Directional Numeral Quantifiers

Modified numerals come in many varieties, such as comparatives (more than forty), superlatives (at least forty), approximatives (almost forty) and prepositional phrases (between forty and fifty, over forty, etc.). In recent literature, it has become clear that not all modifications of numerals are alike and that much can be learned from analysing modified numerals on a case by case basis. (See e.g. Geurts and Nouwen 2007). Prepositional numerals are poorly studied, but an exception is Corver & Zwarts 2006. Interestingly, they claim that only locative aspects of the meaning of prepositions play a role in modified numerals. For instance, the English preposition over has a directional and a locative sense, but only the latter is used in a quantifier like over three hundred men. I argue, however, that there exists a small class of prepositional numerals where a strictly directional interpretation of the preposition is used. I call the resulting quantified phrase a directional numeral quantifier, DNQ for short. Such quantifiers typically contain a P which completely lacks a locative sense. In English, DNQs are exemplified by up to sixteen pages. In many other languages the preposition involved is both a directional spatial preposition and a temporal end-point marker. Prepositions like Dutch tot, Hebrew @ad (cf. Winter 2006) and German bis (zu) all have a spatial, temporal and a numeral usage. (In English, up to cannot be applied in the temporal domain, where durative until is used.)

DNQs have a rather limited distribution. In this paper, I explain this on the basis of a uniform semantics for these prepositions in the spatial, temporal and numeral domain.

In Dutch, the prime example of a DNQ is the class of modified numeral quantifiers that contain the preposition tot. In the spatial domain, tot lacks a locative reading. Consequently, it depends on the expression of some sort of path.

(1) #Ik sta tot hier. (2) Ik ben tot hier gerend. 
I stand TOT here. I am TOT here ran. ‘I ran up to here’

A similar contrast can be found in the numeral domain. DNQs are not generally allowed in positions where other numeral quantifiers are felicitous. For instance, (3-a) is unacceptable (compare with English ??I ate up to four cookies). But the modalised version (3-b) is fine.

(3) a. ??Billy heeft tot veertig vriendjes uitgenodigd.
Billy has TOT forty friends-DIM invited.
b. Van zijn moeder, mag Billy tot veertig vriendjes uitnodigen.
From his mother, may Billy TOT forty friends-DIM invite.
‘Billy’s mother allows him to invite up to forty friends’

The same contrasts occur with English up to, German bis zu and Hebrew @ad. The parallel between the spatial and the temporal function of these prepositions can be fleshed out as follows. (Here, and in what follows, I use tot as the generic example of all these prepositions, but use the corresponding English examples for illustration.)

(4) General semantics for tot (and its kin):
Let I range over intervals (spatial, temporal or other). [X tot a] = true iff for all sub-intervals I of an interval that has a as its end-point X holds for I

According to (4), Syd played chess until midnight means that there is a time interval leading up to midnight such that in each subinterval of that period Syd was playing chess. For spatial use, (4) operates as follows. Syd ran up to the edge of the lake means that
there is a path ending on the edge of the lake such that each subpath is such that Syd ran it. It follows straightforwardly from (4) that \textit{tot} is incompatible with non-homogeneous predicates. Moreover, in the spatial domain, \textit{tot} will need some expression of motion for the interval semantics to apply to.

Crucial to my analysis is that \textit{tot}-PPs modify predicates of intervals. If we view numerals as indicators of cardinality, then it is not immediately clear what such a PP applies to. I propose a silent lift which remedies this type clash. This lift from degree predicates (we liberally view cardinalities as degrees, cf. Hackl 2000) to interval predicates is written as $\Omega$. (Its semantics is trivial. For completeness: $\Omega(P) = \lambda d. \forall d \in I : P(d)$.)

The degree predicate is formed by raising the modified numeral:

\begin{align*}
(5) \quad & a. \text{ Syd is allowed to eat up to four biscuits.} \\
& b. \quad [ \text{up to four} \begin{array}{l} \Omega \begin{array}{l} \lambda \text{ allow [ Syd eats $t$-many biscuits ]] } \end{array} \end{array} ] ] ] ]
\end{align*}

This gives the correct semantics: there is an interval leading up to four (say, $[1, 2, 3, 4]$) such that for each $n$ in that interval Syd has the permission to eat $n$ biscuits. For a simple sentence like \textit{??Syd ate up to four biscuits}, we arrive at a non-sensical meaning: the number of biscuits eaten by Syd is one, two, three and four. (Some people do get one sensible interpretation for this sentence, namely that it is unclear how many biscuits Syd ate, but that he ate no more than four. Such readings, however, once more displays a weak modal element.) The analysis correctly predicts that \textit{up to-DNQs raised over a universal modal are infelicitous}. \textit{??Syd is required to eat up to four biscuits} is interpreted as the non-sensical statement that the number of biscuits Syd is required to eat is one, two, three and four. (Again, the interpretation might be saved by accommodating a weak epistemic modal expression the speaker knows about how many biscuits Syd is required to eat. More on this in the full paper.)

The analysis so far overgenerates, for if \textit{up to-DNQs} can be raised over weak modals, then they should be able to take existential quantifiers in their scope too. As (6) shows, this is wrong. The analysis in (4) wrongly predicts that (6) means that for every number in $\{1, 2, 3, 4\}$ there is a child who ate that amount of biscuits.

\begin{align*}
(6) \quad & \text{Some children ate up to four biscuits.}
\end{align*}

However, the fact that the theory overgenerates actually demonstrates that it is on the right track. Here is why. In order to yield a sensible interpretation, the \textit{up to} numeral has to move to take scope over some quantifying operator. It has been observed however that degree operators cannot take wide scope over nominal quantifiers. This is known as Kennedy’s Generalisation: if the scope of a quantificational DP contains the trace of a degree phrase, it also contains that degree phrase itself (Heim 2000; Kennedy 1997). So, (7) is simply an instance of the observation captured in Kennedy’s generalisation.

\begin{align*}
(7) \quad & \star [ \text{tot} a \begin{array}{l} \Omega \begin{array}{l} \lambda \text{ DPQ } \end{array} \end{array} \begin{array}{l} \ldots t \end{array} ] ] ] ]
\end{align*}