Negative predicates: The quantity metaphor and transformation values
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I propose a novel explanation for three known generalizations, and one new piece of data:

- Negative and positive antonyms are usually bad in between-predicate comparisons (as in *Dan is taller than Sam is short).
- Numerical degree phrases combine with positive, but not with negative, predicates (as in two meters tall versus *two meters short).
- Numerical degree phrases are always fine in comparatives (as in two meters shorter).
- Negative predicates combine less easily than positive ones with ratio modifiers (as in ??Dan is twice as short as Sam versus Dan is twice as tall as Sam). When Google searching for entries of the form twice as ADJ as and half as ADJ as, in 75% of the cases (12 of the 16 antonym pairs I looked at) the number of entries is significantly smaller in negative adjectives, compared to their positive antonyms.

Comparative morphemes have quantity readings (as in more boys than girls smile). Semanticists often describe their extent reading (as in Dan is happier than Sam) in terms of a quantity metaphor, too. Moltmann (2006) argues that the extent to which entities satisfy an adjective, for instance happy, reflects the quantity that they possess of the thing denoted by the adjective's nominalization, happiness (where one’s happiness is an element of the domain D, just like one’s legs or hair). In accordance with this view, when semanticists discuss gradable predicates, they often treat their nominalizations as ‘ordering-dimensions’ (Kennedy 1999 and references therein). Furthermore, quantity functions are additive. For example, the number of apples in two baskets together equals the sum of numbers of the apples in each basket separately. In accordance, semantic theories postulate that predicates’ degree functions are additive (with respect to their nominalization). For instance, the predicate long can be seen as measuring quantities of length. Klein (1991) symbolizes the concatenation (placing end to end) of two rods $d_1$ and $d_2$ as $d_1 \oplus_{\text{length}} d_2$. In Klein (1991), the degree function of long, $f_{\text{long}}$, is additive in the sense that it adequately represents the fact that the length of the concatenation of two rods equals the sum of lengths of the two separate rods ($f_{\text{long}}(d_1 \oplus_{\text{length}} d_2) = f_{\text{long}}(d_1) + f_{\text{long}}(d_2)$). In general, the values of additive functions (wrt a quality Q) represent the ratios between quantities of Q in entities. Because of the quantity metaphor, semantic theories postulate additivity.

Functions from D to $\mathbb{R}$ (the set of real numbers) that map entities to their length quantity transformed by a constant are not additive wrt length. For example, the function $f_{1,\text{t}}$ that maps each d in D to the value $(1 - f_{\text{long}}(d))$ is not additive wrt length. The ratios between the degrees that this function assigns do not adequately represent the ratios between the quantities of length in entities. If, for instance, $f_{\text{long}}(d_1) = f_{\text{long}}(d_2) = 5$, then by additivity $f_{\text{long}}(d_1 \oplus d_2) = 10$. But by the definition of $f_{1,\text{t}}$ ($f_{1,\text{t}}(d_1) = f_{1,\text{t}}(d_2) = -4$) and ($f_{1,\text{t}}(d_1 \oplus d_2) = 1 - f_{\text{long}}(d_1 \oplus d_2) = -9 \neq (2 \times -4)$). The ratio between the degrees of $d_1 \oplus d_2$ and $d_1$ is 9/4 and the ratio between their lengths is 8/4.

When semanticists look at negative predicates like short, they often assume that the values of their degree functions depend on quantities of height that entities possess (Rullmann 1995; Landman 2005) or do not possess (Seuren 1978, 1984; von Stechow 1984; Kennedy 1999, 2001). I submit that the mapping of entities to degrees in short is not known to be additive wrt these quantities in any actual context c. I call an information state c an actual context iff it represents the linguistic and world knowledge of a given community of speakers out of the blue. The information in an actual context c is partial. Let $W_c$ be the set of worlds that given the information in c may still be the actual world (Stalnaker 1975). What do we know about the degree function of short in actual contexts? We know (we have a very strong intuition) about the entity ordering of short that it is reversed compared to that of tall (Dan is taller than
Sam iff Sam is shorter than Dan). Thus, the degrees are reversed (if Dan is mapped to a higher degree in tall Sam is mapped to a higher degree in short). But, crucially, that is about all that we know about these degrees. In other words, we know that they are produced by a reversed function, but we do not know which reversed function. There are many candidates. For any world $w$ of $W_c$, let $f_{tall,w}(d) \in \mathbb{R}^D$ be the function linked with tall in $w$. For any constant $Tran \in \mathbb{R}$, a function $f_{Tran}$ that assigns any $d$ the degree $(Tran - f_{tall,w}(d))$ can properly reverse the degrees. But only when the constant Tran is 0, the function is additive wrt height, as demonstrated with $f_{1-f}$. Do we have intuitions that tell us that the transformation value of short, $Tran_{short,w}$, is 0, in every world $w$ in $W_c$ of any actual context $c$? No, we don’t. Evidence for this is formed by our intuitions concerning the value of entities with zero height (abstract entities; surfaces; points). If, in every world $w$ of $W_c$, $Tran_{short,w} = 0$ and $f_{tall,w}$ is additive (it maps entities with no height to 0), then the degree of entities with no height in short should be known to be 0 in c (because in every $w \in W_c$ it is 0). But is it? Not really. For instance, some well-known semantic theories (von Stechow 1984; Kennedy 1999) endorse the view that entities with no height are mapped to a degree that approximates infinity. But if so, then the degree function of short is not additive; it is a function that transforms height quantities by a non-zero constant, $Tran_{short}$. To be honest, we know nothing about this constant; it may be any number (it varies across worlds in $W_c$).

Interestingly, by representing this constant we can derive the polarity effects. If in $c$, e.g., tall maps an entity $d$ to 2 meters ($\forall w \in W_c, f_{tall,w}(d) = 2$), short maps $d$ to $Tran_{short} - 2$ meters ($\forall w \in W_c, f_{short,w}(d) = Tran_{short,w} - 2$), where the value $Tran_{short}$ is unknown (varies in $W_c$).

First, this produces indeterminacy concerning the degrees short assigns. Lacking knowledge about $Tran_{short}$, we cannot say which entities are $n$ meters short in $c$ ($\neg \exists d \in D: \forall w \in W_c, f_{short,w}(d) = n$), so numerical-degree phrases like two meters can’t be used with short.

Second, when degree-differences are computed, the transformation values of the degrees cancel one another, so negative predicates are felicitous in statements expressing numerical degree differences (as in Dan is 2 meters shorter than Sam). For instance, $d_2$ has 2 meters more length than $d_1$ in $c$ iff $\forall w \in W_c, d_2$ is 2 meters taller (tall maps $d_2$ to some number $n$ and $d_1$ to $n - 2$) and iff $\forall w \in W_c, d_2$ is 2 meters shorter (short maps $d_2$ to $Tran_{short,w} - n$ and $d_1$ to $Tran_{short,w} - (n - 2)$. The difference between these degrees is still 2. The difference is negative, $(Tran_{short,w} - n) - (Tran_{short,w} - (n - 2)) = -2$, since $d_1$ has a higher degree in short.)

Third, in cross-polar comparisons (*Dan is taller than Sam is short) since only the degree assigned by short has a transformation value, this value does not get canceled out. As we lack knowledge of this value, such statements can never be verified, and are considered anomalies.

Fourth, $d_2$ has a double length compared to $d_1$ in $c$ iff in any $w$ in $W_c$ tall maps $d_1$ to some number $n$ (say, 2 meters), and $d_2$ to $2n$ (say, 4 meters). Given that short reverses the degrees, short maps $d_1$ to $n' = Tran_{short,w} - n$ (e.g., $Tran_{short,w} - 2$ meters), and $d_2$ to $m' = Tran_{short,w} - 2n$ (e.g., $Tran_{short,w} - 4$ meters). But $m'$ is not two times $n'$ (unless $Tran_{short,w}$ is 0). As a consequence, twice as short is less acceptable than twice as tall.

Finally, this analysis captures properties of certain positive predicates, like the infelicity of 2 degrees warm and the unclear intuitions concerning the zero point of warm. Presumably, positive predicates like warm have a transformation value too.

My analysis improves upon previous accounts (Seuren 1978, 1984; von Stechow 1984; Kennedy 1999, 2001, etc.) These theories represent degrees as intervals, not numbers (which is counter-intuitive and complex). In addition, Kennedy (1999, 2001) predicts that cross polar comparisons are anomalies by stipulating that different scales (degree types) are incomparable. Yet, often different scales are comparable, as in: more typical of a Leo than of a Virgo; more sour than sweet, and more a cat than a bird (- interestingly, noun pairs are always comparable! This fact wasn't noticed before and its implications were not studied).