Functional finance and intergenerational distribution in a Keynesian OLG model

by

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This paper examines the role of fiscal policy in the long run. We show that (i) dynamic inefficiency in a standard OLG model generates aggregate demand problems in a Keynesian setting, (ii) fiscal policy can be used to achieve full-employment growth, (iii) the required debt ratio is inversely related to both the growth rate and government consumption, and (iv) a simple and distributionally neutral tax scheme can maintain full employment in the face of variations in ‘household confidence’.

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1 Introduction

A standard OLG framework has important weaknesses from a Keynesian perspective. It assumes that desired household saving at full employment is automatically turned into investment; there are no aggregate demand problems. This paper presents an OLG model that includes Keynesian concerns. Households save but investment decisions are made by firms.

It is well known that neoclassical OLG models can produce dynamic inefficiency and that, if this happens, fiscal policy and public debt can be Pareto improving (Diamond 1965). This outcome -- dynamic inefficiency -- is sometimes dismissed as empirically irrelevant (Abel et al. 1989). The standard argument for irrelevance, however, does not apply under imperfect competition (Skott and Ryoo 2014), and what appears as a problem of dynamic inefficiency in a neoclassical version of the model turns into a problem of aggregate demand and unemployment in a Keynesian setting.

The existence of a link from dynamic inefficiency to aggregate demand clearly does not exclude other sources of aggregate demand problems. Indeed, an OLG setting with an implied period length that greatly exceeds a normal business cycle cannot be used to analyze the short-run problems that dominate macroeconomic policy. Why then analyze a stylized OLG model? Our motivation is two-fold. Intergenerational fairness – the distribution of income across generations – has figured strongly in debates on public debt; arguably, an OLG framework can be useful for the analysis of these issues. The obsession with public debt and the alleged long-run dangers of high levels of debt, second, is surprising: dominant macroeconomic models satisfy ‘Ricardian equivalence’ and imply that public debt becomes largely irrelevant. This irrelevance of debt in benchmark theoretical models may explain the prominence in the debate of purely empirical studies.¹ Our analysis contributes a theoretical perspective on this (and other)

¹ Reinhart and Rogoff’s (2010) suggestion that debt-income levels above 90 percent tend to be associated with lower rates of economic growth has been discredited (Herndon et al. 2014), and the broader Reinhart and Rogoff analysis has been challenged by other studies (e.g. Irons and Bivens (2010), Dube (2013), Basu (2013)). But the challenge has been largely empirical.
policy issues, incl. ‘secular stagnation’. We deliberately choose a setting in which public debt matters, and the focus on long-run issues, rather than short-run business cycles, implies that some of the obvious drawbacks of the OLG model become less important.

Adopting a ‘functional finance’ approach, we assume that monetary policy determines the rate of interest and the choice of technique, leaving fiscal policy to ensure a trajectory of aggregate demand that is consistent with full employment. Our steady growth analysis shows a long-run relationship between the required debt ratio and the rate of growth, but the causal link unambiguously runs from growth to debt: a low growth rate generates a high steady-growth ratio of debt to income. The required debt, moreover, is inversely related to government consumption. Similar results have been found in other Keynesian models (e.g. Schlicht 2006, Ryoo and Skott 2013). Obtaining the results using a widely accepted OLG structure strengthens the argument.

Extending the analysis beyond steady growth, we examine the implications of shifts in ‘household confidence’ that lead to fluctuations in saving rates. A simple and distributionally neutral tax scheme can maintain full employment in the face of shifts in confidence that would otherwise lead to problems of aggregate demand and secular stagnation. Moreover, in the special case where households correctly anticipate future taxes, no variations in taxes will be needed: the tax policy effectively functions as an insurance scheme. Concerns over the sustainability of the public debt trajectory, finally, find no support. A fiscal policy based on functional finance may in some cases lead to high levels of public debt. But no scenarios become explosive or otherwise unsustainable in this OLG setting.

The Keynesian literature on functional finance has a long history, going back to Lerner (1943). We know of no other Keynesian studies, however, that use a formal OLG structure to examine the long-run implications of functional finance. OLG models have been used to analyze dynamic

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3 Skott and Ryoo’s (2014) analysis of fiscal policy in an OLG model with imperfect competition includes no Keynesian elements and the focus is on steady states; there are no fluctuations in ‘confidence’, and the paper does
inefficiency and public debt dynamics but not from the perspective of functional finance. Chalk (2000), for instance, assumes a constant primary deficit per worker. He finds that even if the economy is dynamically inefficient when public debt is zero, a constant primary deficit may be unsustainable; moreover, in those cases where a primary deficit is sustainable, convergence is to a steady growth path that is dynamically inefficient. These results invite several questions. Why would a government want to pursue policies of this kind? Why focus on trajectories that keep a constant primary deficit? Economic analysis of monetary policy typically looks for ‘optimal’ policies (or policy rules), given some welfare function and a model of how the economy operates. Our functional finance approach introduces elements that are usually absent in the analysis of monetary policy, but the search for appropriate policies is in a similar spirit.

Section 2 analyzes the choice of technique. Section 3 completes the Keynesian OLG model by introducing household behavior and firms’ investment decisions. We add taxation and public debt and derive the full-employment requirements for fiscal policy in section 4. Fluctuations in ‘confidence’ and their implications for fiscal policy are analyzed in section 5. Section 6 relates the analysis to recent policy debates. Section 7 presents a few concluding remarks.

2 Functional finance and the choice of technique

Keynesian growth models typically assume a Leontief (fixed-coefficients) production function. This seemingly restrictive specification can be justified in a number of ways. The capital controversy criticized the use of a smooth aggregate production function from a theoretical perspective, and the degree and relevance of substitutability can also be questioned empirically. In this paper, however, we take a different approach: a Leontief specification can be justified along lines that are consistent with Lerner’s analysis of functional finance.

Monetary policy, Lerner argued, should be used to set interest rates at levels that induce an
optimal amount of investment or, equivalently, an optimal share of investment in output (assuming that fiscal policy keeps output at full employment). An optimal share of investment in output translates into an optimal capital-output ratio in the long run, that is, an optimal choice of technique. Intuitively, our Keynesian OLG model differs from standard neoclassical versions by separating investment and saving decisions. Firms make employment and investment decisions based on factor prices and demand conditions, and factor prices do not automatically adjust to ensure full employment at all times. Instead the interest rate (the cost of finance) is set so as to make firms choose the capital intensity that is deemed socially optimal. The fixed coefficients of the Leontief production function can be seen as the outcome of this profit maximizing choice of technique, given a policy-determined interest rate.

Following Skott (1989, chapter 5), firms may be able to choose from a range of blueprints when they make an investment decision. But ex post – once an investment has been made in particular plant and machinery – the substitutability between ‘capital’ and ‘labor’ is limited. Assume, for simplicity, that the ex ante production function is Cobb-Douglas,

\[ Y_t = K_t^\alpha N_t^{1-\alpha}, \quad 0 < \alpha < 1 \]  

where \( Y_t, K_t \) and \( N_t \) are the amount of output, capital and employment, respectively. Let \( W_t \) and \( P_t \) be the money wage rate and the price of capital goods, and let \( i \) and \( \delta \) denote the cost of finance (the real rate of interest) and the rate of depreciation. Profit maximizing firms will minimize cost:

\[
\min_{L_t, K_t} \quad W_t N_t + (i + \delta) P_t K_t \\
\text{s.t.} \quad (u^* K_t)^{\alpha} N_t^{1-\alpha} = Y_t
\]

Firms may want to maintain a certain amount of excess capacity on average; the reason for this include lumpiness in investment (a minimum scale of investment), short run volatility of demand at the firm level, and entry deterrence. Thus, the constraint in the minimization problem allows for the desired utilization rate of capital (\( u^* \)) to be less than one. The first order conditions imply that

\[
\frac{Y_t}{N_t} = \lambda_t = \left( \frac{u^* \alpha}{(i+\delta)^{1-\alpha} P_t} \right)^\alpha
\]
The price of capital goods, \( P_t \), is exogenous to the individual firm (and was treated as such in the minimization). In equilibrium, however, this price must be equal to the general price level in a one-good model. Assuming profit maximization, the pricing decision is based on marginal cost, and both the technical coefficients and the stock of capital are predetermined in the short run. Employment and output by contrast are taken to be variable. With excess capital capacity and constant labor productivity, this yields a markup on unit labor cost,

\[ P_t = (1 + m)W_t \frac{1}{\lambda_t} \]  

(5)

where the markup \( (m) \) is determined by the firm’s perceived elasticity of demand, which we take to be constant.

Combining equations (3)-(5)

\[ \lambda_t = \lambda = \left[ \left( \frac{\alpha}{1-\alpha} \right) \left( \frac{u^*}{1+i+\delta} \right) \left( \frac{1}{1+m} \right) \right]^{\alpha/(1-\alpha)} \]  

(6)

\[ \sigma_t = \sigma = \lambda^{-(1-\alpha)/\alpha} = \left( \frac{1-\alpha}{\alpha} \right) \left( \frac{i+i+\delta}{u^*} \right) (1 + m) \]  

(7)

Thus, the choice of technique is fully determined by \( i, u^* \) and \( m \). Intuitively, cost minimization produces one relation between \( \lambda_t \) and \( W_t/P_t \) (for given \( i \)); pricing decisions give another relation. In equilibrium these two relations – equations (3) and (5) – must be mutually consistent; this consistency requirement fixes the real wage and the cost-minimizing input coefficients. In other words, for a given interest rate, the cost minimization pins down the coefficients of a Leontief production function:

\[ Y = \min\{\lambda N, \sigma K\} \]  

(8)

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4 The same qualitative outcome of the analysis – the determination of the choice of technique by the interest rate – could be derived by assuming a markup on total unit cost.

5 The argument is quite general and does not depend on the existence of a smooth ex ante production function. For a given set of input prices, firms will choose a particular technique,

\[ \frac{y}{N} = \phi \left( i, \frac{w}{P} \right) \]

Their pricing decisions, in turn, determines the real wage,

\[ \frac{w}{P} = \psi \left( \frac{y}{N}, i \right) \]
Using (6) and (7), the real wage and the rate of return on capital are determined by

\[ w = \frac{W}{p} = \frac{1}{1 + m} \lambda = \left[ \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{u^*}{i + \delta} \right) \right]^{\alpha/(1 - \alpha)} (1 + m)^{-1/(1 - \alpha)} \quad (9) \]

\[ R(i) = \pi u^* \sigma - \delta = m \left( \frac{1 - \alpha}{\alpha} \right) (i + \delta) - \delta \quad (10) \]

where \( \pi = m/(1 + m) \) is the profit share. We assume that \( R(i) \geq i \) (or equivalently, \( m \geq \alpha/(1 - \alpha) \)). The failure of this condition to be met would imply negative net profits (after interest payments) and there would be no incentive for firms to invest.

3 OLG models

3.1 Steady growth

Following Diamond (1965), all agents live for two periods: they work in the first period and live off their savings in the second. The number of workers \( (L_t) \) grows at the constant rate \( n \geq 0 \),

\[ L_{t+1} = (1 + n)L_t \quad (11) \]

To keep the saving side simple, the utility function for a young agent in period \( t \) is taken to be logarithmic (or Cobb-Douglas):

\[ U = \log c_{1,t} + \frac{1}{1 + \rho} \log c_{2,t+1} \quad (12) \]

where \( c_{1,t} \) and \( c_{2,t+1} \) are the levels of consumption per capita when the agent is young and old.\(^6\) The labor supply is inelastic, and normalizing the supply of an individual worker to one, the budget constraint is given by

\[ c_{1,t} + \frac{1}{1 + r_{t+1}} c_{2,t+1} = w_t \quad (13) \]

where \( r_{t+1} \) is the rate of return on savings and \( w_t \) is the real wage.

Utility maximization implies that

\(^6\) The more general CIES specification \(-U = \frac{c_{1,t}^{-\theta - 1}}{1 - \theta} + \frac{1}{1 + \rho} \frac{c_{2,t+1}^{-\theta - 1}}{1 - \theta}, \theta \geq 0\) – complicates the analysis and does not add much, given the purposes of this paper.
\[ c_{1,t} = (1 - s)w_t \]  
where the young generation’s saving rate \( s \) can be written

\[ s = \frac{1}{2 + \rho} \]  

### 3.1.1 A non-Keynesian version: dynamic inefficiency

In non-Keynesian OLG models, saving decisions directly determine investment. Households save in the form of fixed capital, and the total saving by the young determines the capital stock in the following period

\[ K_{t+1} = S_t = sN_t w_t \]  

Using (15) and (16), and dividing through by \( K_t \), the growth rate of the capital stock is given by

\[ \bar{R}_t = \frac{K_{t+1}}{K_t} - 1 = s(1 - \pi) \frac{Y_t}{K_t} - 1 = \frac{(1 - \pi)u_t \sigma}{2 + \rho} - 1 \]  

where \( u_t = Y_t / (\sigma K_t) \leq 1 \) is the utilization rate of capital and a hat over a variable is used to denote growth rates \((\bar{x}_t = (x_{t+1} - x_t) / x_t)\).

The utilization and accumulation rates are constant in steady growth, full-employment growth requires that \( \bar{R}_t = n \), and by assumption the technical coefficient \( \sigma \) is given. Thus, if the profit share \( \pi \) is determined by the markup, equation (17) determines the steady-growth solution for utilization, \( u^{**} \). There is an upper bound on utilization, \( u^{**} \leq 1 \), and full-employment growth becomes impossible if there are no solutions satisfying this restriction. The restriction can be written

\[ \sigma \geq \frac{1}{1 - \pi} (1 + \pi)(2 + \rho) \]  

Assuming that (18) is met, adjustments in the utilization rate play the same role as movements along the production function in specifications with smooth substitution; the adjustments allow full employment growth. But the dynamic inefficiency problem is brought into stark focus by fixed coefficients: for utilization rates below one, the marginal product of capital is zero, even though capital

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7 With fixed coefficients and a given profit share, the solution is unique, even if \( \theta \neq 1 \). Thus, the analysis can be extended to cover a general CIES specification of the utility function. Appendix A considers the case with perfect competition. When \( u_t = u^{**} = 1 \), the profit share becomes indeterminate, and adjustments in the profit share can allow full employment growth with full utilization; the full utilization solution is unstable, however.
gains a positive rate of return. This outcome illustrates a more general point: having profit rates that exceed the rate of growth does not imply dynamic efficiency under imperfect competition (Skott and Ryoo 2014).

### 3.1.2 A Keynesian version: aggregate demand issues

In a Keynesian economy, firms make the investment and production decisions (and select the production technique). Households do not participate directly in these decisions and typically do not own the physical capital; households' ownership of firms takes the form of equity shares. Thus, in place of (16) we have

\[ S_t = \sum A_{i,t+1} \]  

where \( A_{i,t+1} \) represents the agent’s holdings of asset \( i \) at the beginning of period \( t + 1 \), one of the assets being equity.\(^8\)

Assume for simplicity that firms finance investment exclusively through corporate bonds and that the price of capital goods (= the price of output) is constant. Thus, normalizing the price to one, let

\[ K_t = M_t \]  

where \( M \) denotes the value of the outstanding corporate bonds. The model now contains two financial assets, equity and bonds. It may be reasonable to add another asset, cash. In equilibrium, however, cash holdings will be zero if we disregard risk and any need to hold non-interest bearing cash for transactions purposes.

With these assumptions, equation (19) can be written

\[ S_t = M_{t+1} + V_{t+1} \]  

where \( V_{t+1} \) is the value of corporate equity. Dividing through by employment \( N_{t+1} \) and using (15), we now have

\[ q_{t+1} = \frac{sw}{N_{t+1}} \]  

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\(^8\) We assume complete symmetry among firms. Shares in different firms are perfect substitutes and can be aggregated into a single asset, equity.
where \( k = K/N \) and \( q \) is the valuation ratio (Tobin’s q), i.e.,

\[
q = \frac{M + V}{k}
\]  

(23)

In steady growth \( N_t/N_{t+1} \) is constant and, using (6)-(7), the steady-growth value of \( k \) is given by

\[
k = \frac{u^*K}{Y} \frac{1}{u^* N} = \frac{\lambda}{\sigma} \frac{1}{u^*}
\]  

(24)

Thus, \( q \) must be constant in steady growth:

\[
q = sw \frac{1}{1+\theta} \frac{1}{k}
\]  

(25)

where \( \theta \) is the steady growth rate of \( N_t \).

Another steady-growth condition can be derived from households’ portfolio decisions. The absence of risk implies that equity and bonds become perfect substitutes and must carry the same rate of return. The rate of return is determined by monetary policy: the central bank offers to buy or sell bonds at a price that corresponds to its chosen interest rate. The private sector holds no cash when the return on bonds is positive, and the equilibrium net position of the central bank therefore will also be zero for any positive interest rate.\(^9\)

The rate of return on equity adjusts to the interest rate via the valuation of shares. The equality between the rate of return on corporate assets and the interest rate \( i \) implies that

\[
(1 + i)(M_{t+1} + V_{t+1}) = [Y_{t+1} - w_{t+1}N_{t+1} - \delta K_{t+1}] + M_{t+1} + V^e_{t+2}
\]  

(26)

The term in square brackets represents the sum of dividend and interest payments and \( V^e \) is expected equity valuation. Dividing through by \( K_{t+1} \) and using (20) and (23), equation (26) can be written

\[
(1 + i)q_{t+1} = \frac{Y_{t+1} - w_{t+1}N_{t+1} + (1 - \delta)K_{t+1} + V^e_{t+2}}{K_{t+1}}
\]

\[
= 1 + \frac{V^e_{t+2}}{K_{t+1}} = 1 + \frac{(M^e_{t+2} + V^e_{t+2}) - M^e_{t+2} K^e_{t+2}}{K^{*}_{t+2}}
\]

\[
= 1 + \frac{(q^e_{t+2} - 1) K^e_{t+2}}{K_{t+1}}
\]

(27)

\(^9\) Bank loans and deposits could be used instead of, or in addition to, corporate bonds (Skott 1989).
where the return on capital, \( R = R(i) \), is given by (10).

With a constant growth rate and a constant value of \( q \) (cf. equation (25)), it would be unreasonable to introduce persistent deviations of actual from expected values. Equation (27) therefore reduces to

\[
(1 + \theta)q = 1 + R(i) + (q - 1)(1 + \theta)
\]  

or

\[
q = \frac{R(i) - \theta}{i - \theta}
\]

By assumption \( R(i) \geq i \), and the rate of interest must exceed the growth rate to avoid dynamic inefficiency; with this restriction, equation (29) implies \( q \geq 1 \) and \( \partial q / \partial i < 0 \).

Equations (25) and (29) – together with the constraint \( \theta < i \) – determine a unique steady-growth solution for the growth rate \( \theta \).\(^\text{10}\) Only by a fluke will this solution be equal to the growth of the labor force, \( n \). Formally, a discrepancy between \( \theta \) and \( n \) can be avoided by abandoning the requirement that \( u = u^* \). The utilization and accumulation rates are constant in steady growth, and full employment requires that \( \tilde{K} = n \). For given values of \( \pi, \sigma \) and \( n \), the condition for full-employment steady growth is given by

\[
q(1 + n) = s(1 - \pi) \frac{Y}{K} = \frac{1}{2 + \rho}(1 - \pi) u \sigma
\]

where \( q \) is given by evaluating (29) at \( \theta = n \). The steady-growth solution for utilization, therefore, equals

\[
\tilde{u} = \frac{q(1 + n)(2 + \rho)}{(1 - \pi) \sigma}
\]

Full-employment steady growth is possible through the adjustment of the utilization rate if the feasibility condition, \( \tilde{u} \leq 1 \), is met, i.e.,

\(^{10}\) We have \( 1 + \theta = \frac{s \omega}{K} \frac{t - \theta}{R(i) - \theta} \). The left hand side is increasing and the right hand side decreasing in \( \theta \). Existence and uniqueness now follows from the fact that both sides are continuous for \( \theta \leq i \) and

\[
1 + \theta < \frac{s \omega}{K} \frac{t - \theta}{R(i) - \theta} \quad \text{for} \quad \theta = -1,
\]

\[
1 + \theta > \frac{s \omega}{K} \frac{t - \theta}{R(i) - \theta} \quad \text{for} \quad \theta = i.
\]
\[\sigma > \frac{1}{1-\pi}q(1+n)(2+\rho)\]  

The derivation of \(\bar{u}\) in equation (31) assumes that investment is determined passively by household saving and that, as a result, dynamic inefficiency becomes the only downside to high saving. The problem of dynamic inefficiency is transformed into one of aggregate demand if the level of investment is determined by profit-maximizing firms. The steady-growth requirement \(u_t = u^*\) can be seen as the steady-growth implication of a Harrodian investment function

\[
\frac{d}{dt} R = \mu(u_t - u^*)
\]

Underlying this stylized description of investment behavior lies a simple claim: profit maximizing firms will not maintain a constant rate of accumulation if they have persistent, unwanted excess capacity. Using Harrod’s terminology, \(\hat{R}^*\) – the steady growth solution associated with \(u_t = u^*\) – defines the warranted growth rate. A low saving rate implies that the warranted rate is below the natural rate, \(\hat{R}^* < n\) and accumulation will be insufficient to keep up with the growth in the labor force. More interesting for present purposes is the case of high saving rates and \(\hat{R}^* > n\). In this situation labor constraints imply that in the long-run output cannot grow at the warranted rate. Excess capacity must emerge, and the dynamics depend on the full specification of investment behavior. The likely result – at least from a Harrodian perspective – is downward instability and depression. But whatever the details, if \(\hat{R}^* \neq n\), there is no steady growth path with full employment and equilibrium in the product market. This general conclusion does not depend on our use of a Harrodian specification of investment. The saving side determines the solution \(\bar{u}\): only by chance will this value of the utilization rate be equal to the value determined from the investment side.\(^{11}\)

The implausibility of accommodating variations in utilization does not exclude a neoclassical route to full-employment growth. The warranted growth rate depends on the interest rate \(i\) and capital intensity \(k\). Thus, depending on the set of possible techniques, there may be a ‘natural interest rate’ which

\(^{11}\) A simple Kaleckian specification, for instance, assumes that the accumulation rate is positively related to utilization, \(I/K = f(u)\). With this specification, full-employment growth requires \(u = f^{-1}(n + \delta) \geq \bar{u}\).
induces a choice of technique such that $\bar{R}^* = n$, thereby equalizing the warranted and natural rates of growth. This choice of technique, however, may be dynamically inefficient; the natural interest rate may even be negative. In the latter case the economy suffers from a 'structural liquidity trap': full-employment growth requires a positive inflation rate (Nakatani and Skott 2007, Skott 2001). Whether or not it is negative, the natural interest rate will – in general – deviate from the interest rate that is required to induce the capital intensity that is deemed socially optimal. A single instrument, the interest rate, cannot simultaneously achieve two independent targets.

These results are closely related to the problems of 'secular stagnation' which have gained recent attention (Summers' 2013, 2015). Basically, a failure to reduce the real interest sufficiently leads to sustained aggregate demand problems; a successful reduction, on the other hand, may lead to dynamic inefficiency. Fiscal policy provides a solution.

4 Public debt

In standard OLG models, dynamic inefficiency problems can be overcome by introducing a public sector and public debt. Analogously, fiscal policy makes it possible to escape aggregate demand problems and achieve full-employment growth in our Keynesian OLG model.

4.1 Adding a public sector

The government consumes ($G_t$), levies lumpsum taxes on the young and old generations ($T_t^Y$ and $T_t^O$) and has debt ($B_t$). With an extra financial asset, young households save in the form of corporate and government bonds as well as equity. We assume that these assets are perfect substitutes and have the same rate of return, $r_t$.

The saving equation (19) now takes the form

$$S_t = M_{t+1} + V_{t+1} + B_{t+1}$$

and the public sector budget constraint is given by

$$G_t + (1 + r_t)B_t = B_{t+1} + T_t^Y + T_t^O$$
A young (employed) agent in period $t$ maximizes (12) subject to a modified constraint,

$$c_{1,t} + \frac{1}{1+\tau_{t+1}}c_{2,t+1} = w_t - \tau_t - \frac{1+n}{1+\tau_{t+1}}y_{t+1}$$

(36)

where, $\tau_t = T_t^Y/L_t$, and $y_t = T_t^0/L_t$. The maximization gives the following solution for saving

$$S_t = \left[ s(w_t - \tau_t) + (1-s)\frac{1+n}{1+\tau_{t+1}}y_{t+1} \right]N_t$$

(37)

where $s$ is given by (15). Alternatively, saving can be written

$$S_t = \bar{s}_t(w_t - \tau_t)N_t$$

(38)

where the young generation’s saving rate out of the current disposable income ($\bar{s}_t$) is given by

$$\bar{s}_t = \frac{s(w_t - \tau_t) + (1-s)\frac{1+n}{1+\tau_{t+1}}y_{t+1}}{w_t - \tau_t}$$

(39)

Using (38) and dividing through by $N_t$, (34)-(35) can be rewritten,

$$(1 + \theta_t)(b_{t+1} + q_{t+1}k_{t+1}) = \bar{s}_t(w_t - \tau_t)$$

(40)

$$g_t + (1 + \tau_t)b_t = (1 + \theta_t)b_{t+1} + (\tau_t + y_t)(L_t/N_t)$$

(41)

where $g_t = G_t/N_t$, $b_t = B_t/N_t$, $k_t = K_t/N_t$, and $\theta_t = N_{t+1}/N_t$.

Steady growth with full employment requires that $N_t = L_t$ (thus, $\theta_t = n$), $b_t = b$ and $u_t = u^*$; we have $r_t = i$, and $w, k$ and $q$ are still given by (9), (24) and (29). There are three fiscal instruments, government consumption, taxes on the old and taxes on the young. Taking the taxes on the young to be the active instrument, we assume that $g_t = g$ and $y_t = y$ are exogenous. The analysis would be analogous with $g$ or $y$ as the active policy instrument.

Substituting the steady-growth conditions into (40)-(41), using (39), and rearranging, the solution for $b_t$ becomes

$$b^* = \frac{sw - (1+n)qk^*}{1+n+s(i-n)} + \frac{1}{1+i}y - \frac{s}{1+n+s(i-n)}g$$

(41)

This steady-growth solution for the required debt ratio ($b^*$) depends inversely on public consumption ($g$) and directly on the level of taxes on the old generation ($y$).\footnote{An inverse relation between debt and government consumption is obtained in a non-OLG setting by Schlicht} An increase in $g$ implies that consumption
has to contract in order to maintain equilibrium in the product market. This is achieved by increasing taxes on the young. As a result the desired saving decreases and this, in turn, reduces the need for government debt as an outlet for saving. Analogously, with a given value of \( g \), an increase in \( \gamma \) must be accompanied by a reduction in \( \tau \) in order to maintain the level of consumption and ensure equilibrium in the goods market; the disposable income of the young increases, and the amount of public debt must also increase to meet the rise in saving.

Equation (42) also implies a relation between economic growth and public debt: the required debt is inversely related to the growth rate \( n \) for empirically relevant values of the parameters; the partial \( \partial b/\partial n \) is negative if \( (1 + i)qk + (1 - s)(w - g) > 0 \) and the wage share greatly exceeds the share of government consumption \( (w - g > 0) \) in all OECD countries. Intuitively, debt is required because the young generation wants to save in excess of what is needed to provide fixed capital for the next generation. A higher growth rate raises the need for fixed capital and therefore reduces the need for public debt.

The required debt, finally, depends on the interest rate, directly as well as indirectly via the effect of the interest rate on the choice of technique and the value of \( k^* \). The relationship between \( i \) and \( b^* \) can be interpreted in relation to the ‘natural rate of interest’ (p. 11): the interest rate that is consistent with full-employment growth – the natural rate of interest – depends on fiscal policy and the debt ratio. Putting it differently, functional finance (i) identifies the optimal capital intensity, (ii) sets the interest rate at the associated level through an appropriate monetary policy, and (iii) uses fiscal policy to make the natural interest rate equal to this optimally chosen rate.

Equations (40)-(41) can be used to derive the steady-growth solution for the tax rate \( \tau \):

\[
\tau = \frac{(i-n)[sw-(1+n)qk^*]}{1+n+s(i-n)} - \frac{1+n}{1+\tau} \gamma + \frac{1+n}{1+n+s(i-n)} g \tag{43}
\]

An increase in \( g \) raises \( \tau \) and reduces \( b^* \); an increase in \( \gamma \) reduces \( \tau \) and raises \( b^* \). It follows that shifts in \( g \) or \( \gamma \) produce a negative correlation between the steady-growth values of \( \tau \) and \( b^* \); high

debt in these cases is associated with low tax rates. Other parameter shifts yield a positive correlation between \( \tau \) and \( b^* \); a shift in the markup, for instance, will change \( b^* \) and \( \tau \) in the same direction if \( i > n \).\(^{13}\)

5 Fluctuations in ‘confidence’ and intergenerational fairness

The saving propensity may fluctuate across generations. These fluctuations could be the result of differences in the discount rate across generations, but unfounded variations in ‘confidence’ (or irrational exuberance) can do the trick too. Suppose, for instance, that young agents believe that in addition to the returns on their saving, they will have an after-tax income of \( \varepsilon \). By assumption, the actual after-tax income will be the net tax on the old, that is, the value of \( \varepsilon_{t+1} \) is given by \(- (1 + n) \gamma_{t+1}\). Agents may not have perfect foresight, however, and for present purposes it does not matter whether a high \( \varepsilon \) reflects a mistaken belief about future taxes or an expectation that some other source of income will be available (e.g. an expectation of being able and willing to work in the second period). The state of confidence – the beliefs about future income – may be wrong, but the beliefs alter the perceived budget constraint and affect the saving decisions. The budget constraint now reads:

\[
e_{1,t} + \frac{1}{1+r_{t+1}} e_{2,t+1} = w_t - \tau_t + \frac{1}{1+r_{t+1}} \varepsilon_{t+1} \tag{44}
\]

Assuming, for simplicity, that there is no subjective uncertainty, the maximization of (12) subject to (44) implies that

\[
S_t = \left[ s(w_t - \tau_t) - (1-s) \frac{1}{1+i} \varepsilon_{t+1} \right] N_t \tag{45}
\]

\[
= \bar{s}(\varepsilon)(w_t - \tau_t)N_t \tag{46}
\]

Distributionally neutral intervention

A distributionally neutral policy intervention can be achieved by instituting a transfer to those

\(^{13}\) From (41) it follows that in steady growth \((i - n) b = \tau + \gamma - g\). Thus, \( b \) and \( \tau \) must move in the same direction if \( i, n, \gamma \) and \( g \) are kept constant.
young generations that are unduly pessimistic (tend to consume too little) and finance the transfer by
taxing the same generations when they get old. Conversely, an overly optimistic generation can be
taxed in the first period and compensated by a transfer in the second.

Substituting (45) into (40)-(41), full-employment growth requires that

\[
\tau_t = \frac{(1+n)qk^*g+(1+i)b_t-\gamma_t-sw}{1-s} + \frac{1}{1+i} \epsilon_{t+1}
\]  

(47)

The tax on the old generation \(y_t\) and the public debt \(b_t\) appear on the right hand side of equation (47).

These variables are pinned down by the neutrality requirement.

Distributional neutrality implies that a generation should not be favored (or punished) because of
a shift in the confidence of the succeeding generation. If \(b^*\) and \(\gamma^*\) denote the steady-growth values of
\(b\) and \(\gamma\) when there are no variations in confidence, this condition can be stated formally as

\[
(1+i)(qk^* + b_t)-\gamma_t = (1+i)(qk^* + b^*) - \gamma^*
\]  

(48)

The expression on the left hand side of equation (48) gives the income available to the old generation in
period \(t\). Neutrality requires that this income be equal to the level that characterizes the steady growth
path. Output follows the full-employment path; the stabilization of the consumption of the old generation
therefore implies that the consumption of the young will also be at its steady-growth value.

Using (47) and (48), the equation for the tax on the young at time \(t\) can now be written

\[
\tau_t = \frac{(1+n)qk^*g+(1+i)b^*-\gamma^*-sw}{1-s} + \frac{1}{1+i} \epsilon_{t+1}
\]  

(49)

where \(\epsilon_{t+1}\) is the steady-growth value of \(\tau\) in the absence of variations in confidence.

Using (41), (48) and (49), we have

\[
(1+n)b_{t+1} = g + (1+i)b^* - \gamma^* - \tau^* - \frac{1}{1+i} [\epsilon_{t+1} + (1+n)\gamma^*]
\]  

(51)

14 The transfer stimulates the young generation’s consumption but a part of the transfer will be saved. The
additional saving will be absorbed by the issue of government bonds.
\[ y_{t+1} = y^* + (1 + i)(b_{t+1} - b^*) \]  
\[ = y^* - \frac{1}{1+n}[\varepsilon_{t+1} + (1 + n)y^*] \]  
\[ = -\frac{1}{1+n}\varepsilon_{t+1} \]  

Hence,

\[
\frac{\partial \tau_t}{\partial \varepsilon_{t+1}} = \frac{1}{1+i'} \frac{\partial b_{t+1}}{\partial \varepsilon_{t+1}} = -\frac{1}{(1+i)(1+n)} \frac{\partial y_{t+1}}{\partial \varepsilon_{t+1}} = -\frac{1}{1+n} 
\]

\[
\frac{\partial \tau_{t+k}}{\partial \varepsilon_{t+1}} = \frac{\partial b_{t+1+k}}{\partial \varepsilon_{t+1}} = \frac{\partial y_{t+1+k}}{\partial \varepsilon_{t+1}} = 0 \quad \text{for} \quad k \geq 1
\]

A shock to a generation’s confidence is fully absorbed by adjustments in the taxes for that same generation; there are no persistent effects on the debt ratio.

**Tax expectations**

The above analysis uses systematic variations in \( \tau_t \) and \( y_{t+1} \) to get distributional neutrality across generations. The analysis took the expected after-tax, non-capital income as exogenous, and this exogeneity assumption may seem unreasonable: it is often assumed that systematic variations in taxes will be anticipated. The private sector’s anticipation of future taxes does not, however, negate the possibility of distributionally neutral stabilization.

Consider the other extreme where taxes are perfectly foreseen. In this case, confidence is characterized by the value of the expected pre-tax, non-capital income, \( z_{t+1} = \varepsilon_{t+1} + (1 + n)y_{t+1} \).

Taxes can now be used as an insurance mechanism. Formally, let \( y_{t+1} \) be determined by

\[ y_{t+1} = y^* + \frac{\bar{z}_{t+1}}{1+n} \]  

where \( \bar{z}_{t+1} \) is the actual average non-capital income when the generation is old. By assumption there is symmetry across agents within a generation and the individual agent’s own income \( z_{t+1} \) will be equal to the average income \( \bar{z}_{t+1} \); the specification of taxes in terms of average income is used to preserve the lump-sum character of the tax.

The tax scheme in (57) implies that the budget constraint (44) can be rewritten
\[ c_{2,t+1} = (1 + i)(w_t - \tau_t - c_{1,t}) + (1 + n)[z_{t+1} - (1 + n)y_{t+1}] \]
\[ = (1 + i)(w_t - \tau_t - c_{1,t}) - (1 + n)y^\ast \]  
(58)

The budget constraint becomes independent of ‘confidence’: the conditional tax scheme provides insurance and effectively guarantees that the after-tax, non-capital income will be equal to \(-(1 + n)y^\ast\). Consequently, the tax rate on the young can be set at the steady-growth level, \(\tau_e = \tau^\ast\), no variations in \(\tau\) are required.

The assumptions underlying this example may be implausible: a combination of confidence effects and perfect anticipation of future taxes may seem even more questionable than the more common Ricardian assumptions of perfect foresight with respect to both taxes and future returns. In response to this objection, one can take one of two routes: assume that there are no variations in confidence or, alternatively, accept that variations in confidence do occur and that households do not fully anticipate future taxation. The first route we will leave to others; the second can be approached by examining – as in equation (49) – how variations in expected after-tax, non-capital income can be neutralized by taxation.

**Non-neutral intervention**

The analysis may be subject to another objection. We have assumed that the private sector is subject to swings in confidence; the government, by contrast, correctly infers the private sector’s expectations, correctly anticipates the future incomes, and has the ability to implement fairly sophisticated tax schemes. These assumptions may impose policy demands that no government can meet. Intergenerational neutrality and perfect government foresight are not required, however, for full-employment growth.

To see this, consider the simple case in which capital income is taxed at a constant rate \(\beta\),
\[ y_t = \beta(qk_t + b_t)(1 + r_t) \]  
(59)

Returning to the case without private sector anticipation of future taxes, we take the expected future after-tax, non-capital income as exogenous; thus, the saving rate by the young generation is given by (45). Combining these assumptions with equations (40) and (41) – still assuming that \(\tau_t\) is used as the active
instrument to ensure full-employment growth – the debt dynamics can be written

\[ b_{t+1} = A - Bb_t + \frac{1}{(1+n)(1+i)(1-s)} \varepsilon_{t+1} \]  

(60)

where

\[ A = \frac{s[w-g+\beta(1+i)qk']-qk'(1+n)}{(1+n)(1-s)} \]  

(61)

\[ B = (1-\beta) \frac{1+i}{1+n} \frac{s}{1-s} \]  

(62)

If \( \varepsilon_t = 0 \) for all \( t \) and the tax rate \( \beta \) is sufficiently large, the difference equation (60) has a unique, stable stationary point,

\[ b^{**} = \frac{A}{1+B} \]  

(63)

Random fluctuations in \( \varepsilon \) generate fluctuations in \( b_t \). But if the fluctuations in \( \varepsilon \) are bounded then so are the fluctuations in \( b_t \). \(^{15}\)

6 Discussion

6.1 Public debt, interest rates and economic growth

In OLG models an exogenous rise in debt will be associated with a fall in the capital stock and an increase in the return on capital. A functional finance approach to fiscal policy makes this result irrelevant: debt is allowed to increase if an increase is necessary to maintain both full employment and the optimal capital intensity. A perfectly executed fiscal policy of this kind may show fluctuations in debt (as in section 5), but the capital intensity and the return on capital are kept constant.

Fiscal policy may not always be conducted in accordance with the principles of functional finance – the current obsession with austerity testifies to that – but the result carries important implications for empirical evaluation: observed correlations between interest rates and debt depend on the interaction between policy regimes and private sector behavior. Without knowledge of the sources of changes in the

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\(^{15}\) Other non-neutral schemes could be used, including one with a balanced government budget at all times: tax the pessimistic young and transfer the tax revenue to the currently old generation.
public debt, there is no way to predict the empirical correlation between debt and interest rates. Thus, it is not surprising that the results of empirical studies are weak and inconclusive.\textsuperscript{16}

Disregarding this inconclusiveness, a standard OLG link is between the level of debt and the levels of capital and income. Recently, however, the possibility of a long-run relation between the debt ratio and the rate of economic growth has received great attention following the publication of Reinhart and Rogoff (2010) and Kumar and Woo (2010). The theoretical story behind this relation is unclear, and theoretical ambiguities accentuate the difficulty of interpreting empirical results.\textsuperscript{17} Accepting, for the sake of the argument, that a negative correlation can be found between debt and economic growth, a key question concerns causation. This question has two parts. The first part asks whether past episodes of high debt did in fact \textit{cause} low growth, as opposed to a reverse causal link between the two variables or an explanation in which a third factor accounts for the changes in both debt and growth. Empirical studies by Irons and Bivens (2010), Basu (2013) and Dube (2013) conclude that causation has run from growth to debt. These empirical results could be driven by short and medium term effects of a slowdown in growth on deficits and debt, but the analysis in this paper lends theoretical support to the conclusions, also for the long run. As shown in section 4, functional finance produces a causal link between growth rates and debt: a reduction in the long-run rate of growth raises the long-run debt ratio.

The second part of the question is more radical. One may ask whether it is at all meaningful to look for a general answer to a reduced-form question about the growth effects of public debt. According to Rogoff and Reinhart (2010, p. 6),

\textit{...war debts are arguably less problematic for future growth and inflation than large debts that are...}  

\textsuperscript{16} In the words of Engen and Hubbard (2005, p.83), there is “little empirical consensus about the magnitude of the effect ... some economists believe there is a significant, large, positive effect of government debt on interest rates, others interpret the evidence as suggesting that there is no effect on interest rates”. Bohn (2010, p.14) makes a similar statement about the difficulty of finding significant interest effects of debt. He goes on to suggest that a “leading explanation is Ricardian neutrality”. There is no need for Ricardian neutrality to explain the results, however; our OLG model does not display neutrality. 

\textsuperscript{17} Kumar and Woo mention a number of possible channels, including the effect of higher interest rates on capital accumulation, and the potential effects of debt induced increases in “uncertainty about prospects and policies”. As discussed above, the evidence on a debt - interest rate link is tenuous, at best. The latter effect seems to be a close cousin of what Krugman has been referring to as the ‘confidence fairy’, and it is hard to see how contractionary fiscal policies will enhance confidence in a recession.
accumulated in peacetime. Postwar growth tends to be high as war-time allocation of man-power and resources funnels to the civilian economy. Moreover, high war-time government spending, typically the cause of the debt buildup, comes to a natural close as peace returns. In contrast, a peacetime debt explosion often reflects unstable underlying political economy dynamics that can persist for very long periods.

As pointed out by Michl (2013):

To a Keynesian, the quote above would very sensibly read ‘high recession-time government spending, typically the cause of the debt buildup, comes to a natural close as growth returns.’ In fact, Keynes (1972, p. 144) once aptly described government borrowing as “nature’s remedy” for preventing a recession from deteriorating into a total collapse in production. As for the rest of the quote, who would deny that ‘unstable political dynamics’ can be an obstacle to growth?

A fiscal expansion is intrinsically neither good nor bad. A reckless fiscal expansion can cause overheating, inflation and macroeconomic instability. But sensible fiscal policies are adjusted in the light of prevailing economic circumstances, and the effects of bad policy say little about the growth effects of a fiscal expansion in a deep recession. The general point is simple: reduced-form correlations between debt and growth depend on the underlying sources of the movements in debt.

### 6.2 Public debt and intergenerational distribution

Claims that high public debt hurts future generations have figured prominently in popular debates and also appear in the academic literature. Having found that public debt has at most small effects on interest rates, Engen and Hubbard (2005), go on to caution that public deficits and debt still matter because large levels of government debt “can represent a large transfer of wealth to finance current generations’ consumption from future generations which much eventually pay down federal debt to a sustainable level.” (p. 132)

The possibility that fiscal policy can hurt future generations is not controversial; inappropriate fiscal policy can have negative effects for future as well as for current generations. But our analysis of an OLG model without bequests – the setting that is most favorable to the case for adverse future effects of
public debt – shows that debt need not be a burden on future generations. On the contrary, it can serve to remove dynamic inefficiencies and maintain full employment. Fluctuations in ‘confidence’ can be addressed through policies that are neutral in their effects on the intergenerational distribution. Even when a policy is not fully neutral in this sense, future generations may be better off than without the policy. With a fixed tax rate on capital income, for instance, the required variations in the tax on the young generation will have distributional effects: a pessimistic generation will be favored by a reduction in its taxes (section 5). This result does not imply that future generations would be better off without the reduction. In the absence of fiscal expansion, a lack of demand would affect capacity utilization, reduce investment and the future capital stock, and jeopardize both current and future employment.

6.3 Austerity and long term consolidation

Entitlement programs like social security or medicare are prime targets of most austerity programs. Reductions in these programs have adverse intra-generational effects on distribution (which our model with identical agents within each generation cannot capture). More surprisingly, our analysis demonstrates that these reductions may also be counterproductive, assuming that the aim is to reduce public debt: reductions in social security and medicare correspond to a rise in the tax on the older generation, and as shown in section 4, an increase in the taxation of the old generation will raise the required debt. A reduction in government consumption \( (g) \), likewise, requires an increase in the long-run debt. The general point, once again, is that the desirable level of public debt depends on a range of behavioral and policy variables.

These results suggest that from a functional finance perspective some critiques of austerity may not go far enough. Krugman’s insistence that the slump is not the time to cut the debt is fully in line with functional finance, but he also suggests that the US has long-run budget problems that must be addressed once we are out of recession.\(^{18}\) The nature of the long-run debt problem is not made clear, however. This

\(^{18}\) “Yes, the United States has a long-run budget problem. Dealing with that problem is going to require, first of all, sharply bending the curve on Medicare costs; without that, nothing works. And second, it’s going to require some combination of spending cuts and revenue increases, amounting to at least 3 percent of GDP and probably more, on
is not to say that there can be no adverse consequences of high public debt. But these consequences have to be clearly specified and balanced against the benefits.

7 Conclusions

Are the current debt levels and fiscal deficits sustainable? It is not always clear what is meant by sustainability, but the question may be whether the fiscal requirements for full employment growth will generate an ever-increasing debt-GDP ratio. The analysis in this paper shows that fiscal policy and public debt may be needed to maintain full employment, and that this fiscal policy need not – and in our OLG model does not – lead to any kind of unsustainability.\(^1\)

The equilibrium debt ratio depends on the parameters of the model. Some parameters are of particular interest. Austerity policies, which tend to reduce government consumption and raise net taxes on the old, will raise the debt ratio. Empirical correlations between growth rates and debt ratios, moreover, are consistent with the model, but the causation runs from growth to debt.

In steady growth all generations do equally well. Questions of inter-generational distribution may arise away from steady growth. We address this issue by introducing fluctuations in household ‘confidence’, showing how full employment can be achieved using distributionally neutral policies. A failure to adjust fiscal policy in response to a decline a confidence, on the other hand, leads to aggregate demand problems and secular stagnation.

The analysis has many limitations. We have focused on a closed economy, and the paper says nothing about the problems of open economies with public debt in a foreign currency.\(^2\) The neglect of heterogeneity within generations represents a second limitation. Public debt may have regressive

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\(^1\) As shown by Ryoo and Skott (2013), functional finance can produce unstable debt-income dynamics in settings with intra-generational heterogeneity. These unstable scenarios are closely linked to (intra-generational) distribution effects and can be avoided by changes in the structure of taxation.

\(^2\) Chalk (2000, p. 319) argues that some OECD countries “have seen an explosion in their indebtedness to such an extent that the solvency of the public sector is brought into question.” Solvency questions of this kind may be relevant for countries with debt in foreign currency. But it is unclear how a sovereign state could ever become insolvent if its debt obligations are denominated in a currency that it can print at will.
distributional effects if taxes on wage income are used to finance interest payments to the rich. The incentive effects of taxes, third, have been ignored throughout. As shown in section 4, a higher level of debt need not be associated with higher tax rates but even if it is, the structure of government consumption and the form of taxation may be more important than the level of debt for the public sector’s incentive effects. Fourth, the model only indirectly addresses inflationary concerns. Engen and Hubbard (2005) suggest that “federal government debt may also pose the temptation to monetize the debt, causing inflation”. They point out, however, that “this concern has not been a problem in the United States over the past fifty years” (p. 98); Reinhart and Rogoff (2010) also find no evidence for a link between debt and inflation in advanced economies. The inflation fear essentially boils down to a concern that policy may not in the future be governed by a functional finance criterion: “eliminate both unemployment and inflation” (Lerner 1943, p. 41). Functional finance, fifth, may imply that interest rates should be set to achieve a desired capital intensity. This objective need not exclude short-run variations in interest rates around the level associated with the chosen capital intensity. The level of public debt influences the effectiveness of short-run monetary policy. A contractionary monetary policy raises interest rates and generates an automatic fiscal expansion unless it is matched by an increase in tax rates. Thus, monetary policy is blunted when debt is high and this may complicate short-run economic policy. The simple OLG structure, sixth, may be appealing for an analysis of public debt, but it has peculiar properties that find no support in data. The model implies that the saving rate is inversely related to the profit share: only the young save, and the young get their income as wage income. Empirically, by contrast, saving rates are higher out of profits than wages; the saving assumptions that are at the center of the analysis in OLG models can be questioned.

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21 The importance of incentives for growth is disputed. Fast growth during the ‘golden age’ was associated with high marginal tax rates in the US, and the Nordic welfare states show a very respectable performance, including low unemployment and labor force participation rates that exceed those of the US.

22 Ryoo and Skott (2015) analyze short-run stabilization in an economy characterized by Harrodian instability; see also Franke (2015).

23 We abandon these assumptions in Ryoo and Skott (2013) which builds on the ‘stock-flow consistent’ framework from Skott (1989) and Skott and Ryoo (2008).
Our analysis, finally, has taken as given the level of government spending. Public investment in infrastructure, education, health, and the environment clearly contribute to future welfare, and public consumption and social spending can also have a high future payoff, even in narrow economic terms (by reducing crime or raising future earnings, for instance). The benefits and distributional effects of public spending could justifiably be ignored in a discussion of public debt if this spending were already at an agreed-upon optimal level. A good deal of the debate over public debt, however, may reflect underlying controversies over the desirable level of public spending. These issues are beyond the scope of this paper.

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27
Appendix A: An OLG model with perfect competition and a Leontief production function

Let the production function be

$$Y_t = \min\{\lambda L_t, \sigma K_t\}$$

Assuming inelastic factor supplies, perfect competition implies that

$$w_t = \begin{cases} 0 & \text{if } \lambda L_t > \sigma K_t \\ \frac{\lambda}{\sigma} & \text{if } \lambda L_t < \sigma K_t \end{cases}$$  \hspace{1cm} (64)

$$r_t + \delta = \sigma \quad \text{if } \lambda L_t > \sigma K_t$$
$$0 \quad \text{if } \lambda L_t < \sigma K_t$$  \hspace{1cm} (65)

The economy has two (non-trivial) steady growth paths.\(^{24}\) There is a full-utilization path with $\lambda L_t = \sigma K_t$ and\(^{25}\)

$$w = (2 + \rho)(1 + n)\left(\frac{\lambda}{\sigma}\right) < \dot{\lambda}$$  \hspace{1cm} (66)

$$r + \delta = \sigma - (2 + \rho)(1 + n) > 0$$  \hspace{1cm} (67)

The steady growth path described by equations (66)-(67) is dynamically efficient: any reduction in the capital-labor ratio would produce a return to capital that exceeds the growth in the labor force ($\sigma - \delta > n$). The path is also unstable, however: a negative shock to $w_t$ reduces $S_t$ (equation (16) and implies that $K_t/L_t < \lambda/\sigma$ in the next period; as a result, $w_t$ drops to 0, there is no saving, and the capital stock drops to 0.

In addition to the efficient steady growth path and the trivial path with $K_t = 0$, there is a locally stable steady growth path with less than full utilization of capital. Starting from the efficient path, a positive shock to $w$ raises saving, and capital intensity increases in the next period to give $K_t/L_t > \lambda/\sigma$.

\(^{24}\) In addition to these two paths there is a trivial steady-growth solution with $K_t = Y_t = 0$.

\(^{25}\) The inequalities in (64)-(65) follow from condition (18).
The wage rate then rises to \( w_t = \lambda \) in subsequent periods, and the economy will be following a steady growth path with excess capacity:

\[
k = \frac{\lambda}{(2+\rho)(1+n)} > \frac{\lambda}{\sigma}
\]

This steady-growth path clearly is dynamically inefficient; the net return on capital is negative along this path; we have \( r_t = -\delta \leq n \) Consumption could be increased by reducing investment and eliminating the excess capacity.