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Whom or What Does the Representative Individual Represent?

Alan P. Kirman

A modern economy presents a picture of millions of people, either as individuals or organized into groups and firms, each pursuing their own disparate interests in a rather limited part of the environment. Somehow, these varied individual activities are more or less coordinated and some relative order emerges. Economists commonly explain that this is due to Adam Smith's "invisible hand," and that despite the conflicting interests of individuals, the result of the pursuit of their selfish ends is socially satisfactory. The market provides the mechanism which links and coordinates all the activities being pursued by individuals.

Paradoxically, the sort of macroeconomic models which claim to give a picture of economic reality (albeit a simplified picture) have almost no activity which needs such coordination. This is because typically they assume that the choices of all the diverse agents in one sector—consumers for example—can be considered as the choices of one "representative" standard utility maximizing individual whose choices coincide with the aggregate choices of the heterogeneous individuals.

My basic point in this paper is to explain that this reduction of the behavior of a group of heterogeneous agents *even if they are all themselves utility maximizers*, is not simply an analytical convenience as often explained, but is both unjustified and leads to conclusions which are usually misleading and often wrong. Why is this? First, such models are particularly ill-suited to studying macroeconomic problems like unemployment, which should be viewed as coordination

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failures. Furthermore these models, instead of being a hive of activity and exchange, are frequently, as Varian (1987) points out, ones in which no trade at all takes place. Indeed, one can cite a whole series of “no trade” theorems (Rubenstein, 1975; Hakansson et al., 1982; Milgrom and Stokey, 1982; and others). In such a world there would be no meaningful stock market, distributional considerations could not enter government policy and the very idea of asymmetric information would make little sense.

If, however, it were simply this that was wrong with the representative individual, then the macroeconomist would have an easy defense. Such models, it could be argued, are not intended to study those problems which involve, in an essential way, questions of coordination but are designed to examine some central macroeconomic phenomena. I will argue that this position is untenable for four reasons.

First, whatever the objective of the modeler, there is no plausible formal justification for the assumption that the aggregate of individuals, even maximizers, acts itself like an individual maximizer. Individual maximization does *not* engender collective rationality, nor does the fact that the collectivity exhibits a certain rationality necessarily imply that individuals act rationally. There is simply no direct relation between individual and collective behavior.

Secondly, even if we accept that the choices of the aggregate can be considered as those of a maximizing individual, there is a different problem. The reaction of the representative to some change in a parameter of the original model—a change in government policy for example—may not be the same as the aggregate reaction of the individuals he “represents.” Hence using such a model to analyze the consequences of policy changes may not be valid.

Thirdly, even if we are in the highly unlikely situation in which these two criticisms do not apply, a fundamental difficulty remains. The “representative individual” whose choices coincide with the aggregate choices of the individuals in the economy is a utility maximizer. However it may well be the case that in two situations of which the representative prefers the first to the second, every individual prefers the second to the first. Thus the preferences of the representative individual cannot legitimately be used to decide whether one economic situation is “better” than another.

Lastly, when used as a model for empirical testing, the representative agent presents a peculiar disadvantage. Trying to explain the behavior of a group by that of one individual is constraining. The sum of the behavior of simple economically plausible individuals may generate complicated dynamics, whereas constructing one individual whose behavior has these dynamics may lead to that individual having very unnatural characteristics. Furthermore, if one rejects a particular behavioral hypothesis, it is not clear whether one is really rejecting the hypothesis in question, or rejecting the additional hypothesis that there is only one individual. I will give some examples of these difficulties. I will examine each of these criticisms in turn and then, to relieve the rather negative nature of this discussion, consider some more positive alternatives to the

representative individual approach. In particular, I will argue that heterogeneity of agents may, in fact, help to save the standard model.

However, I would go further and suggest that the way to develop appropriate microfoundations for macroeconomics is not to be found by starting from the study of individuals in isolation, but rests in an essential way on studying the aggregate activity resulting from the direct interaction between different individuals. Even if this is too ambitious a project in the short run, it is clear that the “representative” agent deserves a decent burial, as an approach to economic analysis that is not only primitive, but fundamentally erroneous.

A Basic Question

Before declaring the representative individual defunct, a simple question must be answered. If macroeconomists are interested only in certain basic macroeconomic problems which do not directly involve considerations of distribution or coordination, why do they bother to construct representative individual models?

I claim that economists have been forced into doing this by their insistence on “satisfactory” microfoundations. This has occurred in two stages. First, a widespread and growing conviction developed amongst economists that they have an adequate model for individual behavior, namely that of the constrained maximizer. It was therefore natural to wish to build macro-models based on maximizing individuals. This is not quite as simple as it sounds, since frequently it is required that the equilibrium of the economy be unique and stable. The stability of equilibria justifies, at least superficially,¹ the sort of account often given by economists as to how economies arrive at equilibria. The uniqueness of equilibrium legitimizes the use of comparative statics to evaluate the effects of changes, particularly policy changes, on the economy. These properties, however, depend on characteristics of the aggregate excess demand of the economy. The textbook individual’s excess demand has these properties but the passage from that observation to ensuring that the same properties hold at the aggregate level is more than delicate.

Without any precise results on the relation between the properties of individual and aggregate demand behavior, the easiest way to proceed was simply to assume that the whole economy behaved as one individual. One way to justify this was to suggest that, even though the agents in the economy might be very heterogeneous, aggregate behavior could effectively be described by the behavior of a “representative” individual. The naive expression of this is to say that if all individuals have certain properties then so must the aggregate.

¹I use the word “superficially” since economists have no adequate model of how individuals and firms actually adjust prices in a competitive model. If all the participants are price-takers by definition, then the actor who adjusts prices to eliminate excess demand is not specified.

Versions of this idea date back at least as far as Edgeworth (1881) who, in discussing Jevons, said, of the couple of agents discussed, "Each is in Berkleian phase a 'representative particular'; an individual dealer only is presented, but there is presupposed a class of competitors in the background." At least in Edgeworth's case there were two representative agents who traded with each other. The modern macroeconomic literature commonly assumes only one such individual.

An alternative and more rigorous approach is to make particular assumptions about individuals which guarantee that the collectivity will indeed also act as an individual. These assumptions are so special that few economists would consider them plausible. Typical examples are that all individuals should have identical homothetic utility functions (that is, ones with linear Engel curves); or that all individuals should have homothetic utility functions, not necessarily identical, but that the relative income distribution should be fixed and independent of prices.²

Supposing for a moment that the formal conditions for the existence of a representative agent were satisfied, then we would be back to the no-trade problem. Now economics has a traditional way, not always explicit, of reconciling this "representative" approach with the fact that we do, in fact, observe extensive trading between different agents. This is based on the idea that the economy is essentially in equilibrium all the time, with the same production and consumption taking place. Thus, all basic production and consumption can be subsumed under the activity of one amoeba-like individual who owns the one firm and consumes what it produces. In this view, all the genuinely individual activity, the arbitrage involved in searching for and seizing on profitable opportunities, reflects movement around equilibrium. Thus, the single individual is just a convenient fiction which satisfactorily describes the basic evolution of the economy.

However, this approach cannot be accepted unless the economy, when it moves away from the path that it would follow if it really did consist of one individual, moves back to a stable equilibrium under some "reasonable" adjustment process such as "tâtonnement." If this does not hold—and in general, it will not—then the "representative" individual is being used to *provide* the stability and the uniqueness of equilibria which are not *guaranteed* by the underlying model.³ In fact, it is contradictory to begin with a single

²This is the so-called "exact aggregation problem." See, for example, Gorman (1953), Eisenberg (1961), Lau (1982), Jorgenson et al. (1982) and Lewbel (1989). The latter gives the most general conditions, but even these are extremely restrictive.

³This applies to the standard suggestions that well-informed individuals are constantly doing the necessary arbitrage to bring the economy back to its equilibrium. If this is the case, individuals must differ, at least in their information. Once this is so, one has again to *prove* that the arbitrage activity will lead back to equilibrium. As Stiglitz (1989) points out, simply to assume this is wholly unwarranted.

representative agent and then to envisage different individual actions which lead the economy back to equilibrium.

The Basic Theoretical Problem

The motivations for the extensive use of the representative agent are the desires to provide microfoundations for aggregate behavior, and also to provide a framework in which equilibria are unique and stable. Without stability the force of an equilibrium concept is greatly reduced for, as Morishima (1984) writes:

If economists successfully devise a correct general equilibrium model, even if it can be proved to possess an equilibrium solution, should it lack the institutional backing to realize an equilibrium solution, then that equilibrium solution will amount to no more than a utopian state of affairs which bears no relation whatsoever to the real economy.

If those equilibria which do exist are not unique, then the use of comparative statics makes little sense.

The needs for microfoundations and for a stable and unique equilibrium would not seem, by themselves, to justify the use of such an artificial device as the representative agent. The simple answer would be to find conditions implied by assumptions on the individuals in an economy which guarantee uniqueness and stability. However, a series of results starting with those of Sonnenschein (1972) and Debreu (1974) show unequivocally that no such conditions exist.⁴ Let me explain these results in the context of the simplest of all cases, the exchange economy.⁵ In such an economy we make all the standard assumptions on the individual consumers, so that each agent is characterized by textbook indifference curves and a positive bundle of initial endowments of all goods. From this combination of tastes and endowments is derived a demand function, and subtracting the initial endowments from the demand curve gives the excess demand curve for each individual. Summing

⁴The conditions for uniqueness and stability mentioned previously are not like the standard conditions on individuals, like strict convexity and monotonicity of preferences, but involve assumptions about how individuals are related to each other—for example, that they should have *identical* preferences.

⁵Alert readers may suspect that they are being cheated since, in an exchange economy, the distribution of income may be very directly affected by changes in relative prices and this might be a source of instability. If individuals' holdings are concentrated in certain commodities, the changes in relative prices will have an important effect on them. This means that the aggregate "income effect" matrix may have very positive elements and produce multiple unstable equilibria. However, as Kehoe (1985) points out, this situation is made worse, not better, by the introduction of production.

over all individuals, of whom it is assumed there are only a finite number, provides the excess demand curve for society as a whole. Under certain not-very-restrictive conditions, three properties will carry over from the individual's excess demand curve to the aggregate demand curve: continuity; that the value of total excess demand must equal zero at all positive prices, i.e. that the budget constraint for the economy as a whole be satisfied (Walras' law); and that excess demand is homogeneous of degree zero (only relative prices count).

However, the results in question⁶ also show that these three properties are the *only* properties that carry over from the individual to the aggregate demand function.⁷ In particular this shows that the Weak Axiom of Revealed Preference may not be satisfied at the aggregate level, i.e. the collectivity may choose x when y was available in one situation but then choose y when x was available in another, something which cannot happen in the case of a textbook individual. Yet we know that, if we are to obtain uniqueness and stability of equilibria, some such restriction must be imposed. Even if the class of admissible preferences is restricted even further—for example, if it were required that they be homothetic, i.e. that every individual have linear Engel curves—Mantel (1976) has shown that the same situation obtains. Thus the appropriate properties cannot be obtained from assumptions on the individuals in the economy.

Now if the behavior of the economy *could* be represented as that of an individual, the situation would be saved, since textbook individual excess demand functions *do* have unique and stable equilibria. This is where the representative individual comes into the picture. By making such an assumption directly, macroeconomists conveniently circumvent these difficulties, or put alternatively, since they wish to provide rigorous microfoundations and they wish to use the uniqueness and stability of equilibrium and are aware of the Sonnenschein-Debreu-Mantel result, they see this as the only way out.

Having examined the negative results which have driven those wishing to reconcile rigor, individual maximization, uniqueness and stability into the

⁶To state Debreu's (1974) theorem more precisely, have each agent characterized by textbook indifference curves and a positive bundle, called $e(a)$, of initial endowment of all goods. From these is derived a well-behaved demand $\phi(a, p)$ and excess demand $z(a, p) = \phi(a, p) - e(a)$. Summing over all the individuals a , of whom we assume there are a finite number, gives aggregate excess demand $Z(p)$. If only prices greater than some positive ϵ are considered, which may be chosen as small as we like, three basic properties of individual excess demand carry over to $Z(p)$:

1) $Z(p)$ is continuous.

2) $Z(p)$ satisfies Walras' Law, i.e. $p \cdot Z(p) = 0$.

3) $Z(p)$ is homogeneous of degree 0, i.e. $Z(\lambda p) = Z(p)$ for any positive λ .

Debreu found that given any function $f(p)$ satisfying properties 1–3 we can find individuals with strictly convex and monotonic preferences and positive initial endowments whose aggregate excess demand $Z(p)$ is equal to $f(p)$ for all prices greater than ϵ . This means that the only properties the aggregate excess demand of an economy can have are the three given above.

⁷The reader will observe that, for Debreu's result, prices are bounded away from zero. Balasko (1986) has argued that since we do know something, with our assumptions, about the behavior of excess demand functions when some prices go to zero, the class of such functions is much smaller than would be suggested by the Sonnenschein-Debreu results. This discussion does not seem to me to materially affect the basic point that I am making.

strait-jacket of the representative agent model, one slightly more positive remark is in order. With the standard textbook individualistic assumptions, competitive equilibria are “locally unique,” that is there is no equilibrium allocation “very near” another one (see Debreu, 1970). Thus, if there are many equilibria, one could argue as follows. When the economy starts from one of the equilibria which is stable and a small change occurs, it will go to that equilibrium in the changed economy which is “near” the old one. Therefore uniqueness is not the problem it is sometimes made out to be. This argument, unfortunately, is full of pitfalls and in any event is certainly not one generally used by macroeconomists.

Can the Representative Individual Disagree with the Agents?

Consider the most favorable situation, that in which we can construct an individual whose utility-maximizing choices correspond to the aggregate choices of the individual in an economy. Even in this case the representative agent can lead to misleading policy analyses. In models with a representative consumer, one makes the policy change and then examines the new equilibrium for the representative. However, there is an implicit assumption here that, after the change, the choice of the representative will still coincide with the aggregate choice of the individuals in the economy. The acknowledgement of this assumption usually comes in some caveat, such as “we will ignore distributional considerations.”

Yet, the change involved will frequently affect individuals differently. Indeed, many policy changes have this as their objective. As soon as this is the case, the representative constructed *before* the change may no longer represent the economy *after* the change. Rather than cite examples in which such a procedure is wrongly used, let me give two where the authors themselves point out this difficulty. Geweke (1985) constructs an example in which the effects of subsidies to production are miscalculated if the representative agent approach is used. Another example is given by Kupiec and Sharpe (1991) who look at the suggestion that the introduction of margin requirements will reduce the volatility of stock market prices. They construct an economy with a representative agent. However, the new margin requirements only affect the behavior of certain of the individuals in the economy. In the new modified economy we can again construct a representative agent, but he will be different from the old one, since the latter’s choices no longer coincide with the aggregate choice after the policy change. Thus, as the authors show, to have studied the effect of the introduction of margin requirements on the original representative individual would have led to misleading conclusions.

A standard “trick” in macroeconomics will not work in such cases. Since the equilibrium in the basic competitive model is Pareto optimal, then one can always take the supporting prices at such an equilibrium and make them

tangent to the indifference curves of a textbook individual. This individual's *equilibrium* will then correspond to that of the collectivity. It is clear that this individual is less representative than the one discussed previously, since only his equilibrium, and not his choices in general, coincide with society's. It is not difficult to see that if a policy change is made there is, in general, no reason to believe that his new equilibrium now coincides with that of society.⁸

Now let us ignore the previous problem and suppose that the aggregate choice of society does coincide with that of the representative individual, both before and after that change. This reflects a pious hope, but at least with this heroic assumption we should be able to use the model to make policy recommendations. Since the economy's behavior is properly represented by one individual, it might seem that we merely have to ask, which of the two possible outcomes, that before the change or that after, does the representative agent prefer?

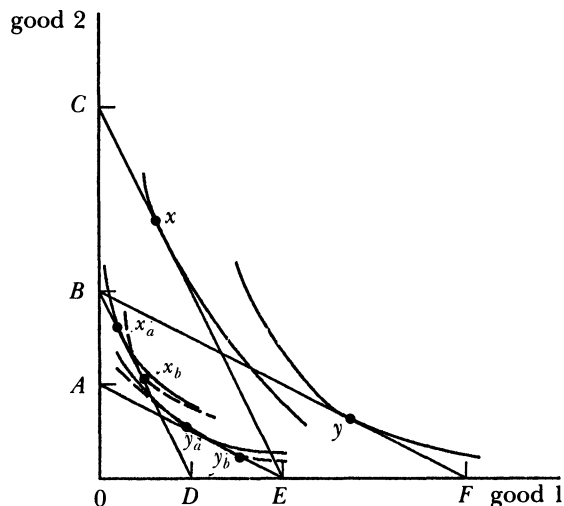
However, this reasoning contains a fatal flaw. It is possible that the representative individual prefers situation *a* to situation *b*, whilst *all* the individuals that are "represented" strictly prefer *b* to *a*. Even though the representative individual makes the same choices as the aggregate choices of the individuals in the economy, the preferences of that agent may be completely at variance with theirs! The first numerical example of this was due to Jerison (1984), and illustrates this very clearly. He considers two individuals with preferences similar to Cobb-Douglas and fixed shares of total income. He then gives a numerical example of two situations *a* and *b* with corresponding total incomes and prices (R_a, p_a) , and (R_b, p_b) . The aggregate individual prefers the bundle he chooses in situation *a* to that in situation *b*, whilst *both* the individuals prefer the bundle they choose in situation *b* to that which they choose in situation *a*.

To understand the intuition behind this result, look at Figure 1. Two individuals, *a* who has solid indifference curves, and *b* who has dashed curves, are faced with the same budget constraint *AE*, and they make choices y_a and y_b respectively. When their budget constraint is given by *BD* they choose x_a and x_b respectively. Their aggregate choice in the first situation is given by *y* lying on the aggregate budget constraint *BF* and in the second *x* lying on the budget constraint *CE*. Now it is easy to see that the "representative individual" whose indifference curves are given by the heavier solid lines does indeed make the same choices as the sum of *a* and *b*'s choices. However, this representative prefers *y* to *x* whilst *a* prefers x_a to y_a and *b* prefers x_b to y_b .

Thus to infer society's preferences from those of the representative individual, and to use these to make policy choices, is illegitimate. It is no challenge to find many examples in the macroeconomic literature in which changes in the representative individual's welfare are interpreted as corresponding to changes

⁸I will pass over the fact that this "trick" will not function at all in those macro models where the equilibrium is not Pareto optimal (Greenwald and Stiglitz, 1986).

Figure 1
Combining Two Agents



in society's welfare. Since they are so easy to find, I will not risk the wrath of my colleagues by citing particular examples of this practice.

Difficulties with "Representative" Analysis: Some Examples

It should be clear by now that the assumption of a representative individual is far from innocent; it is the fiction by which macroeconomists can justify equilibrium analysis and provide pseudo-microfoundations. I refer to these as pseudo-foundations, since the very restrictions placed on the behavior of the aggregate system are those which are obtained in the individual case and, as we have seen, there is no formal justification for this. Thus, when the conclusions of such a model are tested with empirical data (not a particularly frequent occurrence) and should they, by chance, be rejected, this may simply reflect the fact that the assumption that the economy could be represented by a single individual was erroneous.⁹ In other words, whenever a representative agent model is tested, one is testing a joint hypothesis: the particular behavioral hypothesis one is interested in and the hypothesis that the choices of the aggregate can indeed be described as the choices of a single utility-maximizing agent. It is worth looking at a few examples in the literature to have a clear vision of this problem.

Summers (1991) emphasizes the confusion as to what is being tested in representative agent models in his critical study of empirical macroeconomics.

⁹Blanchard (1987), for example, shows explicitly how, when looking at price and wage rigidity, using disaggregated data can give results different from those obtained by looking at purely aggregate phenomena.

He points out, for example, that the well-known work of Hansen and Singleton (1982, 1983) resulted in the rejection of a particular relationship between consumption and asset pricing, yet what was perhaps being rejected is the representation of the consumer sector by one individual with an additively separable utility function and constant relative risk aversion. These authors clearly take their representative agent seriously, since they discuss the “economic plausibility” of the estimated values of the parameters of his utility function.

To take another example, the apparently paradoxical response of consumption to changes in income, the so-called “excess smoothness” of consumption, has been taken to reject the permanent income hypothesis. This paradox has resulted in a large and technically sophisticated literature. If the consumption sector is viewed as an individual, he apparently does not react as much to changes in current income as would be predicted by the permanent income hypothesis. Thus while labor income in the U.S. is a random walk, consumption is much smoother. However, the question of “excess smoothness” has usually been examined in the framework of an economy with a single individual. What may really be involved is the modelling of the economy as one individual. When an unpredictable shock happens to current income, the agent revises his estimate of his permanent income and modifies consumption accordingly. How much he changes his consumption depends on how much of the current change he perceives to be permanent and how much transitory. As Quah (1990) points out, we can always make the representative agent decompose current shocks in such a way so that empirical observations are consistent with the permanent income hypothesis. However, as he and others like Lippi and Reichlin (1990) point out, the decompositions in question are completely arbitrary, so the representative agent can only be resurrected if we have some reason to believe that the agent perceives shocks in this way. Various ingenious efforts have been made to resolve the paradox without losing the representative agent. For example, Diebold and Rudebusch (1991) argue that if this agent takes into account possible low frequency changes—like a change in income in the very long run—this may be sufficient to explain the smoothness of consumption.

However, one has to ask why economists should be interested in the reaction of a consumer with a 100-year horizon to changes in income which are generated by a very complicated process, when aggregating over several heterogeneous and myopic individuals who analyze their income processes in a much simpler but still consistent way may produce a similar result. We return to the essential point. To preserve a tractable maximizing model, economists have clung to the representative agent model. Yet not only the resolution of the paradox of “excess smoothness” but the very existence of the issue may be a result of the choice of this framework. For example, Clarida (1991) has constructed a model which allows for a number of agents—each of whom satisfies the permanent income hypothesis, smooths consumption and saves for

retirement. He gets on the aggregate level the sort of smooth reaction to income change which is actually observed. Aggregate per capita income is a random walk with a drift and the drift in per capita consumption is equal to the drift in total income. The important point is that one only has to allow for agents of different ages, but identical in other respects, to coexist and the assumption of a suitable demographic structure will resolve the so-called paradox.

Stoker (1986) emphasizes that the sort of difficulty I have referred to is intrinsic to the use of single individual models. He points out that the two cases in which the representative agent construction makes some sense are those in which either individuals take their decisions only on the basis of aggregate variables or in which all individuals have the same marginal reactions. He comes to the conclusion: "It makes no sense to insist in general that equations be consistent with the behavior of a single individual, no matter how 'representative,' when the equations are estimated with economy-wide aggregate data." He points out the pitfalls of ignoring distributional effects and, in particular, explains that complicated dynamic behavior may arise from aggregation over heterogeneous agents with simple behavior.

Lippi (1988) develops this point. He takes as one of his examples the work of Davidson et al. (1978), who estimated the aggregate consumption-income equation for the United Kingdom. Their model involves an infinitely distributed lag. One then either tries, as various authors have done, to show that this is the result of the complicated optimization of a single individual, or, like Lippi, one shows that this macrodynamic behavior can arise from aggregation over heterogeneous individuals whose behavior may depend only on variables with one lag, or even on current variables alone. Once one allows for different micro-behavior and for the fact that different agents face differing and independent micro-variables—as is clearly the case with income, for example—then Lippi shows how complex aggregate dynamics may arise from simple, but rational, individual behavior. Rather than proposing ever more complicated dynamic optimization schemes for representative agents, it makes sense to explore the patterns that may develop from aggregation over heterogeneous individuals, who make simpler but still coherent calculations.

Dispersion or Concentration of Characteristics: A Negative Result

Having argued that we cannot hope to obtain stability and uniqueness of equilibrium by making "individualistic" assumptions, one might still ask whether the situation could be rescued without having to resort to the representative individual. A standard objection to the Sonnenschein-Debreu-Mantel results is that they make no restrictions whatsoever on the distribution of agents' characteristics and that this might be the source of the difficulty. This recalls Black's

result that Arrow's Impossibility Theorem no longer holds if attention is restricted to distributions of preferences which are "single peaked."

Let me separate the problem into two possible sets of restrictions: those on the distribution of preferences and those on the income distribution. Consider first the distribution of preferences. It has been suggested that aggregate excess demand functions which lead to instability or multiple equilibria may only occur in economies in which preferences differ widely and implausibly from each other. This suspicion is heightened by the fact that in Debreu's (1974) paper each individual always has positive excess demand for one particular good, regardless of the prices of the other goods.

Thus, the suggestion is that allowing too large a dispersion of characteristics leaves so many degrees of freedom that almost anything can happen. If the distribution of characteristics were to be sufficiently concentrated—or put another way, people were to be sufficiently similar to each other—then maybe stability and uniqueness could be recovered. The underlying idea is a sort of continuity argument. As individuals become "almost identical," then it may seem plausible that a representative agent characterization would be more reasonable, and that the desirable properties of stability and uniqueness appear.

Unfortunately, no matter how close individuals are to each other in terms of their characteristics, there is no hope in this direction. Kirman and Koch (1986) have investigated the situation where all individuals have *identical* preferences, and they show that the Sonnenschein-Debreu result still holds.

Turning now to the distribution of income they then consider the case in which the relative income distribution can be chosen in any way you like, except for complete uniformity. They find that with any variation in the a priori chosen income distribution at all, it is impossible to guarantee well-behaved excess aggregate demand functions that assure uniqueness and stability. The *only* income distribution that is not permissible is the uniform one where all individuals have the same income and thus, since they have the same preferences, they are all the same.¹⁰ Put another way, given an arbitrary excess demand function, no matter how ill-behaved and difficult to work with, I can give you an economy in which people are as close as you like to being identical, i.e. they have the same preferences and almost the same income, which will generate this ugly aggregate excess demand function. Thus, trying to squeeze the economy down to almost one individual does not help to generate the sort of equilibrium properties that macroeconomists would like to have.

¹⁰A relatively informal statement of the Kirman-Koch (1986) theorem would proceed as follows: Given an arbitrary function F satisfying the three conditions mentioned previously in the earlier footnote, and given n different positive numbers with $n \geq$ the number of goods, and a positive ϵ then we can find an economy \mathcal{E} with n individuals all having the *same* preferences and each having α_i of the total income, and such that the aggregate excess demand of that economy $Z_\epsilon(p)$ coincides with F , for all prices greater than ϵ .

It is perhaps worth making a remark at this point. For any excess demand function satisfying the standard properties, the above results demonstrate that it is possible to construct an economy of individuals with identical preferences who generate this function. Thus, there is someone who is eminently qualified to represent the collectivity, a large individual who shares the common preferences. Unfortunately, without homotheticity, such an individual's choices are irrelevant for the agents he supposedly represents; and worse, his choices will not, in general, coincide with their aggregate choice.

Whilst the idea that aggregate behavior will have desirable properties if the economy behaves as one representative individual is obviously correct, it is not very useful. Agents may be as similar as desired, but the economy can still have a large number of unstable equilibria.

Can Dispersion be Part of the Solution?

If close similarity is not an adequate approach to sustaining a well-behaved aggregate demand function, the obvious alternative is to consider whether economics should be seeking heterogeneity, rather than avoiding it.

It might seem at first that the argument we have just made implies that heterogeneity, even in small doses, is fatal to our purposes. But the preceding remarks must be interpreted with care. What they show is that we can restrict the characteristics of the agents in an economy and eliminate dispersion of preference so that the economy is arbitrarily close to being one with a representative agent, and still anything may happen. Trying to reduce the dispersion of characteristics to something approximating a single agent is hopeless. Instability and multiplicity of equilibria may persist, even with a pre-chosen relative income distribution. Yet, as I have suggested, it would be wrong to conclude that distributional restrictions offer no help. As several economists have pointed out, we know quite a lot about empirical income distributions, and it seems unreasonable not to exploit this information (Deaton, 1975), though in the light of Kirman-Koch it may seem unlikely to offer much help. However, this is not really the case.

Recent results by Hildenbrand (1983) and Grandmont (1987, 1991), for example, have shown that *increasing* dispersion of the income or the distribution of preferences may actually improve the situation and have a stabilizing effect. Heterogeneity of agents, even in the standard general equilibrium model, may be useful in making aggregate behavior more regular. This result will come as no surprise to those familiar with Cournot's observation that erratic *individual* demand behavior may give very smooth *aggregate* demand behavior, if individuals are different enough. The basic observation amounts to the following. If consumers do not have textbook preferences, for example if they do not have convex preferences, then their demand will not be continu-

ous. It will therefore jump from one bundle to another at certain prices. If all individuals are the same, these jumps will occur at these prices also for aggregate demand. However, if there are many agents and they all have different preferences, the individual jumps will occur at different prices and will not be enough to disturb the overall continuity of demand. This insight was formalized much later (Trockel, 1984). However, the value to a model and to an economy of having dispersed characteristics has been readily apparent in many important papers. Houthakker's (1955) early result considered a situation in which if all firms have fixed coefficient production functions, but the parameters of these differ over firms sufficiently, then the aggregate production sector behaves as if it had a textbook smooth production function.¹¹ In the context of imperfect competition, Caplin and Nalebuff (1990) and Dierker (1989) have shown how quasi-concavity of the profit function may be obtained from assumptions which imply that the distribution of consumer preferences is sufficiently dispersed.

Hildenbrand (1983) has shown that if the income distribution is decreasing—that is if each successively higher income class contains a smaller proportion of agents—then the so-called “law of demand” holds. This implies that the partial demand curve for each good is downward sloping and that equilibria are unique and stable.¹² This result is particularly interesting, since the “law of demand” does not necessarily hold for the individual. Thus in this case the aggregate satisfies a stronger condition than the individual. Although the particular form of the income distribution that he posited is not acceptable, the result was an important step toward utilizing distributional considerations to obtain desirable properties of aggregate behavior. Grandmont (1987) obtained similar results looking at distributions over preferences. Hildenbrand (1989) later provides an argument to show why the “law of demand” should hold, empirically, in general, in a way that overcomes the objection to the distributional form required for his earlier result.

Grandmont (1991) has recently obtained a rather remarkable result. He gives an example in which if agents have very spread out preferences, then aggregate excess demand will have the well known “gross substitutability” property and equilibria will be unique and stable.¹³ In fact, paradoxically, his result shows the *unimportance* of individual maximizing behavior to building up well-behaved aggregate demand functions; for his result, it is enough that

¹¹Actually he showed that if the parameters were Pareto distributed, then the aggregate production function was Cobb-Douglas.

¹²The reader may be puzzled by the apparent inconsistency of this result with Kirman-Koch. Hildenbrand has a continuum economy with finite per capita income. If we tried to use the Kirman-Koch construction to obtain his sort of income distribution in the limit, per capita income would become infinite.

¹³Actually, by pushing his spreading procedure to the limit, loosely speaking, he would recover a single agent who actually would have Cobb-Douglas preferences and whose behavior would coincide with that of the economy. However, in no meaningful way does this individual represent the behavior of the underlying individual agents.

agents respect their budget constraints. This finding is highly destructive for those who have insisted that well-behaved aggregate demand should be the result of constrained maximization by individual agents.

The open question now is to what extent can dispersion of characteristics help us to regain structure at the aggregate level. The cases just cited show how dispersion may help. It would also doubtless be easy to construct examples in which dispersion had no such positive effects. For those who are attached to the standard model of aggregate demand, this would seem to be a promising line of research.

I would like to go even further than this, however, and suggest that it is not enough to simply introduce heterogeneity into some version of the standard general equilibrium model, which is still the basis for most macroeconomic models. The basic limitation of that model is that the individual's only interaction is through the anonymous forces of the market. As Samuelson (1963) said:

... individualistic atoms of the rare gas in my balloon are not isolated from the other atoms. Adam Smith, who is almost as well known for his discussion of the division of labor and the resulting efficiency purchased at the price of interdependence, was well aware of that. What he would have stressed was that the contacts between the atoms were *organized* by the use of markets and prices.

I would like to suggest that we should avoid interpreting this quotation too narrowly and that we should also consider the macroeconomic behavior of models in which heterogeneous individuals interact directly and consciously, through mechanisms like trading, the passage of information, the building of reputations, organizing into groups for purposes of bargaining, and more. The nature and extent of these contacts will, inevitably, be influenced by market considerations, but agents are much more than anonymous price-takers.

Heterogeneity and Interaction

As the complexity of economic models increases—with the addition of uncertainty, infinite horizons, infinite commodity spaces, and so on—the plausibility of the single representative agent, acting optimally in all markets and at all times, diminishes. An alternative and attractive approach is offered by game theory, where the interaction between heterogeneous individuals with conflicting interests is seriously taken into account.

For a long time, this approach did not seem well adapted to the analysis of the whole game constituted by a large economy, unless many unnatural symmetry conditions were imposed. Most of the papers which apply non-cooperative game theory to a macroeconomic model have only a few

“representative” players. In fact, remarkably enough, there are macroeconomic, imperfectly competitive, models in which the behavior of only one individual is studied. This is possible, since all individuals are assumed to be identical. Thus, there are symmetric equilibria and it is sufficient to look at the strategy choice of any one player. But surely, any such model begs the basic issue of analyzing what will happen when different individuals with conflicting interests are involved. Furthermore, given the exacting informational and rationality requirements of standard non-cooperative game theory, this sort of analysis seems to be replacing one sort of representative agent with another. However, as I will indicate in a moment, recent game theoretic literature does offer some promising developments.

A tentative conclusion, at this point, would be that the representative agent approach is fatally flawed because it attempts to impose order on the economy through the concept of an omniscient individual. In reality, individuals operate in very small subsets of the economy and interact with those with whom they have dealings. It may well be that out of this local but interacting activity emerges some sort of self organization which provides regularity at the macroeconomic level. Lesourne (1991) offers a detailed account of this sort of approach. The analogy with biological and physical systems is obvious.

This sort of interaction in an economic system can be envisaged in several ways. One might assume that agents meet each other at random and that the subsequent activity of the agents is conditioned by these meetings. This approach has been used widely in various branches of economics, as in models of search (Diamond, 1987) or matching (Roth and Sotomayor, 1990). However, most of these models have paid little attention to the resultant dynamics of allocations, distributions or even of populations, concentrating on the existence of stable states or, at best, convergence to such a state. However, some developments in the financial market literature and the game theory literature are beginning to take explicit account of the evolution of the proportions of agents with different characteristics in an economy.¹⁴

In financial market models individuals hold diverse opinions, or expectations, and as a result of encounters or imitation, the proportion of agents holding each view changes. As the state of the system changes so do asset prices.¹⁵ In game theory, models have been developed in which players meet each other at random and the number of people playing each strategy evolves according to the success of that strategy. This may be the result of those people playing more successful strategies having more “offspring,” or of players imitating the strategies played by more successful opponents. These models rely on heterogeneous behavior, and often on less than full-blown individual rationality. Instead, individuals act and learn in a very simple way and, as a

¹⁴This development has already occurred in sociology, where the evolution of the proportions of the population holding different opinions was studied. For example, see Weidlich and Haag (1983).

¹⁵For examples of this sort of model, see Sharfstein and Stein (1990), Kirman (1991), Topol (1991) and the discussion of “popular” models in Shiller (1990).

result of this, the population may evolve to some sort of equilibrium. The analogy with the biological literature on evolutionary games (like Maynard Smith, 1982) is obvious.¹⁶

In the literature just discussed, different individuals meet each other at random. However, a vision which corresponds more to that of a real economy is one in which agents are in direct contact with only a limited subset of other agents. In this case, one can think of a graph-like relation describing the communication between agents which governs their trading relations, their possibilities of forming coalitions, or which individuals might affect which other agent's preferences or opinions. One can then study how local phenomena propagate through the economy. Föllmer (1974) was the first to introduce such models into economics; for more recent work, see Durlauf (1990) and Blume (1991). The communication structure should, of course, be endogenous, but this is a subject for future research.¹⁷

The equilibria of the worlds described by any of these approaches may be conceptually very different from those implied by the artifact of the representative individual. Cycles and fluctuations emerge not as the result of some substantial exogenous shock and the reaction to it of one individual, but as a natural result of interaction, together with occasional small changes or "mutations" in the behavior of some individuals. Such endogenous cycles can, of course, arise, even in a simple deterministic economy, for example one with only two agents alive at any one time (Grandmont, 1985). Indeed, the evolution of the equilibria of such an economy may be chaotic. However, although endogenous fluctuations arise very naturally in a context of many interacting agents, it may still be that the evolution of such an economy may be relatively stable. In a world with many interacting heterogeneous agents, a natural idea of an equilibrium would be not a particular state, but rather a distribution over states, reflecting the proportion of time the economy spends in each of the states.¹⁸ This distribution could exhibit very regular characteristics, while remaining far from reflecting the behavior of a single maximizing agent.

Conclusion

To many macroeconomists, the aggregation problems of the sort implied by the research just described look difficult enough that the simplification of the representative individual looks more attractive, rather than less. In this

¹⁶For examples of this sort of work, see Binmore and Samuelson (1990) and Foster and Young (1990).

¹⁷Kirman, Oddou and Weber (1986), Ioannides (1990) and Haller (1991) investigate the idea that the communication network might itself be stochastic.

¹⁸To be fair, it should be pointed out that several authors are concerned with the convergence of such a system to a single steady state—see Arthur (1987) and Sah (1991)—but this is, of course, a special case of the general distributional idea.

connection Lewbel (1989), in discussing those conditions under which a representative agent does exist, makes two very revealing remarks. First he carefully points out that “the representative consumer is a purely mathematical result and need not have economic content.” Then he goes on to say: “It is a fact that the use of a representative consumer assumption in most macro work is an illegitimate method of ignoring valid aggregation concerns. However, the representative consumer framework vastly simplifies a great deal of macro work and thought, and so is not likely to be abandoned.” This practice, which incidentally Lewbel does not endorse, corresponds to the behavior of the person who, having dropped his keys in a dark place, chose to look for them under a street light since it was easier to see there!¹⁹

Given the arguments presented here—that well-behaved individuals need not produce a well-behaved representative agent; that the reaction of a representative agent to change need not reflect how the individuals of the economy would respond to change; that the preferences of a representative agent over choices may be diametrically opposed to those of society as a whole—it is clear that the representative agent should have no future. Indeed, contrary to what current macroeconomic practice would seem to suggest, requiring heterogeneity of agents within the competitive general equilibrium model may help to recover aggregate properties which may be useful for macroeconomic analysis.

Yet, despite these arguments, I suspect that the representative individual will persist for as long as economists focus on a framework of anonymous individual maximization. Only if we are prepared to develop a paradigm in which individuals operate in a limited subset of the economy, are diverse both in their characteristics and the activities that they pursue, and interact directly with each other, will economics escape from the stultifying influence of the representative agent. Within such models there can and should be considerable aggregate regularity. However, the fact that behavior at the macroeconomic level exhibits regularities does not mean that it is useful or appropriate to treat the economy as a maximizing representative individual.

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¹⁹This analogy has been used in the past to criticize general equilibrium theory. Hahn (1984) mentions this, but his response is, to say the least, elliptic.

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