DEPARTMENT OF ECONOMICS

Working Paper

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Working Paper 2016-14

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Entitlement theory of justice and end-state fairness in the allocation of goods*

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November 18, 2016

Abstract

Robert Nozick allegedly introduced his liberal theory of private ownership as an objection to theories of end-state justice. Nevertheless, we show that, in a stylized framework for the allocation of goods in joint ventures, both approaches can be seen as complementary. More precisely, in such a context, self-ownership (the basis for Nozick's entitlement theory of justice) followed by voluntary transfer (Nozick's principle of just transfer) can lead to end-state fairness (as well as Pareto efficiency). Furthermore, under a certain solidarity condition, the only way to achieve end-state fairness, following Nozick's procedure, is to endorse an egalitarian rule for the initial assignment of rights.

*We thank Roberto Veneziani, as well as the remaining participants of the 2015 QMUL Exploitation Workshop for helpful comments and suggestions. We are also grateful to William Thomson, Richard Bradley, and two anonymous referees for their insightful comments to revise the paper.

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1 Introduction

The last decades have witnessed a considerable effort among political philosophers to analyze the problem of distributive justice. A central impetus for this should be attributed to John Rawls's theory (e.g., Rawls, 1971), probably the most influential endorsement of egalitarianism in the twentieth century. Roughly speaking, Rawls argued that in an "original position," before knowing what our talents, wealth, or education are, we would agree on basic principles about justice and the distribution of the fruits of collaboration in society. Rawls then concluded that justice thus defined depends entirely on the *pattern* or *end-state* distribution at any moment in time. Rawls' conclusion, the so-called *Difference Principle*, is that social and economic inequalities are to be arranged so that they are to the greatest benefit of the least advantaged.

Prompted by Rawls' theory, Robert Nozick presented another (polar) theory of distributive justice (e.g., Nozick, 1973; 1974). The theory is grounded on *self-ownership*, one of the fundamental axioms of liberal political philosophy, represented classically by John Locke (e.g., Locke, 1988). Self-ownership, which is taken to a different level in Nozick's theory, is a somewhat attractive postulate (albeit denied by Rawls himself and other influential political philosophers, such as Ronald Dworkin) declaring some rights to derive from superior skills. Nozick's theory questions Rawls' focus on the end-state distribution in assessing distributive justice. The core of his argument is that, if the initial distribution of property rights is just (i.e., property does not derive from exploitation or theft), and the exchanges that follow this initial situation are voluntary (i.e., there is no coercion), then the resulting distribution, no matter how unequal, would also be just. Nozick argues that justice consists of respecting individual's rights, particularly the right to self-ownership and the freedom to decide how to use one's property. This means that economic inequalities need not be considered unjust nor be rectified to the benefit of the disadvantaged. Nozick stresses that justice corresponds to the respect of individual rights, which are more important than an agreement reached in Rawls' original position (which is just a thought experiment). Thus, if attaining end-state equity requires violating property rights, this cannot be just.

More precisely, the following quote from Nozick (1973, page 47) nicely summarizes the core position regarding his liberal theory of private ownership:

> "If the world were wholly just, the following inductive definition would exhaustively cover the subject of justice in holdings:
>
> 1. A person who acquires a holding in accordance with the principle of justice in

acquisition is entitled to that holding.

2. A person who acquires a holding in accordance with the principle of justice in transfer, from someone else entitled to the holding, is entitled to the holding.

3. No one is entitled to a holding except by repeated applications of (1) and (2)."

Our aim in this paper is to show that, in a stylized framework for the allocation of goods in joint ventures, Nozick's and Rawls' approaches can be seen as complementary.

To wit, we consider a model formalizing three different levels of fairness for the allocation of goods in joint ventures:

1. Fairness in the initial allocation of rights.

2. Fairness in the transaction of rights.

3. Fairness of the end-state allocation.

The formalization of the first two levels will be inspired by Nozick's *procedural* approach. More precisely, we shall focus on a family of rules allocating goods in two successive stages. The first stage (*rights assignment*) determines an initial allocation of rights. The second stage (*exchange*) determines a final allocation from such an initial allocation.

We formulate *self-ownership* as an axiom for the first stage of rights assignment inspired by Nozick's principle of justice in acquisition. In our model, individual claims represent the (objective and verifiable) amounts of goods the person can obtain through her self-ownership, when it does not conflict with the self-ownership of anyone else. Thus, in an economy with abundant social endowment for fully satisfying all individual claims, self-ownership admits that all claims are granted.

Nozick's procedural approach can also provide a useful guideline for the second stage of exchange. More precisely, the second stage implements Nozick's principle of just transfer by imposing the application of a *voluntary exchange* rule, i.e., a rule guaranteeing that agents only exchange when they improve from their endowments.

The formalization of the third level will rely on the notion of *no-envy*, probably the concept with the longest tradition in the theory of fair allocation (e.g., Tinbergen, 1953; Foley, 1967).[1] No-envy is satisfied if no agent prefers the consumption by anyone else to her own. The same

[1]No-envy is also used by Ronald Dworkin as a basic test for resource egalitarian allocations (e.g., Dworkin, 1981, p.285).

comparative notion of fairness, defined through interpersonal comparisons of *net consumptions* (consumptions net of "claims"), gives rise to the notion of *net-no-envy*, which we shall also consider here.[2]

No-envy conceptualizes the impartial spectator's point of view, à la Adam Smith, by requiring that agents place themselves in the situation of other agents. A different, yet related, conceptualization of the impartial spectator is the contractarian construct of veil of ignorance by John Harsanyi (1953, 1955) and John Rawls (1971), which enforces the decision maker to evaluate the outcome through the individual standards of well-being.[3] The main advantage of no-envy, in comparison with the mentioned contractarian theories, is that it does not rely on *cardinal* preferences; it is based purely on *ordinal* preferences.

Our results show that the combination of a rights-assignment rule, satisfying self-ownership, with a voluntary exchange rule, may lead to end-state fairness, as formalized by the no-envy axioms described above, as well as to (Pareto) efficiency. Conversely, we show that the two focal (and somewhat polar) rules, known as *constrained equal awards* and *constrained equal net-awards* are the unique *solidaristic* ones that lead to fair end-state allocations. The two rules have a long tradition of use, which can be traced back to Maimonides (e.g., Thomson, 2003). Although they assign rights in quite different ways, they both achieve equality with different perspectives; namely, equality of the absolute or net amounts.

Therefore, our investigation provides an instance where a principle of end-state fairness can facilitate the search of appropriate procedural principles of justice (in particular, principles of just acquisition), which constitute Nozick's procedural (or historical) theory of justice. Conversely, Nozick's theory can be used to implement a principle of end-state fairness through informationally simple and voluntary procedures. This is why we claim that Nozick's procedural approach, at least in our framework, is complementary to the (Rawlsian) end-state approach.

Our contribution in this paper can also be viewed as an alternative way of extending Locke's theory (at least in a highly stylized framework of joint ventures). Following a similar line of investigation, Roemer (1988, 1989), Moulin (1987, 1990), and Roemer and Silvestre (1993) propose generalizations of Locke's theory in the framework of common resources under a

[2]This is reminiscent of the classical notion of *fair net trades*, introduced by Schmeidler and Vind (1972).

[3]While no-envy provides a specific standard for fair allocations, the contractarian theories only provide the environment of impartial decision making and leave it up to the "rational" decision maker to come up with the exact standard of fair allocations; namely, the utilitarian allocation for John Harsanyi and the difference principle for John Rawls.

decreasing-returns-to-scale technology, which gives rise to the so-called *tragedy of the commons*. The allocation rules proposed in these works respect Locke's thesis based on self-ownership: that is, they coincide with the unlimited appropriation outcome in the case of a constant returns to scale technology, the case satisfying the Lockean proviso. Nevertheless, unlike Nozick's radical generalization, they all have egalitarian features. It turns out that some of the rules highlighted in this literature are similar to the rules derived here.

Somewhat related, Gibbard (1976) and Grunebaum (1987) propose "equal rights" or "public ownership" of unowned properties to be the baseline upon which the appropriation should be judged.[4] Moulin and Roemer (1989), in a production economy model, investigate implications of the baseline of public ownership without denying the thesis of self-ownership.[5] Their axiomatic approach shows that the axioms for public ownership and self-ownership, together with other standard axioms, imply a unique welfare-egalitarian outcome, which disregards any difference in individual talents. Hence slightly strengthening their axiom of self-ownership to rule out the welfare-egalitarian outcomes and admit only less extreme ones will break the coherency of the set of axioms. All their axioms are for end-state rules and they do not deal with the assignment of ownership rights. The egalitarian rules we support here exhibit their egalitarian features only in the assignment of property rights and so diverse end-state allocations may arise through the exchange of the property rights.

Moulin and Roemer (1989) assume a single representative utility function and, due to this feature, their axiom of self-ownership, which is essentially an order preservation property for rights-assignment rules, coincides with no-envy. We do not impose from the outset an order-preservation property because it is implied by other basic axioms. The solidarity axiom we consider (for rights-assignment rules) may be compared to their axiom of public ownership, called "technology monotonicity". However, our axioms are merely requirements in the rights-assignment stage. They are not requirements for end-state rules as in Moulin and Roemer (1989). Hence, it could be argued that our axioms are in a certain sense weaker than theirs; in fact, they are extremely mild allowing for a rich spectrum of rules. In our approach, the baseline of public ownership and the thesis of self-ownership can be met jointly without putting too much restriction on the choice of rules. End-state fairness plays a critical role to pin down

[4]Nozick sets the baseline to be the state where the unowned properties are unowned; their appropriation, according to Nozick, gives the appropriator the entitlement to the properties as long as no one is harmed relative to the baseline. Roemer (1996, Chapter 6) gives a comprehensive overview of the related literature.

[5]Ownership rights in their paper are assumed to be respected when a rule satisfies certain axioms.

a unique egalitarian rule.

Using no-envy as both procedural and end-state principles of fairness, Kolm (1972), Feldman and Kirman (1974), Goldman and Sussangkarn (1980), and Thomson (1982), among others, investigate whether procedural fairness induces end-state fairness. The results are negative. The combination of envy-free initial allocation (equal division) and a sequence of envy-free trades may lead to a core allocation with envy. Our results impose different versions of no-envy as the principle of end-state fairness and obtain no-envy, "with some constraints", of the initial allocation as an implication. We do not impose no-envy as a procedural requirement. Nevertheless, other axioms are used as procedural requirements such as self-ownership for rights assignment rules and voluntary exchange for exchange rules.[6]

In standard exchange economies, Thomson (1983) is also concerned with the three levels of justice: fair initial position (endowment), fair trade (or exchange), and end-state fairness. In his approach, the principle of fair trade plays a central role and the principle of fair initial position is formulated through the possibility of changing the initial positions of agents (as in the definition of no-envy) and their objections based on the principle of fair trade from any reshuffled position. Thus, the key idea of no-envy is behind his notion of fair initial position. He shows that *no-envy* is the unique end-state fairness concept that is obtained from his procedural approach using voluntary exchange as the principle of fair trade (Proposition 1). His main result is that Walrasian trade and the principle of fair initial position defined via Walrasian trade give rise to the same outcomes as the Walrasian rule from equal division (Proposition 2). In a sense, this result says that if one accepts Walrasian trade to be a fair rule of trade, and one also accepts the possibility of changing initial positions among agents, then the only fair initial position is equal division. Our Theorem 1 can be viewed as reinforcing this conclusion in our extended framework, when adopting Nozick's normative perspective. Our approach is informationally simple and guarantees freedom of choice. It is also representative of actual institutions. The first stage of rights assignment allows us to use the findings in the vast literature on rights problems.[7] The second exchange stage is assumed to meet *voluntary exchange*; so not only the Walrasian (perfectly competitive market) trade but also non-Walrasian (imperfectly competitive market) trades are covered. Unlike Thomson (1983), we use no-envy as the end-state fairness axiom and characterize egalitarian rights-assignment rules for the first stage.

[6]Our model and the procedural approach follow the lesson on procedural fairness delineated by Thomson (2011, pp.419-422).

[7]See, for instance, O'Neill (1982) and Thomson (2003, 2015, 2016b).

2 A model of joint ventures

Consider a society N of agents, who share common resources for producing ℓ privately appropriable and infinitely divisible goods. Agents have preference relations defined on the corresponding set of consumption bundles (i.e., real vectors with ℓ coordinates).[8] Each agent $i \in N$ is also endowed with specific capabilities that would allow her to produce certain amounts of the ℓ goods with exclusive access to the common resources. We shall refer to the resulting profile of those amounts, denoted by c_i, as the *claim* of agent i. We assume that claims are objective and verifiable. The classical Lockean "thesis of self-ownership" (e.g., Locke, 1988), to which Nozick (1974) adheres, would state in this context that, in the society consisting of a single person, she is the only one with access to the common resources and, thus, her claim can be granted. We assume that all agents in society collaborate in a joint venture, which allows them to use the common resources cooperatively to produce a *social endowment* Ω of the ℓ goods. When the joint production technology exhibits decreasing returns to scale, the sum of claims exceeds the social endowment, i.e., $\sum_{i \in N} c_i \geqq \Omega$.[9] When the technology exhibits increasing returns to scale, the sum of claims does not reach the social endowment, i.e., $\sum_{i \in N} c_i \leqq \Omega$.[10]

We thus consider the problem of allocating the social endowment among the agents based on their claims. Formally, an *economy* $e \equiv (N, \Omega, c, R)$ is defined by the set of agents, a social endowment Ω, a profile of individual claims $c \equiv (c_i)_{i \in N}$, and a profile of preferences $R \equiv (R_i)_{i \in N}$. Let $\mathcal{E}$ denote the set of all economies. Throughout the paper, we will assume "private goods" economies where each agent is not concerned with how much others consume. Hence agent i's preferences R_i, namely the binary orderings of her well-being can be defined over her consumption bundles. Given any pair of consumption bundles x, y, we write $x \, R_i \, y$ when agent i is at least as well off with consuming x as with consuming y; we write $x \, P_i \, y$ when agent i is better off with consuming x than with consuming y.

An *allocation* for an economy is a profile of individual consumption bundles, denoted by

[8] Preferences satisfy the classical conditions of rationality, continuity, strong monotonicity, and convexity. As usual, we denote by R_i the preference relation of agent i, by P_i the corresponding strict preference relation, and by I_i the corresponding indifference relation.

[9] We rule out decisions in the production-side of the economy from our consideration and focus on the allocation of produced goods among persons. This may be viewed as an economy with a simple output technology where there is a uniquely efficient assignment of the agents into production facilities, which gives rise to a uniquely efficient output combination.

[10] Our mathematical notation $x \leqq y$ to relate vectors $x, y \in \mathbb{R}^{\ell}_{++}$ means that $x_k \leq y_k$ for each $k = 1, \ldots \ell$. Likewise, $x \geqq y$ means that $x_k \geq y_k$ for each $k = 1, \ldots \ell$.

$z \equiv (z_i)_{i\in N}$, which is feasible in the sense that the total consumption of each good equals the total endowment, i.e., $\sum_{i\in N} z_i = \Omega$. It is *Pareto efficient* if there is no other allocation that makes a person better off without making anyone else worse off.

The following examples fit the stylized model just described.

Example 1. *Simple joint production economy.* Consider a production economy with a common capital good (land) that can be used for producing a good in each period. There are ℓ different periods and the good produced at period l is referred to as good l. Each agent has a skill to produce each good. Let $s_i \equiv (s_{il})_{l=1}^{\ell}$ be i's skill vector. All agents supply the same unit labor for the joint production and thus effective labor is identified with skill. The production technology for each good l is represented by a production function $f_l \colon \mathbb{R}_+ \to \mathbb{R}_+$ mapping the total effective labor $\sum_{i\in N} s_{il}$ into the amount of output for good l. Let $c_i \equiv (f_l(s_{il}))_{l=1,\dots,\ell}$ be i's claim vector. When the production technology is *subadditive*,[11] the joint production is below the sum of claims.[12] When the production technology is *superadditive*,[13] the joint production is above the sum of claims.

Example 2. *Property rights disputes* (Ju and Moreno-Ternero, 2016). Consider a society in which each person has initial property rights (claims). Due to an unexpected misfortune, for which no one is responsible, the initial property rights cannot be fully respected; the society does not have enough resources to satisfy all of them. That is, denoting the available social endowment by Ω, and claims as $(c_i)_{i\in N}$, $\sum_{i\in N} c_i \geqq \Omega$.

Example 3. *Surplus sharing.* Consider the alternative case to the previous one in which society not only has enough resources to satisfy all individual property rights, but also has a surplus to be shared among all members of society. That is, $\sum_{i\in N} c_i \leqq \Omega$.

An *allocation rule* associates with each economy a non-empty set of end-state allocations. We shall be mostly interested in allocation rules that are defined by the following two consecutive stages: First, a *rights-assignment stage* to deal with the assignment of rights, mapping the non-preference information into a profile of individual endowments, and second, an *exchange stage* determining final allocations for the exchange economy resulting from such a profile of individual endowments obtained in the first stage. In doing so, we shall be able to scrutinize

[11]That is, $f_l(x) + f_l(y) \geq f_l(x + y)$, for each $x, y \in \mathbb{R}_+$.

[12]We assume that the technology cannot be accessed by individuals separately. Otherwise, given the subadditivity of the technology, each one would be better-off producing on her own.

[13]That is, $f_l(x) + f_l(y) \leq f_l(x + y)$, for each $x, y \in \mathbb{R}_+$.

the relationship between principles of procedural justice (imposed in each of these two stages) and principles of end-state justice (imposed on the final allocations determined by allocation rules).

2.1 Rights assignment

A *rights problem* is defined by a set of persons N, a social endowment Ω, and a profile of individual claims c. Let $\mathcal{C}$ denote the set of all rights problems (N, Ω, c). Good l is in *deficit* if the endowment of good l is not large enough to honor all claims, that is, $\sum_{i \in N} c_{il} \geq \Omega_l$. It is in *surplus* if the endowment is more than sufficient to satisfy all claims, that is, $\sum_{i \in N} c_{il} \leq \Omega_l$. A rights problem may involve both a deficit in one good and a surplus in some other good.

A *rights-assignment rule* φ associates with each rights problem (N, Ω, c) individual property rights over the social endowment, specified by an allocation of individual endowments $\varphi(N, \Omega, c) \equiv (\omega_i)_{i \in N}$ (with the feasibility, $\sum_{i \in N} \omega_i = \Omega$) to be traded in the exchange stage.

We impose from the outset the following mild requirement on rights-assignment rules that models the thesis of self-ownership. It is that each person be assigned ownership rights that fully respect her claim, if allowing her ownership of the claimed resources leaves (as stated in the Lockean proviso) "enough and as good left in common for others" (27 in Chapter 5, Locke, 1988). Since her claim represents her own capabilities, the assigned rights in this case fully respect her self-ownership (the ownership of her own capabilities).

Self-Ownership. For each $(N, \Omega, c) \in \mathcal{C}$ and each $i \in N$, if $\Omega - c_i \geqq \sum_{j \in N \setminus \{i\}} c_j$, then $\varphi_i(N, \Omega, c) \geqq c_i$.

Then, by the resource constraint in the definition of a rights-assignment rule, whenever the sum of individual claims equals the social endowment ($\sum_{i \in N} c_i = \Omega$), the rights-assignment should be determined by the claims ($\varphi(N, \Omega, c) = c$). *Self-ownership* concerns problems where a person's claimed ownership leaves enough of the social endowment to fully honor all the remaining claims. If this requirement is not met, private appropriation of socially endowed goods needs to be restricted.

Additionally, we consider a solidarity axiom, which says that the arrival of immigrants, whether or not accompanied by changes in the available endowment, should affect all original agents in the same direction: either all gain or all lose.[14] Formally,

[14]This axiom has been used in related contexts by Chun (1999) and Moreno-Ternero and Roemer (2006), among others.

Solidarity. Let (N, Ω, c) and $(N', \Omega', c') \in \mathcal{C}$ be such that $N \subseteq N'$ and, for each $i \in N$, $c'_i = c_i$. Then, one of the following statements hold:

$$\varphi_i(N, \Omega, c) \leq \varphi_i(N', \Omega', c') \text{ for each } i \in N,$$

$$\varphi_i(N, \Omega, c) \geq \varphi_i(N', \Omega', c') \text{ for each } i \in N.$$

Two focal rights-assignment rules, each obeying the axioms just presented, are defined next.

The *constrained equal awards rule* φ^{CEA} splits the social endowment as equally as possible, provided no agent is awarded more than his claim in the case of a social deficit and less than his claim in the case of a social surplus. Formally, for each $(N, \Omega, c) \in \mathcal{C}$, each $i \in N$, and each $l \in \{1, \ldots, \ell\}$,

$$\varphi_{il}^{CEA}(N, \Omega, c) = \begin{cases} \min\{c_{il}, \lambda\}, & \text{if } \sum_{i \in N} c_{il} \geq \Omega_l, \\ \max\{c_{il}, \mu\}, & \text{if } \sum_{i \in N} c_{il} \leq \Omega_l, \end{cases}$$

where λ and μ guarantee that the feasibility conditions $\sum_{i \in N} \min\{c_{il}, \lambda\} = \Omega_l$ and $\sum_{i \in N} \max\{c_{il}, \mu\} = \Omega_l$ are met. In particular, if for each $i \in N$, c_{il} is larger than equal division of good l, or c_{il} is smaller than equal division of good l, the constrained equal awards rule divides good l equally (i.e., $\varphi_{il}^{CEA}(N, \Omega, c) = \Omega_l/n$, where n denotes the number of agents in N).

The *constrained equal net-awards rule* φ^{CEN} allocates the social endowment so that both agents end up having as equal net awards as possible, provided no agent gets a negative amount. Formally, for each $(N, \Omega, c) \in \mathcal{C}$, each $i \in N$, and each $l \in \{1, \ldots, \ell\}$,

$$\varphi_{il}^{CEN}(N, \Omega, c) = \max\left\{c_{il} - \nu, 0\right\},$$

where ν guarantees the feasibility condition $\sum_{i \in N} \max\{c_{il} - \nu, 0\} = \Omega_l$. In particular, if, for each $i \in N$, c_{il} is larger than the equal net awards of good l, the constrained equal net-awards rule divides the total net-awards of good l equally (i.e., $c_{il} - \varphi_{il}^{CEN}(N, \Omega, c) = c_{jl} - \varphi_{jl}^{CEN}(N, \Omega, c) = (\sum_{i \in N} c_{il} - \Omega_l)/n$).

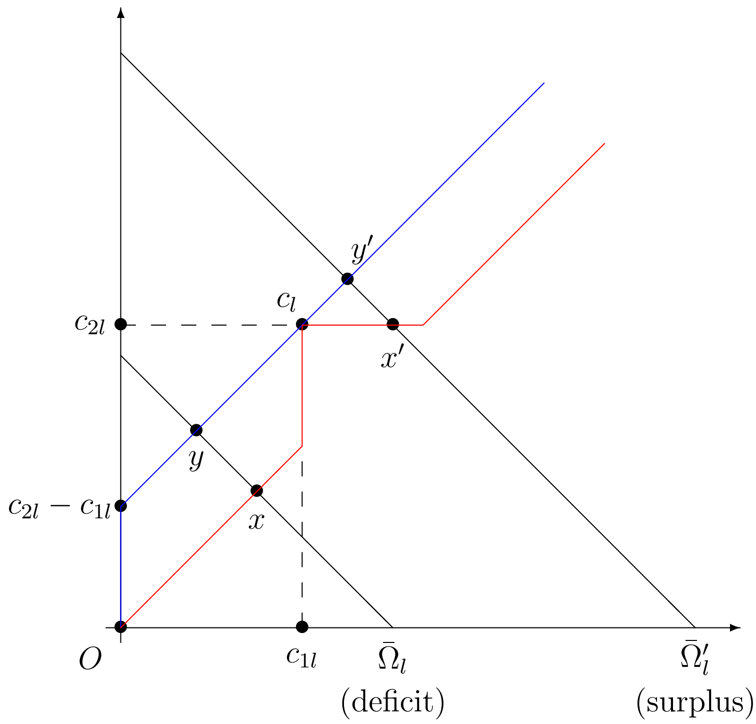

Figure 1: Rules in the two-agent case. This figure illustrates the "paths of awards" (the locus of the awards vector chosen by a rule as the endowment Ω_l varies from 0 to ∞) of the constrained equal awards rule and the constrained equal net-awards rule for $N = \{1, 2\}$ and $c_l \in \mathbb{R}_+^N$ with $c_{1l} < c_{2l}$. The path of awards of the constrained equal awards rule (red) follows the 45^o line until agent 1 obtains her whole claim. Then, it is vertical until it reaches the vector of claims. For endowments above the aggregate claim (i.e., the surplus case), the path is horizontal until agent 1 obtains c_{2l}. From there on it follows again the 45^o line. As for the constrained equal net-awards, its path of awards (blue) is vertical until the average loss coincides with the lowest claim, i.e., until the endowment reaches $\Omega_l = c_{2l} - c_{1l}$. After that, it becomes the line of slope 1 (thus crossing the vector of claims, when moving to the surplus case). In the specific deficit case illustrated in the figure (for endowment $\bar{\Omega}_l < c_{1l} + c_{2l}$), the allocation proposed by the constrained equal awards rule is at the intersection x with the 45^o line, whereas the allocation proposed by the constrained equal net-awards rule is at the intersection y with the parallel line emanating from $(0, c_{2l} - c_{1l})$. In the specific surplus case illustrated in the figure (for endowment $\bar{\Omega}'_l > c_{1l} + c_{2l}$) the allocation proposed by the constrained equal awards rule is at the intersection x' with the horizontal line from c_l, whereas the allocation proposed by the constrained equal net-awards rule is also at the point of intersection y' with the line of slope 1 emanating from $(0, c_{2l} - c_{1l})$.

The next property, which is a useful implication of the combination of self-ownership and solidarity (as shown in Lemma 1 in the Appendix), indicates that in a deficit situation all agents are rationed, whereas in a surplus situation no one is.

Claims Boundedness.

- For each $(N, \Omega, c) \in \mathcal{C}$ and each l, if $\sum_{i\in N} c_{il} \geq \Omega_l$, then $\varphi_{il}(N, \Omega, c) \leq c_{il}$, for each $i \in N$.
- For each $(N, \Omega, c) \in \mathcal{C}$ and each l, if $\sum_{i\in N} c_{il} \leq \Omega_l$, then $\varphi_{il}(N, \Omega, c) \geq c_{il}$, for each $i \in N$.

It is evident by definition that the two focal rights-assignment rules satisfy claims boundedness.

2.2 Exchange

A rights-assignment rule converts each economy into an ordinary exchange economy with individual property rights (individual endowments) compatible with the social endowment. Formally, an *exchange economy* is a triple (N, Ω, c), where ω denotes the profile of individual endowments (adding up to the social endowment Ω). Let $\bar{\mathcal{E}}$ denote the set of exchange economies. An *exchange rule* F associates with each exchange economy a non-empty set of allocations. Exchange rules are studied extensively in the literature. The best known one is the so-called *Walrasian (exchange) rule*, F^W, which associates with each exchange economy its set of Walrasian equilibrium allocations.[15] We shall also consider other rules that are not Walrasian, yet satisfy the following basic condition of *voluntary exchange*, which can be seen as the natural way of implementing Nozick's principle of just transfer. In words, *voluntary exchange* requires that the outcome of the exchange process determined by the exchange rule does not leave any agent within the group worse off (according to the agent's preferences) than in the initial situation, where they were all in possession of their endowments.[16] That is, everyone ends up at least as well off as she initially was. Formally,

[15]Formally, for each vector of market prices p, define the individual budget, delineated by the initial endowment ω_i, as $B(\omega_i, p) = \{z_i : p \cdot z_i \leq p \cdot \omega_i\}$. An allocation z is a *Walrasian equilibrium* allocation if there exists a vector of prices p, such that, for each $i \in N$, and each $z_i' \in B(\omega_i, p)$, $z_i \in B(\omega_i, p)$ and $z_i \, R_i \, z_i'$.

[16]It is reasonable to formulate justice in transfer more strongly, adding, to voluntary exchange, a criterion of fair trades, e.g., fair net trades by Schmeidler and Vind (1972). In fact, as long as we adopt Walrasian exchange rule in the exchange stage, the stronger version of justice in transfer will be satisfied and so numerous market-based allocation rules characterized in our results will also satisfy the stronger version. Further investigation in this direction is worthwhile, which is left for future research.

Voluntary exchange. For each $(N,\Omega,c) \in \bar{\mathcal{E}}$, $z \in F(N,\Omega,c)$, and $i \in N$,

$$z_i \, R_i \, \omega_i.$$

Note that the so-called *no-trade exchange rule*, which recommends the initial profile of endowments as the outcome of the exchange process, is a well-defined exchange rule satisfying *voluntary exchange*. The Walrasian rule also does.[17] An important distinction between the two is that the former does not guarantee Pareto efficiency of the final outcome, whereas the latter does so, by virtue of the First Fundamental Theorem of Welfare Economics (see for instance, p.549, Mas-Colell et al., 1995). Another example is the *Core rule* selecting the allocations upon which no coalition of agents can improve through the exchange of endowments among coalition members excluding non-members. The Core rule also guarantees Pareto efficiency and contains all Walrasian equilibrium allocations (p.654, Mas-Colell et al., 1995).

2.3 End-state fairness

An *allocation rule* S associates with each economy a non-empty set of allocations. We model end-state fairness by means of some classical fairness axioms for allocation rules. One of the fundamental notions in the theory of fair allocation is envy-freeness, which can be traced back to Tinbergen (1953) and Foley (1967). The concept has come to play a central role in the theory of fair allocation.[18] An allocation satisfies *no-envy*, or is said to be *envy-free*, if no agent prefers the allocation of another agent. An allocation rule S satisfies no-envy if it only selects envy-free allocations. Formally,

No-Envy. For each $e \equiv (N,\Omega,c,R) \in \mathcal{E}$, and each $z \in S(e)$, there is no pair of agents $i,j \in N$ such that $z_j \, P_i \, z_i$.

The above notion does not use information on claims to establish envy comparisons. The following one does so. For each allocation and each agent, we can describe an agent's *net awards* at the allocation as the difference between the claim and the awarded amount. An allocation satisfies *net-no-envy*, or is said to be *net-envy-free*, if no agent prefers the net awards of anyone else to her own net awards. An allocation rule S satisfies net-no-envy if it only selects net-envy-free allocations. Formally,

[17] At a Walrasian equilibrium allocation, each agent is maximizing welfare within her individual budget, which is determined by her own endowment.

[18] See, for instance, Kolm (1972), Pazner and Schmeidler (1974), Feldman and Kirman (1974), and recent surveys, such as Fleurbaey and Maniquet (2011) and Thomson (2011).

Net-No-Envy. For each $e \equiv (N, \Omega, c, R) \in \mathcal{E}$, and each $z \in S(e)$, there is no pair of agents $i, j \in N$, such that $c_i - (c_j - z_j)\ P_i\, z_i$.

Net-no-envy is similar to the notion of *fair net trades* (e.g., Schmeidler and Vind, 1972) for exchange economies, in which the no-envy requirement is formalized for agents' net trades, i.e., the differences between their allocations and their endowments.

2.4 Market-based allocation rules

The composition of a rights-assignment rule and an exchange rule gives rise to an allocation rule, associating a set of allocations for each economy. If the rights-assignment rule satisfies *self-ownership* and the exchange rule satisfies *voluntary exchange*, we say that the resulting allocation rule is *market-based.*

Market-based allocation rules. There exist a rights-assignment rule φ, satisfying *self-ownership*, and an exchange rule F, satisfying *voluntary exchange*, such that $S \equiv F \circ \varphi$, i.e., for each $e \equiv (N, \Omega, c, R) \in \mathcal{E},\ S(e) = F(\varphi(N, \Omega, c), R)$.

As trivial examples, each rights-assignment rule satisfying *self-ownership* yields a market-based allocation rule, when combined with the no-trade exchange rule. Focal market-based allocation rules arise when combining a rights-assignment rule satisfying *self-ownership* (such as the two presented above) with Walrasian exchange.

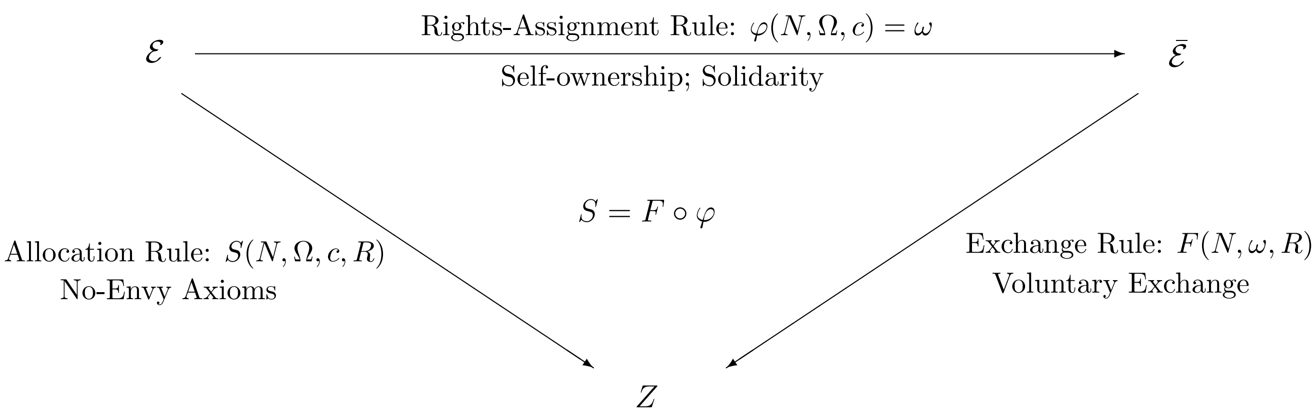

Figure 2. Market-based allocation rules. A market-based allocation rule S is the result of applying a rights-assignment rule φ, satisfying the self-ownership thesis (and, possibly, solidarity), and an exchange rule F satisfying voluntary exchange. For each $e \equiv (N, \Omega, c, R) \in \mathcal{E},\ S(e) = F(N, \varphi(N, \Omega, c), R)$.

3 The results

Our first results illustrate how, in our stylized context, Nozick's theory can indeed be seen as a way of obtaining end-state fairness. More precisely, we show that there exist market-based allocation rules that yield no-envy in each of the two forms described above.

We consider first end-state fairness formalized by no-envy, whose scope needs to be restricted. This is because *claims-boundedness* (a consequence of the combination of self-ownership and solidarity) might make that an unequal choice of endowments, followed by any voluntary exchange from these endowments, will render an agent with small claims envy agents with large claims. To rule out such cases, we restrict our attention to the domain of economies where equal division satisfies claims boundedness.

The first result says that there exist market-based allocation rules that yield *efficient* and *envy-free* allocations at any economy within such a domain. In other words, *self-ownership* followed by *voluntary exchange* (formalizing Nozick's principles of just acquisition and just transfer) guarantee end-estate fairness of *no-envy*.

Proposition 1. *There exist market-based allocation rules satisfying efficiency and no-envy (on the domain of economies where equal division satisfies claims boundedness).*

Proof. We consider the (market-based) allocation rule arising when combining the constrained equal awards rights-assignment rule with Walrasian exchange. Such a rule guarantees equal allocation of initial rights for the domain of economies where equal division satisfies claims boundedness. The Walrasian exchange from equal endowments guarantees *no-envy* and *efficiency*. The former follows because individual budget sets are identical across agents and, thus, each one selects her optimal bundle within such a budget set. The latter follows by the First Fundamental Theorem of Welfare Economics. □

We now switch to net-no-envy, whose scope needs to be limited too. Note that, when agents have sufficiently disparate claims (e.g., an agent with claims larger than the social endowment, and the others with negligible claims), it may not be possible to satisfy net-no-envy. Thus, we restrict our attention to economies without disparate claims. More precisely, we focus on the domain of economies for which equal net division is feasible. The next result states that there exist market-based allocation rules that yield *efficient* and *net-envy-free* allocations at any economy within such a domain. Again, *self-ownership*, followed by *voluntary exchange* can also guarantee end-state fairness, formalized as *net-no-envy*.

Proposition 2. *There exist market-based allocation rules satisfying efficiency and net-no-envy (on the domain of economies where equal net division is feasible).*

Proof. We consider the (market-based) allocation rule arising when combining the constrained equal net-awards rights-assignment rule with Walrasian exchange. Such a rule guarantees equal net awards among agents at the allocation of initial rights for the domain of economies where equal net division is feasible. As the Walrasian budget sets provide equal opportunities of trades across agents, when the endowment is chosen at the allocation with equal net awards, they provide equal opportunities for final net awards across agents (note that the final net award of each agent results from the sum of the equal net award at the endowment and her Warasian trade). Therefore, all equilibrium allocations satisfy net-no-envy. Furthermore, Walrasian exchange guarantees efficiency of the final outcomes, by virtue of the First Fundamental Theorem of Welfare Economics. □

The previous results have illustrated how self-ownership, followed by voluntary exchange, can be invoked to guarantee end-state fairness. In what follows, we focus on the opposite implication, i.e., we search for rights-assignment rules that lead to market-based allocation rules yielding (end-state) fair outcomes. For such an implication, the notion of solidarity introduced above becomes relevant. More precisely, we show that the only way to derive end-state fairness, under *solidaristic* market-based allocation rules (composing a rights-assignment rule satisfying self-ownership and solidarity, and a voluntary exchange rule), is to use an egalitarian rights-assignment rule.

As we show in the next result, if one focuses on *solidaristic* market-based allocation rules satisfying no-envy (on the domain of economies where equal division satisfies claims boundedness) only one rights-assignment rule survives.

Theorem 1. *A market-based allocation rule, generated by a solidaristic rights-assignment rule, satisfies no-envy (on the domain of economies where equal division satisfies claims boundedness) only if the rights-assignment rule is the constrained equal awards rule.*

The technical proof of this result can be found in the appendix. The intuition goes as follows. Suppose first, by contradiction, that the rights-assignment rule does not yield the same outcome as the constrained equal awards rule (such as allocation A in Figure 3). If so, an economy can be constructed for which any market-based allocation rule, arising from such a rights-assignment rule, produces envy. An illustration for the case of two agents and two goods, assuming the Walrasian exchange for the exchange procedure, is provided in Figure 3.[19] Thus, if we want to obtain no-envy of the market-based allocation rule, we are forced to allocate initial rights as equally as possible bounded by claims for all the corresponding rights problems.

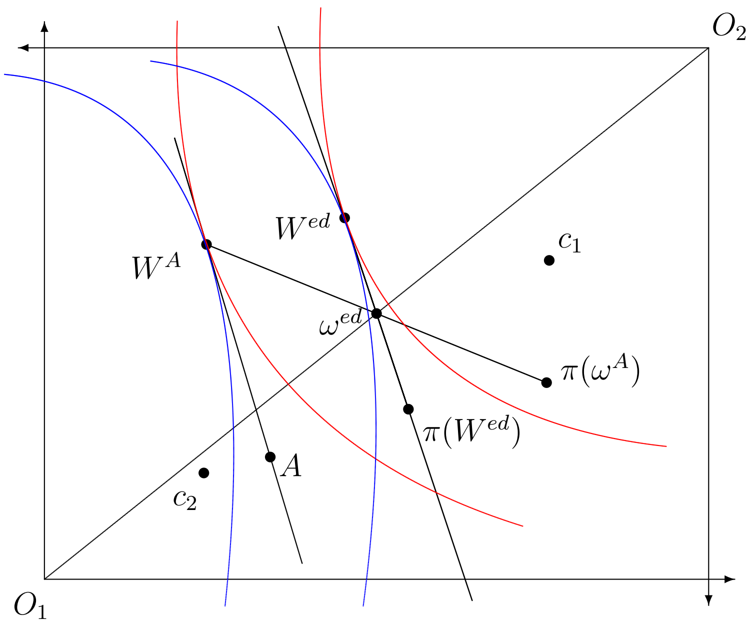

Figure 3. No-Envy in the Edgeworth box. Let $\omega^{ed} \equiv \varphi^{CEA}(N, \Omega, c)$ be equal division and $W^{ed} \equiv F^W(N, \omega^{ed}, R)$ the Walrasian equilibrium from equal division. Let $\pi(W^{ed})$ be the allocation obtained by swapping the two bundles at W^{ed}. Note that for each i, $W^{ed}\, R_i\, \pi(W^{ed})$. Thus, W^{ed} satisfies *no-envy.* Consider any rights-assignment rule $\varphi(\cdot)$ that yields $A \equiv \varphi(N, c, \Omega) \neq \omega^{ed}$. Let $W^A \equiv F^W(N, A, R)$. Let $\pi(W^A)$ be the allocation obtained by swapping the two bundles at W^A. Under the above preferences, $\pi(W^A)\; P_1\, W^A$, that is, agent 1 prefers 2's bundle to his own. Thus, W^A violates *no-envy.*

A parallel result is obtained for net-no-envy and the constrained equal net-awards rule.

Theorem 2. *A market-based allocation rule, generated by a solidaristic rights-assignment rule, satisfies net-no-envy (on the domain of economies where equal net division is feasible) only if the rights-assignment rule is the constrained equal net-awards rule.*

The proof of this result can also be found in the appendix. Its intuition goes parallel to that of the previous one. More precisely, suppose first, by contradiction, that the rights-assignment rule does not yield the same outcome as the constrained equal net-awards rule. If

[19]For such a case, illustrations can be made at the so-called Edgeworth box, an intuitive tool to describe bilateral exchange processes (e.g., Mas-Colell et al., 1995).

so, an economy can be constructed for which any market-based allocation rule, arising from such a rights-assignment rule, produces net-envy. Thus, if we want to obtain net-no-envy of the market-based allocation rule, we are forced to allocate net-awards as equally as possible bounded by claims for all the corresponding rights problems. An illustration for the case of two agents and two goods, assuming the Walrasian exchange for the exchange procedure, appears in Figure 4.

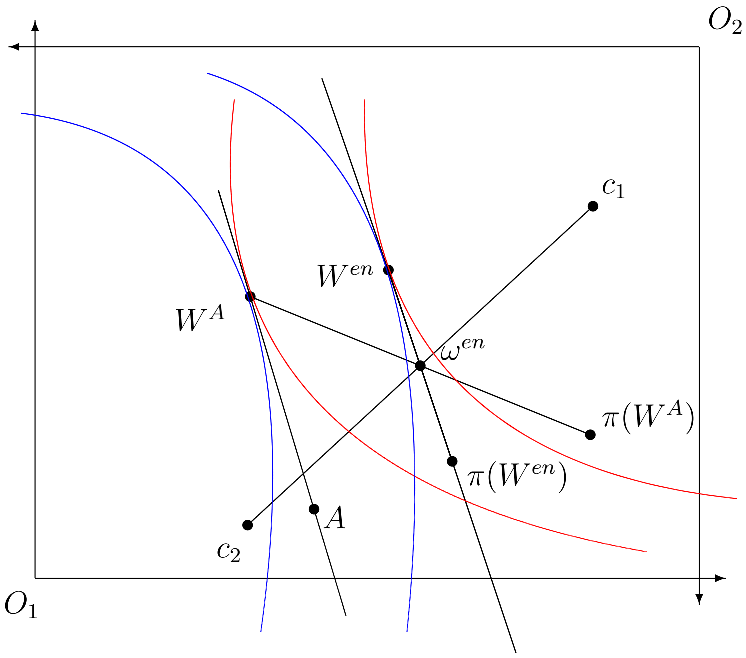

Figure 4. Net-No-Envy in the Edgeworth box. Let $\omega^{en} \equiv \varphi^{CEN}(N, \Omega, c)$ be the equal net division and $\omega^{en} \equiv F^W(N, c^{en}, R)$ the Walrasian equilibrium from the equal net division. Let $\pi(\omega^{en})$ be the allocation obtained by swapping the two bundles at ω^{en}. Note that, for each i, $\omega^{en} \, R_i \, \pi(\omega^{en})$. Thus, ω^{en} satisfies *net-no-envy*. Consider any rights-assignment rule $\varphi(\cdot)$ that yields $A \equiv \varphi(N, c, \Omega) \neq \omega^{en}$. Let $\omega^A \equiv F^W(N, A, R)$. Let $\pi(\omega^A)$ be the allocation obtained by swapping the two net awards in ω^A. Under the above preferences, $\pi(\omega^A) \; P_1 \, \omega^A$, that is, agent 1 prefers agent 2's net awards instead of his own. Thus, ω^A violates *net-no-envy*.

4 Discussion

We have revisited Nozick's entitlement theory of justice in a stylized context for the allocation of goods in joint ventures. We have considered a general model of exchange economies that accommodate the three levels in which fairness can be scrutinized in this context; namely, fairness in the initial allocation of rights on the social endowment, fairness in the transaction of allocated rights, and fairness of the end-state allocation. We have focused, in such a context, on what we dubbed market-based allocation rules, which arise after the combination of rights-assignment rules satisfying self-ownership and voluntary exchange rules.

Self-ownership (and, therefore, Nozick's extension of Locke's theory) is too weak to provide a useful guideline for the resolution of problems such as the ones modeled in this paper. In particular, it gives a green light to any resolution satisfying some minimal respect of the claims. For the case of joint ventures exemplified in our stylized model, Nozick's principle of just acquisition is, consequently, not only modeled by self-ownership, but by the rights-assignment rules we consider. We also keep Nozick's principle of just transfers (voluntary exchange) and show that end-state fairness (formalized by the no-envy conditions we consider) is obtained as a result of combining both principles. More importantly, we show that the only way to derive end-state fairness when composing a solidaristic rights-assignment rule, and a voluntary exchange rule, is to consider an egalitarian rights-assignment rule.

Our approach also resembles *Dworkin's insurance mechanism* (e.g., Dworkin, 1981). Dworkin wished to hold persons responsible for their risk preferences, but not for each person's talent. Thus, behind the veil of ignorance he constructed, the soul representing a person knows its person's utility function, but does not know its person's talent. Behind the veil, the souls purchased insurance against bad luck in the birth lottery. Equality enters importantly into Dworkin's view, as he assumes that the souls have equal purchasing power for insurance. This means that the only way to purchase insurance for indemnity in one state is to sell insurance for the other's indemnity in the other state.[20] In our setting, if claims are interpreted as individuals' purchasing power, then equality is not imposed from the outset. Nevertheless, we obtain equality (in one of the two focal forms considered) of the end-state allocations via the market-based allocation rules. Likewise, we derive equality of the initial allocation of rights as a necessary condition for the end-state fairness of market-based allocation rules.

Finally, we elaborate further on the connection between our work and the theory of exploitation. Most philosophers agree that exploitation should be understood as taking advantage of another person in a way that is unfair or degrading. Classical liberals distinguish between exploitation that is *mutually advantageous*, and exploitation that is *harmful* (e.g., Wertheimer and Zwolinski, 2015). Mutually advantageous exploitation occurs when parties come away from a transaction better off than they would have been without it, but one party considers the distribution of the benefits as unfair. In the parlance of our paper, and if we define unfairness

[20]The reader is referred to Roemer (1996, chapter 7) for a more leisurely discussion of Dworkin's insurance rule).

(of the end-state allocations) as envy (in one of the two forms defined in our model), this is equivalent to saying that, once the allocation of initial property rights has been addressed, voluntary exchange might not preclude the existence of envy (in the end-state allocations). The theoretical implications of our formal approach convey that a just allocation in Nozick's terms is an allocation without mutually advantageous exploitation. In other words, we have shown that there exist market-based allocation rules that yield allocations without mutually advantageous exploitation. Furthermore, we have also shown that, under a solidaristic assignment of initial rights, just allocations in Nozick's terms exist only if such an assignment is egalitarian.

5 Appendix

We collect in this appendix most of the technical parts of our analysis, as well as some auxiliary results.

First, we show some implications of our axioms for our analysis.

The *solidarity* axiom implies the axiom of *resource monotonicity*, which says that when there is more to be divided, other things being equal, nobody should lose.[21] Formally,

Resource Monotonicity. For each pair (N, Ω, c) and $(N, \Omega', c) \in \mathcal{C}$, such that $\Omega \leqq \Omega'$, $\varphi(N, \Omega, c) \leqq \varphi(N, \Omega', c)$.

Resource monotonicity captures the public ownership of the external world (social endowment) in our model. When the public ownership is respected, it is required that no one's property rights should decrease when the external resource increases.

Resource monotonicity allows us to assign rights good by good. More precisely, for each pair of rights problems with identical claims, if the endowment of one good is the same in both problems, then the rights-assignment for such a good should be the same. Formally,

Decomposability. For each pair (N, Ω, c) and $(N, \Omega', c) \in \mathcal{C}$, and each $l \in \{1, \ldots, \ell\}$ such that $\Omega_l = \Omega'_l$, $(\varphi_{il}(N, \Omega, c))_{i \in N} = (\varphi_{il}(N, \Omega', c))_{i \in N}$.

We first show that the combination of self-ownership and resource monotonicity implies claims boundedness: hence, in a deficit situation all agents are rationed, whereas in a surplus situation no one is.

Lemma 1. *Self-ownership and resource monotonicity together imply claims boundedness.*

[21] This axiom was first formalized by Roemer (1986).

Proof. Let φ be a rights-assignment rule satisfying *self-ownership* and *resource monotonicity*. Let $(N, \Omega, c) \in \mathcal{C}$. Let $\Omega^{[0]} \equiv \sum_{i \in N} c_i$. Consider good 1. Let $\Omega^{[1]}$ be such that $\Omega_1^{[1]} \equiv \Omega_1$ and, for each $l \neq 1$, $\Omega_l^{[1]} \equiv \Omega_l^{[0]}$.

Suppose $\sum_{i \in N} c_{i1} \geq \Omega_1$. By *self-ownership* and feasibility, $\varphi\left(N, \Omega^{[0]}, c\right) = c$. Then, by *resource monotonicity*, for each $i \in N$, $\varphi_{i1}(N, \Omega^{[1]}, c) \leq \varphi_{i1}(N, \Omega^{[0]}, c) = c_{i1}$ and, by *decomposability*, $\varphi_{i1}(N, \Omega, c) = \varphi_{i1}(N, \Omega^{[1]}, c)$. Therefore,

$$\varphi_{i1}(N, \Omega, c) \leq c_{i1}.$$

The inequality is reversed when $\sum_{i \in N} c_{i1} \leq \Omega_1$.

The same argument applies for all other goods $k = 2, \ldots, \ell$. □

Solidarity also requires that the application of a rule to each subproblem derived by imagining that some agents leave with their corresponding awards in the original problem, and reassessing the situation from the viewpoint of the remaining agents, produces precisely the allocation that the subgroup obtained in the original problem. This is normally known in the literature as the axiom of *consistency*, which has played a crucial role in axiomatic work (e.g., Thomson, 2012). Formally,

Consistency. For each $(N, \Omega, c) \in \mathcal{C}$, each $M \subset N$, each $j \in N \backslash M$, and each $l \in \{1, \ldots, \ell\}$,

$$\varphi_{jl}(N \backslash M, \sum_{k \in N \backslash M} \varphi_k(N, \Omega, c), c_{N \backslash M}) = \varphi_{jl}(N, \Omega, c).$$

The last property we consider is the *converse* to the previous one. It allows us to deduce the desirability of a proposed awards vector for a given problem from the desirability of its restriction to each two-agent subgroup for the reduced problem obtained by imagining the departure of the members of the complementary subgroup with their awards. The property says that if an awards vector is such that for each problem and each proper two-agent subgroup, the rule chooses the corresponding awards of the vector to this subgroup for the reduced problem it faces, then the rule should choose the awards vector for the initial problem. Formally, for each $(N, \Omega, c) \in \mathcal{C}$ and each rule φ, let $cv.cs(N, \Omega, c; \varphi) \equiv \{\omega : \sum_{i \in N} \omega_i = \Omega$ and, for each $M \subset N$ with $|M| = 2, \omega_M = \varphi(c_M, \sum_{i \in M} \omega_i)\}$.

Converse Consistency. For each $(N, \Omega, c) \in \mathcal{C}$, there is ω such that $\{\omega\} = cv.cs(N, \Omega, c; \varphi)$ and $\omega = \varphi(N, \Omega, c)$.

For (unidimensional) rights problems with deficit, *resource monotonicity* and *consistency* imply *converse consistency*. The same result holds in our model.[22] Thus, a rule satisfying *solidarity* also satisfies *converse consistency*. This is the case for the two rules introduced above.

The previous implication has important consequences. As stated by the so-called Elevator Lemma (e.g., Thomson, 2016a), if a conversely consistent rule coincides with a consistent rule in the two-agent case, coincidence holds in general. Thus, it suffices to characterize the *constrained equal awards rule* and the *constrained equal net-awards rule* in the two-agent case, to derive characterizations in the general case appealing to consistency.

Finally, we introduce additional notation. Let $\mathcal{E}^0$ denote the domain of economies in which equal division satisfies claims boundedness. Formally, $\mathcal{E}^0 \equiv \{(N, \Omega, c, R) \in \mathcal{E} :$ for each $l = 1, \ldots, \ell$, either, for each $i \in N$, $\Omega_l/n \leq c_{il}$, or for each $i \in N$, $\Omega_l/n \geq c_{il}\}$. Let $\mathcal{C}^0$ be the corresponding domain of claims problems, i.e., $\mathcal{C}^0 \equiv \{(N, \Omega, c) \in \mathcal{C}$: for some preferences profile R, $(N, \Omega, c, R) \in \mathcal{E}^0\}$.

We are now ready to prove Theorem 1.

Proof of Theorem 1. Let φ be a rights-assignment rule satisfying *self-ownership* and *solidarity*, F be an exchange rule satisfying *voluntary exchange*, and $S \equiv F \circ \varphi$ be the corresponding market-based allocation rule satisfying *no-envy* on $\mathcal{E}^0$. We will prove that $\varphi = \varphi^{CEA}$ on the class of 2-person problems. Then, the coincidence extends to all other problems with more than 2 persons by the Elevator Lemma. In what follows, and without loss of generality, we fix the set of two persons to be $N \equiv \{1, 2\}$. We skip N from the notation.

For each $l = 0, \ldots, \ell$, let $\mathcal{C}^0(l) \equiv \{(\Omega, c) \in \mathcal{C} :$ for each $k \geq l + 1$, either $c_{ik} \geq \Omega_k/2$ for each $i = 1, 2$, or $c_{ik} \leq \Omega_k/2$ for each $i = 1, 2.\}$ Then, $\mathcal{C}^0(0) \equiv \mathcal{C}^0(N)$ and $\mathcal{C}^0(\ell) = \mathcal{C}(N)$. We show that φ coincides with φ^{CEA} on $\mathcal{C}^0(k)$ for each $k = 0, 1, \ldots, \ell$, using mathematical induction.

We show first that $\varphi = \varphi^{CEA}$ on $\mathcal{C}^0(0)$. Consider any problem $(\Omega, c) \in \mathcal{C}^0(0)$. Suppose, by contradiction, that $\varphi(\Omega, c) \neq (\Omega/2, \Omega/2)$. Then, as illustrated in Figure 3, there is an economy with a preferences profile R and rights problem (Ω, c) such that ω is the only efficient allocation satisfying *voluntary exchange* from endowment ω and one of the two agents envies the other at ω. Then, the market-based allocation rule necessarily chooses ω and no-envy is violated.

[22]This result for (unidimensional) rights problems with deficit, also known as bankruptcy problems, is due to Chun (1999). The proof presented by Thomson (2016b) is easily extended in our multi-dimensional setting due to decomposability.`[see the supporting note for referees]`

Suppose, on our induction basis, that φ coincides with φ^{CEA} on $\mathcal{C}^0(k)$ for each $k \le l-1$. We now prove that φ coincides with φ^{CEA} on $\mathcal{C}^0(l)$. Let $(\Omega, c) \in \mathcal{C}^0(l) \backslash \mathcal{C}^0(l-1)$.

Case 1: $c_{1l} + c_{2l} \ge \Omega_l$. Without loss of generality, suppose $c_{1l} < c_{2l}$. Then $c_{1l} < \Omega_l/2 \le c_{2l}$ and, for each $k \ge l+1$, either for each $i = 1, 2$, $c_{ik} \ge \Omega_k/2$ or for each $i = 1, 2$, $c_{ik} \le \Omega_k/2$. Thus, $\varphi_l^{CEA}(N, \Omega, c) = (c_{1l}, \Omega_l - c_{1l})$. Let $\omega \equiv \varphi(N, \Omega, c)$. Let Ω' be such that $\Omega'_l \equiv 2c_{1l}$ and for each $k \ne l$, $\Omega'_k \equiv \Omega_k$. Then, $(\Omega', c) \in \mathcal{C}^0(l-1)$ and, by the induction hypothesis,

$$\varphi(\Omega', c) = \varphi^{CEA}(\Omega', c). \tag{1}$$

In particular, $\varphi_l(\Omega', c) = (\Omega'_l/2, \Omega'_l/2) = (c_{1l}, c_{1l})$. As $\Omega' \leqq \Omega$, then, by *resource monotonicity*, $\omega = \varphi(\Omega, c) \geqq \varphi(\Omega', c)$. By *boundedness*, $\omega_{1l} = c_{1l}$. Then, $\omega_{2l} = \Omega_l - c_{1l}$. Therefore, $\varphi_l(\Omega, c) = \varphi_l^{CEA}(\Omega, c)$. As $\Omega'_k = \Omega_k$ for each $k \ne l$, then by *decomposability* of both φ and φ^{CEA}, $\varphi_k(\Omega', c) = \varphi_k(\Omega, c)$ and $\varphi_k^{CEA}(\Omega', c) = \varphi_k^{CEA}(\Omega, c)$. Hence, using (2), for each $k \ne l$, we conclude the proof.

Case 2: $c_{1l} + c_{2l} < \Omega_l$. Without loss of generality, suppose $c_{1l} < c_{2l}$. Then $c_{1l} < \Omega_l/2 \le c_{2l}$ and, for each $k \ge l+1$, either for each $i = 1, 2$, $c_{ik} \ge \Omega_k/2$ or for each $i = 1, 2$, $c_{ik} \le \Omega_k/2$. Thus, $\varphi_l^{CEA}(\Omega, c) = (\Omega_l - c_{2l}, c_{2l})$. Let $\omega \equiv \varphi(\Omega, c)$. Let Ω' be such that $\Omega'_l \equiv 2c_{2l}$ and, for each $k \ne l$, $\Omega'_k \equiv \Omega_k$. Then $(\Omega', c) \in \mathcal{C}^0(l-1)$ and, by the induction hypothesis,

$$\varphi(\Omega', c) = \varphi^{CEA}(\Omega', c). \tag{2}$$

In particular, $\varphi_l(\Omega', c) = (\Omega'_l/2, \Omega'_l/2) = (c_{2l}, c_{2l})$. As $\Omega' \geqq \Omega$, then, by *resource monotonicity*, $\omega = \varphi(\Omega, c) \leqq \varphi(\Omega', c)$. By *claims boundedness*, $\omega_{2l} = c_{2l}$. Then, $\omega_{1l} = \Omega_l - c_{2l}$. Therefore, $\varphi_l(N, \Omega, c) = \varphi_l^{CEA}(\Omega, c)$. As $\Omega'_k = \Omega_k$ for each $k \ne l$, then, by *decomposability* of both φ and φ^{CEA}, $\varphi_k(\Omega', c) = \varphi_k(\Omega, c)$ and $\varphi_k^{CEA}(\Omega', c) = \varphi_k^{CEA}(\Omega, c)$. Hence, using (2), $\varphi_k(\Omega, c) = \varphi_k^{CEA}(\Omega, c)$, for each $k \ne l$. □

For the proof of Theorem 2, we need additional notation. Let $\mathcal{E}^*$ denote the domain of economies in which equal net division is feasible. Formally, $\mathcal{E}^* \equiv \{e = (N, \Omega, c, R) \in \mathcal{E} :$ for each $i \in N$, $0 \leqq c_i - (\sum_{j \in N} c_j - \Omega)/n\}$. Let $\mathcal{C}^*$ be the corresponding domain of claims problems.

Proof of Theorem 2. Let φ be a rights-assignment mechanism satisfying *self-ownership* and *resource monotonicity*, F be an exchange rule satisfying *voluntary exchange*, and $S \equiv F \circ \varphi$ be the corresponding market-based allocation rule satisfying *net-no-envy* on $\mathcal{E}^*$. We will prove that $\varphi = \varphi^{CEN}$ on the class of 2-person problems. Then the coincidence extends to all other problems with more than 2 persons by the Elevator Lemma. In what follows, and without loss of generality, we set $N \equiv \{1, 2\}$ and skip N from the notation.

For each $l = 0, \ldots, \ell$, let $\mathcal{C}^*(l) \equiv \{(\Omega, c) \in \mathcal{C}$: for each $k \geq l+1$, and for each $i = 1, 2$, $(c_{1k} + c_{2k} - \Omega_k)/2 \leq c_{ik}\}$. Then, $\mathcal{C}^*(0) \equiv \mathcal{C}^*$ (the domain of claims problems for which equal net division is feasible) and $\mathcal{C}^*(\ell) = \mathcal{C}$. We show that φ coincides with φ^{CEN} on $\mathcal{C}(k)$ for each $k = 0, 1, \ldots, \ell$, using induction.

We first show that $\varphi = \varphi^{CEN}$ on $\mathcal{C}^*(0)$. Let $(\Omega, c) \in \mathcal{C}^*(0)$. Suppose, by contradiction, that $\varphi(\Omega, c) = \omega \neq \varphi^{CEN}(\Omega, c)$. Then, as illustrated in Figure 4, there is an economy for which ω is the only efficient allocation satisfying *voluntary exchange* (from endowment ω) and therein one of the two agents envies the net consumption of the other. Then the market-based allocation rule necessarily chooses ω and net-no-envy is violated.

Let $l \in \{1, \ldots, \ell\}$. Suppose, by induction, that $\varphi = \varphi^{CEN}$ on $\mathcal{C}^*(k)$ for each $k \leq l-1$. We now prove that $\varphi = \varphi^{CEN}$ on $\mathcal{C}^*(l)$. Let $(\Omega, c) \in \mathcal{C}^*(l) \backslash \mathcal{C}^*(l-1)$. Without loss of generality, suppose $c_{1l} \leq c_{2l}$. Then, since $(\Omega, c) \notin \mathcal{C}^*(l-1)$, $(c_{1l} + c_{2l} - \Omega_l)/2 > c_{1l}$ (i.e., $c_{2l} - c_{1l} > \Omega_l$). Hence, $\varphi_l^{CEN}(\Omega, c)= (0, \Omega_l)$ and $\varphi_l^{CEN}(\Omega, c) \leq (0, c_{2l} - c_{1l})$. Let $\Omega'_l \equiv c_{2l} - c_{1l}$ and, for each $k \neq l$, $\Omega'_k = \Omega_k$. Then $(\Omega', c) \in \mathcal{C}^*(l-1)$ and, by the induction hypothesis,

$$\varphi(\Omega', c) = \varphi^{CEN}(\Omega', c). \tag{3}$$

Note that $\varphi_l^{CEN}(\Omega', c) = (0, c_{2l} - c_{1l})$. Since $\Omega \leqq \Omega'$, by *resource monotonicity* and non-negativity, $\varphi_{1l}(\Omega, c) = 0$, which implies $\varphi_{2l}(\Omega, c) = \Omega_l$. Therefore, $\varphi_l(\Omega, c) = \varphi_l^{CEN}(\Omega, c)$. As $\Omega'_k = \Omega_k$ for each $k \neq l$, then applying *resource monotonicity* to both φ and φ^{CEN}, we have $\varphi_k(\Omega, c) = \varphi_k(\Omega', c)$ and $\varphi_k^{CEN}(\Omega', c) = \varphi_k^{CEN}(\Omega, c)$. Hence, using (3), $\varphi_k(\Omega, c) = \varphi_k^{CEN}(\Omega, c)$. □

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Supporting Note for Referees

The proof of the claim that resource monotonicity and consistency together imply converse consistency.

Let φ be a resource monotonic and consistent rule. Let $(N, \Omega, c) \in \mathcal{C}$ and $\omega = \varphi(N, \Omega, c)$. By consistency, $\omega \in cv.cs(N, \Omega, c; \varphi)$. Suppose, by contradiction, that there is $y \in cv.cs(N, \Omega, c; \varphi) \setminus \{\omega\}$. Then, as $y \neq \omega$, there is $M \subseteq N$ such that $\sum_{i \in M} y_i \neq \sum_{i \in M} \omega_i$. Without loss of generality, suppose that $\sum_{i \in M} y_i > \sum_{i \in M} \omega_i$. By resource monotonicity, $y_i \geq \omega_i$, for each $i \in M$. As $\sum_{i \in M} y_i > \sum_{i \in M} \omega_i$, it follows that there exists $i_0 \in N$ such that $y_{i_0} > x_{i_0}$. Now, for each $j \in N \setminus \{i_0\}$, let $N_j \equiv \{i_0, j\}$. By resource monotonicity applied to $(N_j, y_j + y_{i_0}, c_{N_j})$ and $(N_j, \omega_j + \omega_{i_0}, c_{N_j})$, we obtain that $y_j \geq \omega_j$ (for each $j \in N \setminus \{i_0\}$). Thus, $\sum_{i \in N} y_i > \sum_{i \in N} \omega_i = \Omega$, a contradiction.

The Elevator Lemma.

If a rule is consistent, another conversely consistent, and they coincide in the two-agent case, then they coincide for any number of agents. Formally, let φ be a consistent rule and ϕ be a conversely consistent rule. If, for each $(M, \Omega, c) \in \mathcal{C}$, such that $|M| = 2$, $\varphi(M, \Omega, c) = \phi(M, \Omega, c)$, then $\varphi(N, \Omega, c) = \phi(N, \Omega, c)$, for each $(N, \Omega, c) \in \mathcal{C}$.