Labor Time, Commodity, and State Money: Complementary Approaches to Marxian Value Theory

Erik K. Olsen\textsuperscript{1} Ph.D.
Associate Professor
Department of Economics
University of Missouri Kansas City

Abstract: The New Interpretation includes a set of accounting identities, definitions, and laws involving labor time, prices, and money. It preserves the labor theory of value and Marxian theory of exploitation from a range of critiques. It achieves a high level of generality by being agnostic with regard to price formation, accommodating any monetary system, and subsuming the concept of value to price. Conversely, Roberts’s single system interpretation provides a comprehensive theory of commodity value and price of production on a labor time standard, but says little about monetary phenomena. This paper demonstrates that these are complementary approaches and integrating them provides a labor theory that rigorously addresses value and price without subsuming one to the other, and is applicable to a commodity or state money using economy. It also demonstrates that modern approaches to Marxian value theory are characterized by large areas of agreement and convergence rather than rivalry. (149 words)

Keywords: Marxian value theory, transformation problem, New Interpretation, Roberts, single system, commodity money, state money.

\textit{JEL Codes:} B510, D460, E420 \quad \text{Word count: 7,543}

Draft for 3-7-17 Political Economy Seminar, Univ. of Massachusetts Amherst.

\textsuperscript{1}Email address for correspondence: Olsenek@umkc.edu. Comments appreciated.
Several different approaches to Marxian value theory emerged in response to the critiques of the 1960’s and 1970’s. For reasons that are not at all obvious, contributors to this literature often emphasize strongly what is novel or different rather than what is consistent or complementary with other approaches. This development makes the literature on Marxian value theory since the early 1980’s seem disparate and fragmented, characterized by rivalry rather than convergence.

This paper takes a different perspective. It identifies similarities and complementarities between two different approaches. Specifically, it considers the New Interpretation (NI), originally associated with the work of Gérard Duménil (1980, 1984-85) and Duncan Foley (1982, 1986), and an approach that has come to be referred to—somewhat inaccurately—as a "single system interpretation" (SSI). The SSI was initially developed by Bruce Roberts (1981) and subsequently elaborated by Wolff, Roberts and Callari (1982), Wolff, Callari and Roberts (1984), and Roberts (1987, 1997, 2009). These two approaches have characteristics that make them quite different, but should be seen as complementary rather than rival.

The basic features of the NI can be expressed as a set of weak axioms that provide a very general framework of a labor theory. The NI is rigorous and internally consistent, but it also has important limitations. Notable among these limitation are the absence of a theory of price formation and the subsumption of value under price. Indeed, Foley (2000, 20) argues that the central insight of the NI involves the monetary expression of labor time (MELT), which dispenses...

---

2In dealing with single-system value theory I only consider this version, which, in light of what came later, has come to be called the ‘simultaneity’ version. The Temporal Single System Interpretation (TSSI) is a later development that, because of its distinctive emphasis on sequential periods, is not easily compared with other non-temporal theories. Since the objective of this paper is to identify similarities and complementarities between the NI and the SSI, discussing the TSSI is beyond its scope.
with any need for a separate accounting based upon embodied labor coefficients at all (see also Mohun 2004, 77).

While its generality can be considered a strength, the NI certainly achieves less than what Marx endeavors to achieve in his treatment of values and prices of production in volume three of *Capital* (1967b). It is also only a very partial response to the criticisms of the type leveled by Bortkiewicz (1952, (1906/7); 1949 (1907)), Steedman (1977), and others. Duménil and Foley are clear that they see Marxian value theory as fundamentally a theory of exploitation and argue that issues arising from consideration of prices of production do not affect this higher-order issue. They dismiss this by arguing that "(t)he specific property expressed in the equality of the profit rate among industries cannot play any role in the theory of exploitation (2008, 410). For them the NI preserves Marx’s theory of exploitation "in any set of prices". Mohun, a prominent exponent of the NI, articulates a similar position:

The (NI) is not in itself a ‘solution’ to anything. It is rather an approach to the labour theory of value that provides an *ex post* accounting system that is theoretically coherent, and compatible with accounting practices in capitalist society. As an accounting system, no relations of determination are expressed. In particular, that profits are an exact measure of unpaid labour is *not* a deduction from more primitive assumptions. Rather, the labour theory of value is itself defined so that profits are an exact measure of unpaid labour. This means that the (NI) is a very general one, and remains valid whatever prices happen to be (2004, 77).

There are a range of Marxist critiques of the NI. Shaikh and Tonak (1994, 179) argue that the NI is a version of Adam Smith’s ‘labor commanded’ the-
ory of value rather than a Marxian theory of value in which it is the labor in production that adds value to commodities. They argue that the NI definition of the value of labor power is simply the labor commanded in exchange by the wage and surplus value is simply the labor commanded in exchange by profit income. Similarly, Saad-Filho (1996) criticizes the NI for its reliance on empirical market prices to define the value of money and thereby all value quantities. In the NI the effects of circulation on market prices, such as supply and demand imbalances or monopoly power, will impact monetary quantities, and this is transferred back onto the value quantities through the value of money. Labor time in production loses explanatory priority over these circulation issues in the NI. Saad-Filho (1996) and Moseley (2000) also criticize the NI for redefining the aggregate equalities that Marx requires to hold between the price and value measures of the social product to apply to the net rather than gross or total product.\(^3\)

Given that the NI offers only a very partial response to the criticisms of Marxian value theory, the question remains whether other approaches respond more comprehensively. Furthermore, since the NI is a set of weak axioms defining a labor theory at a very high level of generality, many more specific theories of value and price may be consistent with it. A basic objective of this paper is to demonstrate that the SSI is one such theory, and that the NI and SSI are complementary rather than competing theories. For some reason this point has been overlooked by readers of early drafts of this paper, who see in it instead an effort to prove that one of these theories is superior to the other. It is not. This paper proceeds from the premise that the NI is a set of axioms with a high degree of generality, which allows it to be compatible with any number

\(^3\) "... the sum of the profits in all spheres of production must equal the sum of the surplus-values, and the sum of the prices of production of the total social product equal the sum of its value (Marx 1976b, 173)."
of theories of price formation and commodity value. An objective here is to demonstrate that this includes the SSI.

A separate objective is to clarify a number of points about the SSI. Despite having first been presented decades ago and subsequently elaborated in a number of papers, the SSI remains very poorly understood. One point that seems to be universally misunderstood is that all of the primary variables—values, prices of production, surplus value, and profit—in the SSI are denominated in units of socially necessary abstract labor time. This use of labor magnitudes is not surprising for a Marxian theory, and in this regard the SSI is consistent with Marx’s treatment of prices of production in volume three of *Capital* (1967b, ch. IX). But this raises important questions about the role of money and money-denominated magnitudes in this approach. These monetary issues, which are central to the NI, are underdeveloped in the SSI, and potentially represent an important area of complementarity between the two theories.

Contrast helps to clarify, and contrasting the NI with the SSI should help to clarify the features of both. This has been attempted at least twice previously, but neither can be considered successful. Foley (2000, 30-34) discusses the SSI under the heading of the Temporal Single System Interpretation (TSSI), and by equating it with the temporal approach he fails to grasp the important differences between them. He also makes the significant error of mistaking the dimension of the prices of production in the SSI, assuming money-denominated prices (as in the TSSI and NI) instead of labor time prices of production. Mohun (2004, 79-83) also makes this same mistake regarding the labor time dimension of the SSI price of production. This dimensional error may seem a minor detail in the comparison of these theories, but both the coherence and innovativeness of the SSI, as well as an important point of fidelity to Marx’s own approach to values and prices of production, rely on this point. Leaving this issue uncor-
rected in the literature promises to lead to significant confusion among readers (as it did at one time for the author of this paper).

The remainder of the paper is divided into three sections. The first presents a basic outline of the NI that draws heavily from Mohun’s (1994) exceptionally clear and concise presentation. The second section develops an outline of the SSI that relies heavily on Roberts (1997), and, to a lesser degree, on its antecedents Roberts (1981, 1987) and Wolff, Roberts, Callari (1982) and Wolff, Callari, Roberts (1984). This second section also considers similarities and differences between the SSI and the NI, clarifies some points of confusion over the SSI, and introduces monetary variables into the SSI by drawing on the NI theory of money. A final section offers concluding remarks.

**The New Interpretation**

Marxian theory takes as a premise that the transformation of inputs into outputs in production adds value, which is subsequently preserved in the circulation process. Foley (2012, 447) refers to a "law of the conservation of value" in which value (socially necessary abstract labor time) is created in production and conserved in exchange. This is central to the NI because it links labor time to the subsequent expression of value through the price system. Duménil (1984-85) refers to value as a "social substance" created in production as the result of new labor being incorporated with existing commodities, which is then distributed through the price system to the newly-produced commodities. He argues,

The act of pricing does not create the social substance, but merely distributes it. . . . The price system can only arrange the distribution between individuals and among classes . . . This is the core of Marx’s theory (1983-84, 436).
And further,

Whatever the competitive regime, the price-form always corresponds to the same process of the expression of social labor time in the body of another commodity or in a symbol of value (1983-84, 440).

Likewise, Mohun proposes that a price system is a method of distributing labor time to individual commodities and that "prices are always . . . representations or forms of value, or abstract labour (1994, 404)." He concisely describes this relationship between labor time, commodities, and prices in the NI using two equations, which are interpreted here as axioms A1 and A2. The first establishes equality between labor performed during a time period and its expression in the labor requirement of the net product:

\[ L = l \mathbf{x} = \lambda \mathbf{y} \]  

\( L \), a scalar, is the total hours of socially-necessary labor time in production\(^4\); \( l > 0 \) is the (row) vector of direct labor inputs per unit output; \( x > 0 \) is the (column) vector of gross outputs. If \( A \geq 0 \) is the (square) matrix of unit production coefficients \( a_{ij} \), each representing the quantity of the \( i \)th commodity per unit of output \( j \), then \( y = (I - A) x \geq 0 \) is the (column) vector of net outputs, and \( \lambda = (I - A)^{-1} l \) is the vector of direct and indirect labor requirements per unit output. There is nothing distinctly Marxist about \( A1 \), and it has the status of an accounting identity that should hold in any number of fixed-coefficient production models.

The second axiom expresses formally the law of the conservation of value by equating the labor embodied in the net product with its subsequent expression  

\(^4\)For simplicity, throughout this paper hours of labor time are understood to be hours of socially necessary abstract labor time.
by the price system:

\[ \lambda y = py \lambda_m \]  \hspace{1cm} A2

The (row) vector \( p \) is any price vector denominated in units of ‘money’ per unit commodity, and \( \lambda_m \) is the (scalar) value of money, measured in labor time per unit money (see also Foley 1986, 14-15). Since \( \lambda, y \) and \( p \) are all exogenously determined, \( A2 \) defines the value of money as \( \lambda_m = \frac{\lambda y}{py} \). The dimension of \( \lambda_m \), labor time per unit money, also serves as a conversion factor to establish dimensional homogeneity in \( A2 \).

Note that while the unit for prices is defined in terms of money, money itself is not defined as commodity, state, or some other form of money. Foley (1982, 39) passes over this point by stating that the relation characterized in \( A2 \) will hold in the case of either commodity or state money. Presumably this can be extended to other forms of money, which expands the generality of the NI to include not only different theories of price setting, but also various monetary systems.\(^5\) There is also at least one theoretical price system in which \( \lambda_m \) is unnecessary to establish dimensional homogeneity in \( A2 \). That is a system in which \( p \) is denominated in units of abstract labor time per unit commodity. In that case the scalar product \( py \) is itself a measure of labor time and can be equated directly with the quantity of labor in the net product without the need for \( \lambda_m \) to provide dimensional homogeneity. It is demonstrated below that the SSI is a system of this type.

\( A1 \) and \( A2 \) establish the chain of equivalence,

\[ L = lx = \lambda y = py \lambda_m \]  \hspace{1cm} (1)

This formalizes the basic insight of the NI, which is that the net product of the

\(^5\)See Bell (2001) and Wray (1998) for discussions of varieties and hierarchies of money.
economy is the same whether it is measured in terms of labor time ($L$, $lx$, or $\lambda y$) or measured in commodity or state money prices ($py\lambda m$).

Mohun (1994) completes the NI with this definition of the value of labor power ($VLP$):

$$VLP = w\lambda m$$  \hspace{1cm} A3

The wage rate $w$ has dimension units of money per unit labor time. Using $A2$ and $A1$ it is clear that $VLP$ in this approach is the share of net output claimed by aggregate money wages,

$$w\lambda m = \frac{wlx}{py}$$  \hspace{1cm} (2)

Mohun describes this unconventional interpretation of $VLP$ in the following way:

The value of labor power is the share of wages in net output. This defines the value of labor power in terms of a share of aggregate money value added. Rather than the concrete labor embodied in commodities the workers consume, the value of labor power is a proportion of abstract labor performed (Mohun 1994, 403).

It can be shown that the system defined by $A1 – A3$ satisfies Marx’s two aggregate equalities, but only for macroeconomic aggregates and even this requires a partial reinterpretation of the equalities. The equality between the sum of the prices and the sum of the values in the economy is satisfied directly by ($A2$), but only for the net product of the economy, not the total product. The aggregate equality between the sum of the profits and the sum of the surplus value for the gross product can be shown to follow from the system defined by $A1 – A3$. 

9
A "Single System" Interpretation

The NI is, by design, a set of weak axioms that largely avoid the issues at the center of much controversy over Marxian value theory. In contrast, the SSI endeavors to directly address them. This approach emerged contemporaneously with the NI, but has received far less attention, and a good deal of the attention it has received demonstrates how poorly understood it is. Even some of its most basic aspects seem to be misunderstood by sophisticated readers. In part this is because there exists no relatively accessible and detailed close description of this approach. This section is intended to remedy this by providing a close reading of the SSI, with particular attention to its similarities and differences with the NI.

The following discussion of the SSI assumes the simplest case of an economy with \( n \) single-product industries, a single annual production period, and no fixed capital. These assumptions are not necessary for this approach, but they greatly simplify exposition.\(^6\)

*Labor Time Price of Production*

In the NI the price system is the means whereby newly performed labor time is distributed across the net product. This is also true in the SSI, but in a much more direct way. Let \( b \geq 0 \) be a (column) vector containing the wage bundle of commodities per unit labor. The (square) matrix \( M \) of unit commodity advances by capitalists in physical terms is then,

\[
M = A + bl \geq 0
\]

(3)

The elements of \( M, m_{ij} \), have the same dimensions as the elements of \( A \), and

\(^6\)Roberts (1997, appendix 2) analyzes joint production in the SSI.
in the simplest case this matrix is assumed to be irreducible. The vector of prices of production (competitive prices) $p$ is given by,

$$pM(1 + r) = p \tag{4}$$

where $r$ is the dimensionless, scalar rate of profit. The dimension associated with $p$ is discussed below, but the price equation (4) exhibits dimensional homogeneity for prices in any denomination, including commodity or state money, or some other unit. Without further specification $p$ is best understood as any member of the set of possible price vectors, each of which differs according to its denomination.

For convenience define $\lambda^M = \frac{1}{1 + r} \Rightarrow 1 + r = \frac{1}{\lambda^M}$. Substituting this into the (4) and rearranging gives,

$$p[M - \lambda^M I] = 0 \tag{5}$$

Clearly the scalar $\lambda^M$ is an eigenvalue of $M$, and assume both that it is the maximum eigenvalue of $M$ and the economy under consideration is viable, that is it is capable of producing positive profits, which requires that $\lambda^M < 1$ and ensures $r = (1/\lambda^M) - 1 > 0$. Because (5) is homogenous the system has at least one degree of freedom. It also requires further specification of the denomination of $p$.

The SSI price equation (5) is essentially identical to the Marxian price of production system presented by Pasinetti (1977, 127, equation V.A.21), which is a basic Sraffian price system modified to use $M$ rather than $A$. But there is a fundamental and almost universally overlooked difference between Roberts’s method of solving (5) and Pasinetti’s solution to this same system. Pasinetti states that prices of production in this system are determined up to the choice of numeraire, and "adding whatever further equation we choose in order to define
a *numeraire* for the price system, we can determine all the ‘prices of production’ in terms of the chosen *numeraire* (1977, 127). In doing this, Pasinetti assumes a commodity money system, and consequently, his approach leads directly to commodity money prices, with price $p_j$ denominated in units of the *numeraire* commodity per unit commodity $j$.

Roberts’s SSI solution proceeds differently, and readers accustomed to completing price systems like (5) in the manner outlined by Pasinetti should take note that this is *not* how this system is solved. The SSI assumes, as Marx did in his analysis of prices of production in volume three of *Capital* (1967b, chapter IX), that the prices are denominated in units of labor time.  

7 Designate that particular price vector of labor time prices of production as $\rho$. Using $\rho$, the degree of freedom in (5) is eliminated with the condition:

$$l x = \rho y$$

(6)

Since $\rho$ has dimension labor time per unit, the scalar product $\rho y$ is the quantity of labor time in the net product. The scalar product $l x$ has this same dimension, and hence (6) is dimensionally homogenous. Since $l$, $x$ and $y$ are given quantities, $\rho$ is absolutely determined. This net product normalization both makes (5) determinate and establishes an absolute scale for the prices of production by requiring the labor time price of production of the net product to equal the total labor performed during the period.

---

7 Consider Marx’s approach to prices of production in chapter IX. What he keeps constant in his controversial ‘transformation’ of values (labor times) to competitive prices is total labor time, and each of his prices represent re-allocated parts of this total labor time. They are then prices denominated in hours of labor time, not money. Duménil and Foley remark that Marx’s tabular presentation of prices of production is “not rigorous” because the “values and surplus-value are measured in labour time and prices and profits in money (2008, 406).” They note that this can be corrected by defining a general level of prices. From its earliest statements in Roberts (1981) the SSI addressed this by assuming that Marx’s analysis is dimensionally homogenous and all quantities contained in it have the same labor time dimension. It establishes this in precisely the way that Duménil and Foley (2008) propose.
These labor time denominated prices of production $\rho$ are a distinctive aspect of the SSI that seems to be widely overlooked or misunderstood. In effect, instead of choosing one of the $n$ produced commodities as the standard for price, the SSI uses labor time and arrives at prices that are determined absolutely. These prices are the part of the total labor time that must be allocated to each commodity to ensure a uniform rate of profit in a system of labor time prices of production. This approach realizes the NI description of production prices as "re-allotted labor time (Duménil 1983-84, 440)."

It is a basic tenet of the NI that a price system distributes the total labor performed during the period to the net product of that period. This is stated formally in $A_2$ of the NI, and this is also precisely what (6) achieves in the SSI. The resulting price vector is a set of labor-denominated prices that satisfy the requirement for an equalized rate of profit among producers. But it is important to note that because these prices are denominated in labor per unit commodity the two scalar quantities $l_x$ and $p_y$ can be directly equated because both are quantities of labor time, with no need to multiply the price of the net product by a value of money parameter to ensure dimensional homogeneity.\footnote{Both Foley (2000, 31) and Mohun (2004, 80) mistake this basic point in their discussions of the SSI.}

Thus the SSI also establishes equivalence between aggregate labor time and the aggregate price of the net product, but in a system of labor-denominated prices that needs no unit conversion factor. In this case the concept of a ‘value of money’ associated directly with $\rho$ makes little sense.\footnote{In an economy using labor time as the money commodity the value of money would, of course, be unity and a ‘value of money’ parameter would be dimensionless (the dimensions of hour per hour cancelling). So while it is possible to attribute a "value of money" to the system defined by (5) and (6) it makes little sense to do so.} The parameter $\lambda_m$ in the NI provides dimensional homogeneity in $A_2$ by converting the money quantity $p_y$ into a quantity of labor time that can be equated with the quantity of hours $\lambda y$. The vector $\rho$, on the other hand, is measured in labor per unit commodity
and thus $\rho y$ is a quantity of labor time that can directly be equated with $lx$. But while the value of money is not useful in conjunction with $\rho$, this price vector is consistent with the basic principle that the price system distributes the labor time in the economy to the aggregate net product. This principle is common to both the NI and the SSI and was arrived at independently by Roberts (1981) as well as by Duménil (1980) and Foley (1982).

The rate of profit in the SSI is determined by the eigenvalue equation (5), and thus by the conditions of production and distribution, $A, b, l$ and $\lambda^M$. It can be shown that the system exhibits the familiar negatively sloped wage-profit frontier, and that increases in the intensity of labor or length of the working day increase the rate of profit (see Appendix for proofs of these propositions). Thus the rate of profit in the SSI is the outcome of class struggle over the wage, the intensity of labor, and the length of the working day.

It can also be demonstrated that the rate of profit in the SSI is the ratio of two labor time quantities by using $\rho$. The labor time in the gross product is expressed equivalently as either $\rho x \equiv (\rho Ax + \rho blx) (1 + r)$ or $\rho x \equiv \rho y + \rho Ax$. Equating these two definitions, and solving for $r$ yields,

$$r = \frac{\rho y - \rho blx}{\rho Ax + \rho blx}$$

(7)

The numerator of this ratio is the share of the net product that accrues as profit. Since $\rho y = lx$ (from (6)), this equals the total labor performed ($lx$) net of the labor necessary to produce the total wage bundle ($\rho blx$). This is labor time price of production measure of aggregate surplus labor performed. The denominator is the sum of nonlabor production inputs ($\rho Ax$) and the aggregate wage bundle ($\rho blx$), all evaluated as quantities of labor time. This is consistent with Marx’s proposition (1967b, 42) that the rate of profit is the ratio of the
unpaid labor (surplus labor) to paid labor (labor content of labor and nonlabor inputs). It is obvious from inspection of (7) that there is an inverse relation between the rate of profit and the real wage. Increasing the real wage bundle \( b \) reduces the profit rate by both reducing the profit in the numerator and increasing input cost in the denominator.

\[ \text{Commodity Money or State Money in the SSI} \]

The labor time dimension of \( \rho \) is a distinctive feature of the SSI among modern approaches to prices of production. But this leaves the theory still in the realm of labor time rather than money-denominated prices. But any discussion of money prices raises the question of what kind of monetary system is assumed to exist in the economy. Any money price must be associated with a monetary system.

Typically price of production theories assume a commodity money system and commodity money prices. Since these prices represent physical quantities of a \textit{numeraire} commodity, they have a well-defined relation with labor time prices of production. For an economy using the \( k \)th produced commodity as money, each commodity money price \( p_j \) is the physical quantity of the \( k \)th commodity that exchanges for a unit of the \( j \)th commodity. The labor time associated with this quantity of commodity \( k \) under conditions of profit rate equalization is \( \rho_k p_j \). This must equal \( \rho_j \), the labor time associated with commodity \( j \) in exchange\textsuperscript{10}:

\( \rho_k p_j = \rho_j \quad (8) \)

\textsuperscript{10}Explicit dimensional analysis makes this clear. With commodity \( k \) as money the commodity price of \( j \), \( p_j \), has dimension \((\text{units } k/\text{unit } j)\). The labor time price of production of the \( k \)th commodity, \( \rho_k \), has dimension \((\text{labor/units } k)\). The product \( \rho_k p_j \) has dimension \((\text{labor/units } k)(\text{units } k/\text{unit } j) = (\text{labor/units } j)\), which is the dimension of \( \rho_j \). So the labor time associated with the \( k \) price of a unit of \( j \) equals the labor time associated with a unit of \( j \).
This implies that for any commodity $j$ its commodity money price $\bar{p}_j$ is given by the ratio,

$$\bar{p}_j = \frac{\rho_j}{\rho_k}$$

(9)

Using $\bar{p}$, the vector of commodity money prices, it is possible to determine the NI equivalent of the value of commodity money $\bar{m}$ in the SSI. Substituting $\rho = \rho_k \bar{p}$ from (8) into (6) gives,

$$lx = \rho_k \bar{p}y$$

(10)

or,

$$\bar{m} = \rho_k = \frac{lx}{\bar{p}y}$$

(11)

In this expression total labor is defined by $lx$ rather than $\lambda y$ (as in A2), though a substitution could be made as long as $\lambda$ is recognized to contain accounting parameters with no theoretical content beyond that. Since $A1$ establishes an accounting identity between $lx$ and $\lambda y$ (and nothing more), the first equality in (11) is also more accurately described as an identity, and $A2$ is maintained in the SSI. In other words, in a commodity money system the concept of the ‘value of money’, which the NI derives from macroeconomic aggregates, is identical to the labor time price of production of the money commodity in the SSI. This is the amount of social labor time that one unit of the money commodity exchanges for in this system of competitive prices.

It is also important to note that while the NI is agnostic with respect to price setting, and asserts that (A2) holds definitionally no matter how prices are determined (Foley 1982, 38), the SSI does have a specific theory of production prices. The SSI is then much less general than the NI, but this loss of generality supports a great many of the basic arguments of Marxian theory. In particular, labor time production prices are determined endogenously from
the conditions of production and distribution in a competitive economy rather than being exogenous. But clearly both \( \rho \) and \( \bar{p} \) should be included in the set of price vectors consistent with the NI because both sustain the fundamental proposition that value is created in production and preserved in exchange (the conservation of value principle), as well as the corollary that the price system simply distributes this value across the commodity output.

This result is also present in the SSI when assuming a state money system. The vector of state money production prices \( \hat{\mathbf{p}} \) (with dimension currency units per unit commodity) is,

\[
\hat{\mathbf{p}} = \frac{1}{\lambda_m} \rho
\]

The scalar \( \lambda_m \) is the value of state money and its reciprocal is the MELT. In this instance the MELT coefficient serves two purposes. The first is dimensional. The dimension for this version of the MELT is units of state money per unit labor, and as such it converts the labor time production prices to a quantity denominated in currency units per unit commodity.

The second purpose of the MELT in this case is to scale the elements of \( \rho \) relative to the elements of \( \hat{\mathbf{p}} \). State money is not convertible at a fixed rate into any commodity, so it has no intrinsic value. Therefore \( \lambda_m \) cannot be determined except in reference to an exogenously given vector of state money prices. The structure of relative prices is established by \( \rho \) and is invariant with respect to changes in \( \lambda_m \), but \( \lambda_m \) itself depends on the exogenously given vector of state money prices. The definition of \( \lambda_m \) also follows from A2:

\[
\mathbf{l} = \hat{\mathbf{p}}^\omega \lambda_m
\]
or

\[ \hat{\lambda}_m = \frac{\mathbf{l}x}{\mathbf{p}^\circ y} \]  

(14)

In these two expressions \( \mathbf{p}^\circ \) designates any arbitrary vector of state money prices. One possible choice of theoretical interest for \( \mathbf{p}^\circ \) is a vector whose elements are identical to \( \mathbf{p} \) except for their dimension (one a vector of state money prices the other a vector of labor time prices of production). In that case \( \mathbf{l}x \) is numerically (though not dimensionally) equal to \( \mathbf{p}^\circ y \), and \( \hat{\lambda}_m \) equals one hour of labor time per unit money (one currency unit equals one hour of labor). This gives \( \mathbf{p} \) the same elements as \( \mathbf{p} \), while maintaining their different dimensions. Alternatively, \( \mathbf{p}^\circ \) could be an estimate of an empirical price vector. The vector \( \mathbf{p} \) could also be estimated for this economy, as well as \( \hat{\lambda}_m \) and thereby \( \mathbf{p} \). This provides a bridge between empirical phenomena in a state money using economy and labor time prices of production. Estimating the correlation between observed state money prices and labor time prices of production is one obvious research program made possible by this.

Value

Turning now to Marx’s two aggregate equalities, the SSI assumes that these hold as postulates rather than theorems. These are defined as:

\[ \mathbf{v}x \equiv \mathbf{\rho}x \]  

(15)

\[ \mathbf{s}x \equiv \mathbf{\pi}x \]  

(16)

The elements of the (row) vector \( \mathbf{v} \) are commodity values for the \( n \) commodities; \( \mathbf{s} \) is a (row) vector of surplus value per unit; and \( \mathbf{\pi} \) is a (row) vector of profit per unit.

It is important to note that \( \mathbf{v}, \mathbf{s} \) and \( \mathbf{\pi} \) all have the same dimension as \( \mathbf{\rho} \),
hours of labor time per unit. This makes it possible to write Marx’s aggregate equalities as identical scalar products, with no need to convert \( \rho \) or \( \pi \) from money to labor time magnitudes. However, Roberts is clear that these labor-time quantities have monetary analogs,

As stated, both (15) and (16) are understood here as expressions in labor time terms; both could, of course, also be measured in terms of money, as Marx often does, once the monetary unit is defined and expressed as a particular magnitude of labor time, but the labor-time unit of account, which Marx (1967a, p. 38) establishes prior to considering money, is the principal focus of interest . . . . (1997, 486)

The vector \( \pi \) represents the profit per unit on the capital advanced evaluated at prices \( \rho \) and marked-up at profit rate \( r \), or \( \pi = \rho M r \). An alternative expression for \( \pi \) can be derived using (5). Profit is the difference between the selling price of a commodity and the cost of inputs \( \rho (1 - M) \). Equation (5) becomes \( \rho M = \rho \lambda^M \) when (6) is assumed, and from this,

\[
\rho (1 - M) = (1 - \lambda^M) \rho
\]

or,

\[
\pi = \left(1 - \lambda^M\right) \rho
\]  

(17)

Roberts (1997, 2009) shows that \( v \) is determined by the conditions of production and distribution, that is \( A, l, b \) and \( \lambda^M \) along with the scale of output \( x \), but it is possible to derive a useful expression for this vector directly from
Subtracting these two identities gives,

\[ v \mathbf{x} - s \mathbf{x} = \rho \mathbf{x} - \pi \mathbf{x} \]

Substituting \( \rho \mathbf{M} \mathbf{x} = \rho \mathbf{x} - \pi \mathbf{x} \) and rearranging gives,

\[ v \mathbf{x} = (\rho \mathbf{M} + s) \mathbf{x} \]

or,

\[ v = \rho \mathbf{M} + s \tag{18} \]

Value \( v \) in the SSI is, thus, the sum of the wage and commodity inputs into production, evaluated in terms of their labor time price of production \( \rho \), and surplus value \( s \). This is consistent with Marx’s argument that in a competitive economy "commodity value = cost price + surplus value (1967b, 26)".

Equation (18) is the reason that this interpretation of Marxian value theory is referred to as a 'single system'. It shows a relation between values and prices, rather than the strict separation of them required by dual system interpretations. Kliman and McGlone (1999) apply the term ‘single system’ to this approach and remark that what the SSI and the temporal single system interpretation (TSSI) share in common is the idea that prices and values are interdependent, and hence form a single system. They present a definition of value that is similar to (18) in that it measures the constant capital advanced using prices rather than values.\(^{11}\) Similarly Wolff, Roberts, Callari (1982) state,

\[ \ldots \] the quantity of labor time in money form which each capitalist must actually advance to get his constant capital goods

(Their respective prices of production) becomes a constituent part of

\(^{11}\)Their value equation differs in a number of ways from (18), not least of which is that it uses "actual market prices" rather than the labor time price of production vector \( \rho \).
the value of the commodities produced with those constant capital
goods (574. See also Wolff, Callari, Roberts 1984, 126).

The aspect of the TSSI and SSI that makes them both 'single systems' is this
determination of value by price. But it is also important to note that Roberts
(1997) goes to great lengths to prove that $\rho$ is determined by $A, l, b$ and $\lambda^M$, while $v$ is determined by these same things plus $x$. He emphasizes that $v$ can be
found from the data describing production and distribution without reference
to prices at all (489), and furthermore that while competitive prices can be
derived without reference to values (as shown above), they can also be derived
from values using a linear operator that transforms $v$ values into $\rho$ prices (491).
This position is maintained in his later work (2009). These points are important
because they make it clear that it is labor time in production that adds value
to commodities, not things occurring in circulation. Some version of equation
(18) is present in all of the SSI literature, but Roberts's later work shows a
change in interpretation away from the functional determination of $v$ by $\rho$ in
the early literature to a simple relation between them, with each determined
independently by the conditions of production and distribution. So while it
is appropriate to characterize the early SSI literature as describing values and
prices in terms of a 'single system', it appears to be inaccurate to refer to the
later, fully-developed, statements of this approach in this way.

To complete the description of the basic quantities in the SSI, an expression
for $s$ can be derived by first noting that aggregate profit is identically equal to
aggregate surplus value (according to (16)). This implies that the surplus value
per unit commodity can be expressed as total profit distributed to the output
commodities in proportion to the labor used in their production. The fraction
of the total direct labor that commodity $j$ embodies is $[(1/1x)l_j]$, and using this,
the gross product vector \( \mathbf{x} \), and (17), \( \mathbf{s} \) is,

\[
\mathbf{s} = (1 - \lambda^M) \rho \mathbf{x} l(1/\mathbf{l})
\]  

(19)

This is one way to describe \( \mathbf{s} \) in the SSI. Like the definition of \( \mathbf{v} \) this also relies on \( \mathbf{p} \), but this is a matter of convenience rather than necessity as \( \mathbf{s} \) can also be determined without reference to price magnitudes.

\textit{Labor Time Equivalents and the Value of Labor Power}

An expression for the SSI similar to the NI chain of equivalence (1) extends only to the various price measures of the net product,

\[
L \equiv \mathbf{l} \mathbf{x} = \rho \mathbf{y} = \rho_k \mathbf{p} \mathbf{y} = \hat{\lambda}_m \hat{\mathbf{p}} \mathbf{y}
\]  

(20)

The identity \( L \equiv \mathbf{l} \mathbf{x} \) is a definition; \( \mathbf{l} \mathbf{x} = \rho \mathbf{y} \) is established by (6); \( \mathbf{l} \mathbf{x} = \rho_k \mathbf{p} \mathbf{y} \) is established by (10); \( \mathbf{l} \mathbf{x} = \hat{\lambda}_m \hat{\mathbf{p}} \mathbf{y} \) is established by (13). All of the terms in (20) are quantities of labor time and it clearly demonstrates the principle that a price system distributes the total labor performed during the period to the net product of that period is clearly present in the SSI in its labor time, commodity money, and state money price variants. Unlike the NI chain of equivalence no measure of the value of the net product appears in (20). This is an important difference between the NI and the SSI, and it occurs because the SSI maintains that the value-price identity \( \mathbf{v} \mathbf{x} \equiv \rho \mathbf{x} \) holds for the total product rather than the net product. Since \( \mathbf{v} \neq \rho \) the value and price of any commodity bundle that is a subset of \( \mathbf{x} \) will only be equal for commodity vectors that are scalar multiples of \( \mathbf{x} \), and this will not generally be true of \( \mathbf{y} \).

Finally, regarding the \( VLP \), equivalent expression to A3 and (2) can be derived for the SSI using either commodity or state money prices. With com-
modity money, for example, the $VLP$ is,

$$VLP = \bar{w} \lambda_m = \frac{\bar{m} l x}{p y}$$

(21)

As with the NI, this defines the value of labor power as the share of wages in net output. Here $\bar{w}$ is the wage rate as a quantity of commodity money per unit labor time and $\bar{w} \equiv \rho b \left( \frac{1}{\lambda_m} \right)$. The labor time expression for $VLP$ using commodity money follows immediately from (21) and the definition of $\bar{w}$, or, equivalently, by substituting the definition of $\bar{w}$ and (6) into the third term of (21),

$$VLP = \rho b$$

(22)

This is a much more orthodox interpretation of the value of labor power, which interprets it as the labor time associated with the commodity bundle advanced to workers per unit of labor power, but it is also consistent with the NI proposition that the $VLP$ is the wage share of net output. The result is the same when using state money quantities $\hat{p}, \hat{w}$ and $\hat{\lambda}_m$. The value of labor power in terms of values $vb$ will, in general, differ from the value of labor power in terms of labor time prices of production $\rho b$ because of the differences between $v$ and $\rho$.

As with any commodity, or set of commodities, this value-price deviation (or difference between value and value-form) is a consequence of the re-allocation of labor time among commodities that occurs in exchange through the price system.

Conclusions

The NI and SSI should be interpreted as complementary rather than competing approaches to Marxian value theory. The NI consists of a small set of weak axioms with a high degree of generality, but it is incomplete as a response to the traditional critiques of Marxian value theory. The NI is also subject to
criticism on the grounds that it subsumes the concept of value to the concept of price, and satisfies Marx’s aggregate identities by reinterpreting one as applying to the net rather than the gross product.

The SSI is a consistent theory of value and price for individual commodities that is consistent with the three axioms of the NI. Indeed A2—arguably the most important contribution of the NI—is present in the SSI from its inception. A unique labor-denominated price vector allows the SSI to develop a comprehensive price accounting using labor time as the fundamental unit of account. This aspect of the SSI seems to have been largely overlooked or misunderstood. Why this occurred is not obvious, but it seems to involve either confusing the SSI with the TSSI, which assumes conventional money-denominated prices, or misunderstanding the method of determining the labor time price of production vector \( p \).

The SSI also provides a theory of value that is not subject to the neo-Ricardian critique, while satisfying both of Marx’s aggregate equalities as stated by him. In short, the SSI provides a theoretically consistent Marxian theory of value and price of production, as well as an extensive response to the neo-Ricardian critique, which is something the NI does not do on its own. But the connection between the labor-denominated quantities in the SSI and monetary systems has not previously been developed, which made it difficult to use in interpreting money-using capitalist economies. This paper demonstrates that integrating money, either commodity or state, into the SSI is possible. Further, it demonstrates that the basis for the money-denominated price vectors (\( \mathbf{p} \) and \( \mathbf{\hat{p}} \)) is labor time manifesting through the price system in economies using commodity or state money. This treatment formalizes the relationship between commodities, commodity money, state money, and labor time in a much more comprehensive and consistent manner than has been achieved previously.
Future work should undertake to apply this synthetic approach to empirical applications.

After surveying developments in recent decades on Marxian value theory Foley remarks "I see as a large degree of practical and operational agreement on the labor theory of value emerging (2000, 34)". This paper concurs with that assessment, though it has been necessary to correct some important misinterpretations about the SSI to see this clearly. But Foley’s assessment that the central insight of the NI is that the MELT makes a separate accounting using embodied labor coefficients unnecessary, seems potentially misguided if this is interpreted to mean that any value system not tied to process is superfluous. This would be true if embodied labor values were necessary for a comprehensive accounting using labor time, but this is not the case. The SSI provides a consistent method of doing this, and therefore the important contributions of the NI are found elsewhere, notably the way that it elaborates Marx’s synthesis of the labor theory of value with a theory of money. Drawing from both of these approaches makes it clear that the fundamental Marxian labor time accounting provides a viable means to interpret the monetary phenomena that are characteristic of a capitalist economy.

*Acknowledgements:* Bruce Roberts patiently answered my many questions about his work on single system value theory. His assistance has been invaluable in the writing of this paper. Fred Moseley generously provided comments on an early draft of this paper. Comments from the audience at the 2015 URPE at ASSA session, where an early version of this paper was presented, are also gratefully acknowledged.
Appendix: Proofs

The following is Mohun’s (1994, 403-404) proof of the equality between aggregate surplus and aggregate profit in the NI.

It is shown in 2 that \( w\lambda_m \) is the wage share of net output. Designating aggregate money profits as \( \Pi \) the money measure of the net product is \( py = \Pi + wlx \), and the profit share \( (1 - w\lambda_m) \) is,

\[
1 - w\lambda_m = \frac{py - wlx}{py} = \frac{\Pi}{py}
\]

The ratio of the value of labor power to the profit share is,

\[
\frac{1 - w\lambda_m}{w\lambda_m} = \frac{\Pi}{wlx} = \frac{\Pi}{W}
\]

where \( W \) is aggregate wages. The portion of aggregate labor time that is surplus is \( S = L(1 - w\lambda_m) \) and the portion that constitutes variable capital is \( V = Lw\lambda_m \). Thus,

\[
\frac{S}{V} = \frac{L(1 - w\lambda_m)}{Lw\lambda_m}
\]

Cancelling \( L \) in the numerator and denominator of this expression, and using \( \lambda_m \) to create dimensional homogeneity between the labor-denominated terms and the money denominated ones, it is possible to write,

\[
\frac{S}{V} = \frac{\Pi\lambda_m}{W\lambda_m}
\]

or

\[
S = \Pi\lambda_m
\]
Proposition 1: The rate of profit declines when the wage bundle increases.

Proof: By definition \( r = (1/\lambda^M) - 1 \) so \( r \) is monotonically decreasing in \( \lambda^M \). By assumption \( M \) is an indecomposable, nonnegative, square matrix, and \( \lambda^M \) is its maximum eigenvalue. According to the Perron-Frobenius theorems for indecomposable nonnegative matrices (see Kurz and Salvadori (1995, 517) Theorem A.3.5 (f)) \( \lambda^M \) is an increasing function of each of the elements of \( M \). By (3) increasing any element of \( b \) increases at least some of the elements of \( M \), and hence increases \( \lambda^M \) while reducing \( r \), and Proposition 1 is true.

□

Corollary 1: Increasing the intensity of labor or length of the working day (with \( b \) constant) increases the rate of profit.

Proof: Increasing either the intensity labor or length of the working day reduce the elements of \( l \). Reducing \( l \) reduces the elements of \( bl \) and \( M \), and it is shown in the proof of Proposition 1 that \( r \) increases when any element of \( M \) declines.

□

Works Cited


to Karl Marx and the close of his system, by Eugen von Böhm-Bawerk; and Böhm-Bawerk’s criticism of Marx, by Rudolf Hilferding; together with an appendix consisting of an article by Ladislaus von Bortkiewicz on the transformation of values into prices of production in the Marxian system, Paul M. Sweezy (Ed.). New York: Augustus M. Kelley.


