde{\epsilon}(\tau) \in (0, 1]$  and (10.3) are satisfied. Let  $T \subset S$  be the set of strategies such that if  $\tau \in T$ , then there is at least one pure strategy used in  $\sigma$  that is not used in  $\tau$ . Clearly,  $T$  is closed and bounded,  $\sigma \notin T$ ,  $\tilde{\epsilon}(\tau)$  is continuous, and  $\tilde{\epsilon}(\tau) > 0$  for all  $\tau \in T$ . Hence,  $\tilde{\epsilon}(\tau)$  has a strictly positive minimum  $\epsilon^*$  such that (10.3) holds for all  $\tau \in T$  and all  $\epsilon \in (0, \epsilon^*)$ .

If  $\tau$  is a mixed strategy and  $s$  is a pure strategy, we define  $s(\tau)$  to be the weight of  $s$  in  $\tau$  (that is, the probability that  $s$  will be played using  $\tau$ ). Now consider the neighborhood of  $s$  consisting of all strategies  $\tau$  such that  $|1-s(\tau)| < \epsilon^*$  for all pure strategies  $s$ . If  $\tau \neq s$ , then  $\epsilon^* > 1-s(\tau) = \epsilon > 0$

for some pure strategy  $s$ . Then  $\tau = (1 - \epsilon)s + \epsilon r$ , where  $r \in T$ . But then (10.3) gives  $\pi_{r\tau} < \pi_{s\tau}$ . If we multiply both sides of this inequality by  $\epsilon$  and add  $(1 - \epsilon)\pi_{s\tau}$  to both sides, we get  $\pi_{\tau\tau} < \pi_{s\tau}$ , as required. The other direction is similar, which proves the assertion. ■

**THEOREM 10.7** *If  $\sigma \in \Delta S$  is a completely mixed evolutionarily stable strategy (that is, it uses all pure strategies with positive probability), then it is the unique Nash equilibrium of the game and  $\pi_{\sigma\tau} > \pi_{\tau\tau}$  for all  $\tau \in \Delta S$ ,  $\tau \neq \sigma$ .*

**PROOF:** If  $\sigma$  is completely mixed, then for any  $\tau \in S$ ,  $\pi_{\sigma\sigma} = \pi_{\tau\sigma}$ , because any pure strategy has the same payoff against  $\sigma$  as  $\sigma$  does against  $\sigma$ . Therefore, any mixed strategy has the same payoff against  $\sigma$  as  $\sigma$  has against  $\sigma$ . For similar reasons,  $\pi_{\sigma\tau} = \pi_{\sigma\sigma}$ . Thus,  $\sigma$  is an ESS and if  $\tau$  is any other strategy, we must have  $\pi_{\sigma\tau} > \pi_{\tau\tau}$ . ■

#### 10.4 A Symmetric Coordination Game

Consider a two-player pure coordination game in which both players win  $a > 0$  if they both choose Up, and they win  $b > 0$  if they both choose Down, but they get nothing otherwise. Show that this game has a mixed-strategy equilibrium with a lower payoff than either of the pure-strategy equilibria. Show that this game is symmetric, and the mixed-strategy equilibrium is not an ESS. Show that there are, however, two ESSs. This example shows that sometimes adding the ESS requirement eliminates implausible and inefficient equilibria.

#### 10.5 A Dynamic Battle of the Sexes

The battle of the sexes (§3.9) is not symmetric, and hence the concept of an evolutionarily stable strategy does not apply. However, there is an obvious way to recast battle of the sexes so that it becomes symmetric. Suppose when two players meet, one is randomly assigned to be player 1, and the other player 2. A pure strategy for a player can be written as “xy,” which means “if I am Alfredo, I play x, and if I am Violetta, I play y.” Here x stands for Opera and y stands for Gambling. There are thus four pure strategies, OO, OG, GO, GG. This game is symmetric, and the normal form matrix (only the payoff to player 1 is shown) is

	OO	OG	GO	GG
OO	3/2,3/2	1,1/2	1/2,1	0,0
OG	1/2,1	0,0	1,2	1/2,1/2
GO	1,1/2	2,1	0,0	1,1/2
GG	0,0	1,1/2	1/2,1	3/2,3/2

Let  $\alpha \geq 0$ ,  $\beta \geq 0$ ,  $\gamma \geq 0$  and  $\delta = 1 - \alpha - \beta - \gamma \geq 0$  be the fraction of players who use strategy OO, OG, GO, and GG, respectively (or, equivalently, let  $(\alpha, \beta, \gamma, \delta)$  be the mixed strategy of each player). Show that there are two pure-strategy Nash equilibria, OO and GG, and for each  $\alpha \in [0, 1]$ , there is a mixed-strategy Nash equilibrium  $\alpha\text{OO} + (1/3 - \alpha)\text{OG} + (2/3 - \alpha)\text{GO} + \alpha\text{GG}$ . Show that the payoffs to these equilibria are 3/2 for the pure-strategy equilibria and 2/3 for each of the mixed-strategy equilibria. It is easy to show that the first two equilibria are ESSs, and the others are not—they can be invaded by either OO or GG.

### 10.6 Symmetrical Throwing Fingers

Similarly, although throwing fingers (§3.8) is not a symmetric game, and hence the concept of an evolutionarily stable strategy does not apply, there is an obvious way to recast throwing fingers so that it becomes symmetric. Suppose when two players meet, one is randomly assigned to be player 1, and the other player 2. A pure strategy for a player can be written as “xy,” which means “if I am player 1, I show x fingers, and if I am player 2, I show y fingers.” There are thus four pure strategies, 11, 12, 21, and 22. Show that this game is symmetric, and derive the normal form matrix (only the payoff to player 1 is shown)

	11	12	21	22
11	0,0	-1,1	1,-1	0,0
12	1,-1	0,0	0,0	-1,1
21	-1,1	0,0	0,0	1,-1
22	0,0	1,-1	-1,1	0,0

Let  $\alpha \geq 0$ ,  $\beta \geq 0$ ,  $\gamma \geq 0$  and  $\delta = 1 - \alpha - \beta - \gamma \geq 0$  be the fraction of players who use strategy 11, 12, 21, and 22, respectively (or, equivalently, let  $(\alpha, \beta, \gamma, \delta)$  be the mixed strategy of each player). Show that a Nash

equilibrium is characterized by  $\alpha = 1/2 - \gamma$ ,  $\beta = \gamma$  (which implies  $\delta = \alpha$ ). It is easy to show that any such Nash equilibrium can be invaded by any distinct strategy  $(\alpha', \beta', \gamma', \delta')$  with  $\alpha' = 1/2 - \gamma'$ ,  $\beta' = \gamma'$ , so there is no evolutionarily stable strategy for throwing fingers.

### 10.7 Hawks, Doves, and Bourgeois

**THEOREM 10.8** *The mixed-strategy equilibrium in the hawk-dove game (§3.10) is an ESS.*

**PROOF:** The payoff to  $H$  is  $\alpha(v - w)/2 + (1 - \alpha)v = v - \alpha(v + w)/2$ , and the payoff to  $D$  is  $(1 - \alpha)(v/2) = v/2 - \alpha(v/2)$ . These are equated when  $\alpha = v/w$ , which is  $< 1$  if  $w > v$ . To show that this mixed-strategy equilibrium is an ESS, note that  $\pi_{11} = (v - w)/2$ ,  $\pi_{21} = 0$ ,  $\pi_{22} = v/2$ , and  $\pi_{12} = v$ . Thus  $\pi_{11} = (v - w)/2 < 0 = \pi_{21}$  and  $\pi_{22} = v/2 < v = \pi_{12}$ , so the equilibrium is an ESS. ■

Note that in the hawk-dove-bourgeois game (§6.41), the bourgeois strategy is a strict Nash equilibrium, and hence is an ESS.

### 10.8 Trust in Networks II

We now show that the completely mixed Nash equilibrium found in trust in networks (§6.23) is not an ESS and can be invaded by trusters. In case you think this means this equilibrium is dynamically unstable, think again! See Trust in Networks III (§12.10).

For specificity, we take  $p = 0.8$ . You can check that the equilibrium has inspect share  $\alpha^* \approx 0.71$  trust share  $\beta^* \approx 0.19$ , and defect share  $\gamma^* \approx 0.10$ . The payoff to the equilibrium strategy  $s$  is  $\pi_{ss} \approx 0.57$ . The payoff to trust against the equilibrium strategy is of course  $\pi_{ts} = \pi_{ss} \approx 0.57$ , but the payoff to trust against itself is  $\pi_{tt} = 1$ , so trust can invade.

### 10.9 Cooperative Fishing

In a certain fishing village, two fisherman gain from having the nets put out in the evening. However, the fishermen benefit equally whether or not they share the costs of putting out the nets. Suppose the expected catch is  $v$ , the cost of putting out the nets to each is  $c_1$  if each fisherman does it alone, and the cost to each is  $c_2 < c_1$  if they do it together. We assume  $v/2 > c_1$ , so it