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By

Roberto Veneziani and Naoki Yoshihara

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Exploitation and Profits: A General Axiomatic Approach in Convex Economies with Heterogeneous Agents*

Roberto Veneziani[†]and Naoki Yoshihara[‡]

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Abstract

This paper provides an innovative axiomatic analysis of the notion of exploitation as the unequal exchange of labour, focusing on the relation between exploitation and profits. General convex economies with heterogeneous agents endowed with unequal amounts of physical and human capital are considered. An axiomatic characterisation of the class of definitions that preserve the Fundamental Marxian Theorem (FMT) in this general context is derived. It is shown that none of the main received definitions preserves the FMT. Instead, a definition related to the 'New Interpretation' (Duménil, 1980; Foley, 1982) is presented which preserves the FMT and allows one to generalise a number of key insights of exploitation theory to complex advanced economies.

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[†]Department of Economics, Queen Mary University of London, Mile End Road, London E1 4NS, UK. E-mail: r.veneziani@qmul.ac.uk

[‡](Corresponding author) The Institute of Economic Research, Hitotsubashi University, Naka 2-1, Kunitachi, Tokyo 186-0004, Japan. Phone: (81)-42-580-8354, Fax: (81)-42-580-8333. E-mail: yosihara@ier.hit-u.ac.jp

1 Introduction

What is exploitation? In political philosophy, the most general definition affirms that agent A exploits agent B if and only if A takes unfair advantage of B. Despite its intuitive appeal, this definition leaves two major issues in need of a precise specification, namely the source of the unfairness and the structure of the relationship between A and B that allows A to take advantage of B. There is considerable debate in the economic and philosophical literature concerning both issues. Although both aspects of exploitative relations are arguably crucial (see Veneziani, 2008; Yoshihara and Veneziani, 2009; and Yoshihara, 2010, for a thorough defense of this claim), the analytical focus of this paper is on the unfairness, or more precisely, on the economic inequalities involved in the concept of exploitation.

To be specific, this paper analyses the Marxian theory of *exploitation as an unequal exchange* (hereafter, UE) *of labour*, according to which exploitative relations are characterised by a difference between the amount of labour that an individual provides and the amount of labour contained in some relevant bundle that she does (or can) purchase with her income. There are at least two reasons to focus on *labour* as the measure of the injustice of exploitative relations. First, in a number of crucial economic interactions, the notion of exploitation is inextricably linked with some form of labour exchange. Second, the UE definition of exploitation captures some inequalities in the distribution of material well-being and free hours that are - at least *prima facie* - of normative relevance. For instance, they are relevant for *inequalities of well-being freedom*, as discussed by Rawls (1971) and Sen (1985, 1985a),[1] because material well-being and free hours are two crucial determinants of individual well-being freedom. Further, it can be proved that in a private-ownership economy with positive profits, class and UE exploitation status are strictly related, and they accurately reflect an unequal

[1]The notion of well-being freedom emphasises an individual's ability to pursue the life she values. In the Rawls-Sen theory, inequalities in the distribution of well-being freedom are formulated as *inequalities of capabilities*, whereas they are formulated as *inequalities of (comprehensive) resources* in Dworkin's theory (Dworkin, 2000). The resource allocation problem in terms of equality of capability is analysed in Gotoh and Yoshihara (2003), whereas Roemer (1986) and Yoshihara (2003) analyse it in terms of equality of resources.

distribution of assets (as formalised in the *Class-Exploitation Correspondence Principle*; see Roemer, 1982). That is, in equilibrium the wealthy emerge as exploiters and members of the capitalist class, whereas the poor are exploited and members of the working class. From this perspective, exploitative relations are relevant because they reflect unequal opportunities of life options, due to differential ownerhsip of productive assets.

Although the definition of UE exploitation is seemingly intuitive, it has proved surprisingly difficult to provide a fully satisfactory general theory of exploitation. In fact, outside of standard Leontief economies, the appropriate definition of the amount of labour 'received' by an agent is not obvious, and indeed a number of approaches have been proposed (see Yoshihara, 2010). Further, outside of stylised, linear two-class economies, the core insights of exploitation theory do not necessarily hold. A key tenet of the standard Marxist theory, for example, concerns the relation between profits and exploitation: profits represent the way in which capitalists appropriate social surplus and social labour. This has been incorporated into the Fundamental Marxian Theorem (hereafter, FMT) which states that exploitation is synonymous with positive profits. The relevance of FMT is such that although it is proved as a result, its epistemological status is that of a postulate: the appropriate definition of exploitation is considered to be one which preserves FMT. Various definitions of exploitation have been proposed precisely in an attempt to generalise FMT to economies with joint production, heterogenous labour, and so on (see Morishima, 1974; Roemer, 1981; Krause, 1982. For recent debates, see Veneziani, 2004; Flaschel 2010). Yet in more general economies, a number of counterexamples have been found and no fully satisfactory definition that preserves FMT has been provided so far.

In this paper, exploitation is analysed in general economies with a convex production technology and with maximising agents endowed with heterogeneous preferences and with different amounts of both physical and human capital, as outlined in section 2. The economies analysed are thus significantly more general than those usually considered in exploitation theory.[2] One substantive contribution of this paper is to provide a definition of exploitation, which extends some of the core insights of exploitation theory, such as the FMT, and allows one to characterise the exploitation status of all agents in such general economies. This definition focuses on the aggre-

[2]An interesting analysis of nonconvexities in Marxian economic theory can be found in Negishi (1998). The latter paper does not focus on exploitation, though.

gate amount of social labour performed and on its distribution to agents via market mechanisms, and it is conceptually related to the 'New Interpretation' (Duménil, 1980; Foley, 1982; Duménil and Foley, 2008; Duménil, Foley, and Lévy, 2009). Unlike the main received approaches, this definition is firmly anchored to the actual data of the economy and it has an inherently social dimension, because it takes the aggregate allocation of the economy as the starting point. Further, it defines exploitation as a feature of the (competitive) allocation of social labour rather than the result of productive inefficiencies, or imperfections in the labour market. Thus, in addition to preserving FMT in general economies, it captures some key features of the Marxian theory of exploitation, and has a clear empirical content.

Methodologically, this paper extends the axiomatic analysis of UE exploitation developed by Yoshihara and Veneziani (2009) and Yoshihara (2010). This novel approach to exploitation theory provides a general framework to compare the most important definitions in the literature. An axiomatic approach was long overdue in exploitation theory, where the proposal of alternative definitions has sometimes appeared as a painful process of adjustment of the theory to anomalies and counterexamples. The definitions of exploitation thus constructed have progressively lost the intuitive appeal, normative relevance, and even connection with the actual, observed variables emerging from a competitive mechanism. By adopting an axiomatic approach, this paper suggests to start from first principles, thus explicitly discussing the intuitions behind exploitation theory.

To be precise, in section 3, an axiom is introduced, called *Labour Exploitation for the Working Class* (hereafter, **LEW**), which restricts the way in which the set of exploited agents is identified. This axiom is interpreted as the minimal necessary condition to capture the core intuitions of exploitation theory, and it is shown that all of the main definitions of exploitation in the literature satisfy it (see Morishima, 1974; Foley, 1982; Roemer, 1982. See also Yoshihara and Veneziani, 2009; and Yoshihara, 2010). Then, the epistemological role of FMT as a postulate in Marxian exploitation theory is explicitly recognised and FMT is stated in axiomatic terms as requiring that, in equilibrium of a convex economy, propertyless agents be exploited if and only if profits are positive. Theorem 1 provides the first rigorous characterisation of the class of definitions satisfying **LEW** which preserve FMT. Based on this characterisation, it is shown that, among all the main definitions, the 'New Interpretation' is the only one that preserves FMT.

Theorem 1 is an important result for two reasons. Methodologically, it provides the first general axiomatic analysis of the relation between exploitation and profits, and a starting point for further research in general convex economies with heterogeneous agents. Substantively, given the theoretical relevance of FMT in exploitation theory, Theorem 1 provides strong support for the 'New Interpretation' as the appropriate formulation of UE exploitation. Thus, it confirms and extends the analysis of Yoshihara and Veneziani (2009), who have shown that in the class of convex subsistence economies - which may be taken as a subset of the economies analysed in this paper - the 'New Interpretation' is uniquely characterised by a small number of weak axioms capturing the key insights of UE exploitation.

Two extensions of the analysis are also presented, which provide further support for the 'New Interpretation.' First, a focus on the poorest segment of the working class, namely agents without any physical assets, is appropriate from the axiomatic viewpoint: focusing on a strict subset of the set of agents makes the axiomatic framework rather weak. Yet one may argue that this is reductive and some key characteristics of advanced capitalist economies should be explicitly considered, which make the issue of Marxist exploitation a contentious one today - such as the fact that many workers do own some non-labour assets, and even stock in firms, through their pension funds. Second, although FMT is traditionally analysed by focusing on equilibrium allocations (see Morishima, 1974; Roemer, 1981), this perspective might be deemed too narrow. In the context of exploitation theory, one may question general equilibrium-type constructions as representations of allocation and distribution in market economies because they depend on the often tacit assumption of equal-treatment, or equivalently, that transactions take place only at equilibrium prices.[3] In a general theory of exploitation, it would therefore be important to take account of transactions at disequilibrium prices and the resulting horizontal inequity in distribution endogenous to market allocation. In section 4 the generality of the model is exploited to show that the 'New Interpretation' can be extended, first, to analyse the exploitation status of all agents, in economies with heterogeneous preferences, physical assets, and skills (Theorem 2), and then to establish a relation between exploitation and profits outside of equilibrium allocations (Theorem 3).

It is important to note at the outset that exploitation theory is not sim-

[3]This issue has been brought to our attention by Duncan Foley in a private exchange. For an analysis of the implications of trading at disequilibrium prices, see Foley (2010).

ply about proving FMT. Yet the relation between profits and exploitation is crucial and it plays a prominent role in the literature (in addition to the contributions already mentioned, see the recent works by Mori, 2008; Flaschel, 2010; Fujimoto and Opocher, 2010). Further, the 'New Interpretation' provides the foundations for a *general* theoretical framework, which can deal with many unresolved issues in exploitation theory, including the analysis of unequal exchange and international relations. (See Veneziani and Yoshihara, 2009. For a critique of the standard Marxist analysis, see Negishi, 1999.) Some extensions of the analysis are briefly discussed in section 5 below.

Finally, the existence of a general equilibrium is proved in Appendix 1. This proof completes the analysis by showing the consistency of the economic framework, but it is also interesting per se because both the structure of Marxian economies and the equilibrium concept adopted are different from the standard Walrasian framework. Indeed, Appendix 1 generalises the existence results derived by Roemer (1981).

2 The Model

An economy consists of N agents. Let $\mathbb{R}_+$ be the set of nonnegative real numbers. Production technology is freely available to all agents, who can operate any activity in the production set P, which has elements of the form $\alpha = (-\alpha_l, -\underline{\alpha}, \overline{\alpha})$ where $\alpha_l \in \mathbb{R}_+$ is the *effective* labour input of the process; $\underline{\alpha} \in \mathbb{R}^n_+$ are the inputs of the produced goods used in the process; and $\overline{\alpha} \in \mathbb{R}^n_+$ are the outputs of the n goods. Thus, elements of P are vectors in $\mathbb{R}^{2n+1}$. The net output vector arising from α is denoted as $\widehat{\alpha} \equiv \overline{\alpha} - \underline{\alpha}$. P is assumed to be a closed convex cone containing the origin in $\mathbb{R}^{2n+1}$. Let $\mathbf{0} = (0, ..., 0)'$. The following assumptions on P hold throughout the paper.[4]

Assumption 1 (A1). For all $\alpha \in P$, if $\overline{\alpha} \geq \mathbf{0}$ then $\alpha_l > 0$.
Assumption 2 (A2). For all $c \in \mathbb{R}^n_+$, there is a $\alpha \in P$ such that $\widehat{\alpha} \geqq c$.
Assumption 3 (A3). For all $\alpha \in P$, and for all $(-\underline{\alpha}', \overline{\alpha}') \in \mathbb{R}^n_- \times \mathbb{R}^n_+$, if $(-\underline{\alpha}', \overline{\alpha}') \leqq (-\underline{\alpha}, \overline{\alpha})$ then $(-\alpha_l, -\underline{\alpha}', \overline{\alpha}') \in P$.

A1 implies that labour is indispensable to produce any non-negative output vector. A2 states that any non-negative commodity vector is producible as

[4]For all vectors $x, y \in \mathbb{R}^n$, $x \geqq y$ if and only if $x_i \geqq y_i$ $(i = 1, \ldots, n)$; $x \geq y$ if and only if $x \geqq y$ and $x \neq y$; $x > y$ if and only if $x_i > y_i$ $(i = 1, \ldots, n)$.

a net output. A3 is a *free disposal* condition, which states that, given any feasible production process α, any vector producing (weakly) less net output than α is also feasible using the same amount of labour as α itself.

Given P, the set of production activities feasible with $\alpha_l = k$ units of effective labour can be defined as follows:

$$P(\alpha_l = k) \equiv \{(-\alpha_l, -\underline{\alpha}, \overline{\alpha}) \in P \mid \alpha_l = k\},$$

and the set of net output vectors feasible with k units of effective labour is:

$$\widehat{P}(\alpha_l = k) \equiv \{\widehat{\alpha} \in \mathbb{R}^n \mid \text{there is } \alpha \in P(\alpha_l = k) \text{ such that } \overline{\alpha} - \underline{\alpha} \geqq \widehat{\alpha}\}.$$

For any set $X \subseteq \mathbb{R}^n$, $\partial X \equiv \{x \in X \mid \nexists x' \in X \text{ s.t. } x' > x\}$ is the frontier of X, and $SX \equiv \{x \in X \mid \nexists x' \in X \text{ s.t. } x' \geq x\}$ is the efficient frontier of X.

This paper investigates exploitation when heterogeneous agents are endowed with unequal amounts of physical and human capital. In the economy, agents produce, consume, and trade labour. On the production side, they can either sell their labour-power or hire workers to work on their capital, or they can be self-employed and work on their own assets. More precisely, for all $\nu \in N$, let $s^\nu \in \mathbb{R}_{++}$ be agent ν's skill level and let $\omega^\nu \in \mathbb{R}^n_+$ be the vector of productive assets inherited by ν. Then, $\alpha^\nu = (-\alpha_l^\nu, -\underline{\alpha}^\nu, \overline{\alpha}^\nu) \in P$ is the production process operated by ν as a self-employed producer, with her own capital, where $\alpha_l^\nu = s^\nu a_l^\nu$ and a_l^ν is the labour *time* expended by ν; $\beta^\nu = \left(-\beta_l^\nu, -\underline{\beta}^\nu, \overline{\beta}^\nu\right) \in P$ is the production process that ν operates by hiring (effective) labour β_l^ν; $\gamma^\nu = s^\nu l^\nu$ is ν's effective labour supply, where l^ν is the labour *time* supplied by ν on the market. Thus, let $\lambda^\nu = (a_l^\nu + l^\nu)$ be the total amount of labour time expended by ν, and let $\Lambda^\nu = \alpha_l^\nu + \gamma^\nu = s^\nu \lambda^\nu$ be the total amount of effective labour performed by ν, either as a self-employed producer or working for some other agent.

On the consumption side, let $C \subseteq \mathbb{R}^n_+$ be the consumption space of each agent with generic element c^ν as a consumption vector of agent ν, and assume that total labour hours expended by each agent do not exceed the common endowment L^ν, where units are normalised so that $L^\nu = 1$, for all ν. Agent ν's welfare is representable by a function $u^\nu : C \times [0,1] \to \mathbb{R}_+$, which is monotonic on $C \times [0,1]$ (increasing in consumption and decreasing in labour time). The function u can be interpreted either as a standard subjectivist neoclassical utility function or as an objectivist index of individual well-being, or status. The latter view is more in line with exploitation theory, but the two interpretations are formally equivalent.[5] For the sake of simplicity, and with no loss of generality, in what follows, u is assumed to be strictly monotonic on C in at least one argument. The conclusions of the paper do not depend on this assumption, and some extensions of the analysis and the relation with other models in the literature are discussed in section 5 below.

Let p denote the $1 \times n$ vector of commodity prices and let w denote the wage rate *per unit of effective labour*. Given the assumption of perfect contracting in the labour market, the latter is indeed the relevant wage. Given (p, w), each ν is assumed to choose a plan $(\alpha^\nu, \beta^\nu, \gamma^\nu, c^\nu)$ to maximise her welfare subject to the constraint that net income is sufficient for consumption plans; wealth is sufficient for production plans; production plans are technically feasible; and total labour hours expended do not exceed $L^\nu = 1$. Formally, each ν solves the following programme MP^ν.[6]

$$\max_{(\alpha^\nu, \beta^\nu, \gamma^\nu, c^\nu)} u^\nu\left(c^\nu, \lambda^\nu\right)$$

subject to

$$\begin{aligned} \left[p\left(\overline{\alpha}^\nu - \underline{\alpha}^\nu\right)\right] + \left[p\left(\overline{\beta}^\nu - \underline{\beta}^\nu\right) - w\beta_l^\nu\right] + \left[w\gamma^\nu\right] &= pc^\nu, \\ p\left(\underline{\alpha}^\nu + \underline{\beta}^\nu\right) &\leqq p\omega^\nu, \\ \alpha^\nu &\in P; \, \beta^\nu \in P, \, \lambda^\nu \leqq 1. \end{aligned}$$

MP^ν is a suitable way of modelling agent ν's decision problem. On the one hand, it can be interpreted as a generalisation of standard Marxian accumulation economies (see, e.g., Roemer 1981, 1982; Yoshihara, 2010), in which heterogeneous individuals are explicitly considered in their double role as producers and consumers. In Yoshihara (2010), for example, $\mathbf{s} = (1, \ldots, 1)$, $C \equiv \mathbb{R}^n_+$, and there is a continuous, quasi-concave, and strictly monotonic real-valued function $f : C \to \mathbb{R}_+$ such that $u^\nu(c, \lambda) = f(c)$, for all ν and for any $(c, \lambda) \in C \times [0, 1]$. Further, as shown below, although agents are not assumed to maximise profits, profit maximisation is a corollary of MP^ν. On the other hand, individuals are not assumed to be simply 'agents of capital' and unlike in traditional Marxian models (e.g., Roemer, 1982, ch.4),

[5] For a discussion of subjective and objective approaches, see Roemer and Veneziani (2004) and, in the context of Marxist theory, Yoshihara and Veneziani (2010).

[6] The first constraint is written as equality without loss of generality, given the assumptions on the monotonicity of u.

capitalists are not assumed to maximise accumulation per se and production does not take place "for production's own sake" (Luxemburg, 1951, p.333).

Let $\mathcal{O}^\nu(p,w)$ be the set of plans $(\alpha^\nu, \beta^\nu, \gamma^\nu, c^\nu)$ that solve MP^ν at prices (p,w). Let $\Omega = \left(\omega^1, \omega^2, ..., \omega^N\right)$, $\mathbf{u} = \left(u^1, u^2, ..., u^N\right)$, and $\mathbf{s} = \left(s^1, s^2, ..., s^N\right)$. Let $E(P, N, \mathbf{u}, \mathbf{s}, \Omega)$, or as a shorthand notation E, denote the economy with technology P, agents N, utility functions $\mathbf{u}$, labour skills $\mathbf{s}$, and productive endowments Ω. Let the set of all such economies be denoted by $\mathcal{E}$. Let $c = \sum_{\nu=1}^N c^\nu$ be aggregate consumption; and let a similar notation hold for all other variables. The equilibrium concept can now be defined.

Definition 1: A *reproducible solution* (RS) for $E(P, N, \mathbf{u}, \mathbf{s}, \Omega) \in \mathcal{E}$ is a price vector (p,w) and an associated set of actions such that:

(i) $(\alpha^\nu, \beta^\nu, \gamma^\nu, c^\nu) \in \mathcal{O}^\nu(p,w)$ for all ν (optimality);
(ii) $\widehat{\alpha} + \widehat{\beta} \geqq c$ (reproducibility);
(iii) $\underline{\alpha} + \underline{\beta} \leqq \omega$ (feasibility);
(iv) $\beta_l = \gamma$ (labour market equilibrium).

In other words, at a RS (i) every agent optimises; (iii) there are enough resources for production plans; and (iv) the labour market clears. Condition (ii) states that net outputs should at least suffice for aggregate consumption. This is equivalent to requiring that the vector of social endowments does not decrease component-wise, because (ii) is equivalent to $\omega + \left(\widehat{\alpha} + \widehat{\beta} - c\right) \geqq \omega$, which states that stocks at the beginning of next period should not be smaller than stocks at the beginning of the current period. Indeed, although the RS is defined as a temporary equilibrium in a static general equilibrium framework, it can be seen as a one-shot slice of a *stationary equilibrium* in a dynamic general equilibrium framework.[7]

Some properties of RSs should immediately be noted. First, by the assumptions on u, it immediately follows that $p \geq \mathbf{0}$ and $w \geqq 0$ at a non-trivial RS. Next, let $\pi^{\max} = \max_{\alpha \in P} \frac{p\widehat{\alpha} - w\alpha_l}{p\underline{\alpha}}$: by the assumptions on P, $\pi^{\max}$ is well-defined. Hence, let $P^\pi(p,w) = \left\{\alpha \in P \mid \pi^{\max} = \frac{p\widehat{\alpha} - w\alpha_l}{p\underline{\alpha}}\right\}$. It is proven in a straightforward way at any non-trivial RS, that the maximum profit rate is nonnegative, that only processes yielding the maximum rate of profit are activated, and that the profit rate is equalised.

[7]See Veneziani (2007) and Veneziani and Yoshihara (2009) for a thorough analysis.

Lemma 1: *Let (p, w) be a non-trivial RS for $E \in \mathcal{E}$ such that $c \geq \mathbf{0}$. Then,* $p\widehat{\alpha} - w\alpha_l \geqq 0$ for some $\alpha \in P \backslash \{\mathbf{0}\}$*, and $\alpha^\nu, \beta^\nu \in P^\pi(p, w)$ for all ν.*

3 Labour exploitation: an axiomatic approach

In Marxian theory, exploitation is conceived of as the unequal exchange of labour between agents: considering an agent $\nu \in N$, exploitative relations are characterised by systematic differences between the labour contributed by ν to the economy and the labour 'received' by ν, which is given by the amount of labour contained, or embodied, in some relevant consumption bundle(s). Therefore, for any bundle $c \in \mathbb{R}^n_+$, it is necessary to define the labour value (or labour content) of c. Unlike in standard Leontief economies, the definition of the labour content of c is not obvious, and various definitions have, in fact, been proposed. In this section, a general condition - called the axiom of Labour Exploitation for the Working Class, or **LEW** - is proposed which every definition of labour exploitation should satisfy in order to capture the core insights of the theory of exploitation as the UE of labour.

First of all, note that the set of agents N can be partitioned, for analytical purposes, into two disjoint subsets, namely the core of the working class, denoted as W, which comprises agents with no initial endowments; and the set O of agents, including some segments of the working class, who own a positive amount of at least some productive assets. Thus, $W = \{\nu \in N \mid \omega^\nu = \mathbf{0}\}$ and $O = \{\nu \in N \mid \omega^\nu \geq \mathbf{0}\}$. The economies analysed in this paper are more general than the polarised, two-class societies usually considered in the literature, and in the next section the exploitation status of all agents is derived, including those in intermediate class positions. Yet the set of proletarians W is of clear focal interest in exploitation theory: theoretically, if *any* agents are exploited, then those in W should be definitely among them, if they work at all. It is therefore opportune, from a formal viewpoint, to focus on the set W in order to provide a domain condition defining a minimum requirement that all definitions of exploitation as the UE of labour should satisfy.[8]

[8] It might be argued that the appropriate definition of proletarians relates to their financial wealth, rather than their vector of endowments. If this view is adopted, then $W' = \{\nu \in N \mid p\omega^\nu = 0\}$ and $O' = \{\nu \in N \mid p\omega^\nu > 0\}$. This distinction is relevant only if $p \not> \mathbf{0}$ and it does not make any significant difference for the results of this paper. In fact, since axiom **LEW** aims to provide a weak domain condition to define the set of exploited agents, it is theoretically appropriate to focus on the set of agents $W \subseteq W'$.

Let $B(p, w\Lambda) \equiv \left\{c \in \mathbb{R}^n_+ \mid pc = w\Lambda\right\}$ denote the set of consumption bundles that can be (just) afforded, at prices p, by an agent in W, who supplies Λ units of labour at a wage rate w. Let $\phi(c) \equiv \{\alpha \in P \mid \widehat{\alpha} \geqq c\}$ denote the set of activities that produce at least c as net output. A basic axiom can now be introduced that every formulation of labour exploitation should satisfy.

Labour Exploitation for the Working Class (LEW): *Consider any economy $E \in \mathcal{E}$. Let (p, w) be a RS for E. Given any definition of exploitation*, the set of exploited agents $N^{ted} \subseteq N$ *is identified at (p, w). Then, the set N^{ted} should have the following property: there exists a profile $(\overline{c}^1, ..., \overline{c}^W)$ such that for any $\nu \in W$, $\overline{c}^\nu \in B(p, w\Lambda^\nu) \cap \mathbb{R}^n_+$, and for some $\alpha^{\overline{c}^\nu} \in \phi(\overline{c}^\nu) \cap \partial P$ with $\widehat{\alpha}^{\overline{c}^\nu} \not> \overline{c}^\nu$:*

$$\nu \in N^{ted} \Leftrightarrow \alpha_l^{\overline{c}^\nu} < \Lambda^\nu.$$

Axiom **LEW** requires that, at any RS, the exploitation status of every propertyless worker $\nu \in W$ be characterised by identifying a nonnegative reference commodity vector $\overline{c}^\nu$. This reference vector is technically feasible and can be purchased by ν, and it identifies the amount of labour that ν receives. Thus, if $\nu \in W$ supplies Λ^ν, and Λ^ν is more than the labour socially necessary to produce $\overline{c}^\nu$, then ν is regarded as contributing more labour than ν receives. According to **LEW,** all such agents belong to N^{ted}.[9]

As a domain condition for the admissible class of exploitation-forms, **LEW** captures some key insights of the UE theory of exploitation that are shared by *all* of the main approaches in the literature.[10] In the UE theory, the exploitation status of an agent ν is determined by the difference between the amount of labour that ν 'contributes' to the economy, in some relevant sense, and the amount she 'receives', in some relevant sense. In the convex economies considered in this paper, the former quantity is given by the amount of labour supplied, Λ^ν, whereas there are many possible UE views concerning the amount of labour that each agent receives, which incorporate different normative and positive concerns. As a domain condition, **LEW**

[9]In axiom **LEW** the case $N^{ted} = N$ is not ruled out: this is theoretically appropriate, given the nature of **LEW** as a minimum domain condition, and even some of the classic definitions of exploitation - such as Morishima's (1974) - do not exclude this case.

[10]It should be stressed that **LEW** only applies to labour-based definitions of exploitation. It is not relevant, for example, for Roemer's (1982) property-relations definition of exploitation. Similar versions of **LEW** are analysed by Yoshihara and Veneziani (2009) and Yoshihara (2010), in the context of different economies.

provides some minimal, key restrictions on the definition of the amount of labour that a theoretically relevant subset of agents receives.

First, according to **LEW**, the amount of labour that $\nu \in W$ receives depends on her income, or more precisely, it is determined in equilibrium by some reference consumption vectors that ν can purchase. In the standard approach, the reference vector corresponds to the bundle actually chosen by the agent. **LEW** is weaker in that it only requires that the reference vector be *potentially* affordable.

Second, **LEW** captures another key tenet of the UE theory of exploitation by stipulating that the amount of labour associated with the reference bundle - and thus 'received' by an agent - is related to the production conditions of the economy. More precisely, **LEW** states that the reference bundle be technologically feasible as net output, and it defines its labour content as the amount of labour socially necessary to produce it. It is worth noting that **LEW** requires that the amount of labour associated with each reference bundle be uniquely determined with reference to production conditions, but it does not specify how such amount should be chosen, and there may be in principle many (efficient) ways of producing $\overline{c}^{\nu}$, and thus of determining $\alpha_l^{\overline{c}^{\nu}}$.

Third, **LEW** is weak also because it does not provide comprehensive conditions for the determination of exploitation status. It only focuses on a subset of agents, namely those who own no physical assets, and it is silent on the exploitation status of all other agents. Further, given any definition of exploitation, and any RS, the *set of exploiters* $N^{ter} \subseteq N$ is also defined, where $N^{ter} \cap N^{ted} = \varnothing$, but axiom **LEW** imposes no restrictions on the determination of N^{ter}.

Finally, it is worth noting that the vector $\overline{c}^{\nu}$ in **LEW** need not be uniquely fixed, and may be a function of (p, w). Further, once $\overline{c}^{\nu}$ is identified, the existence of $\alpha^{\overline{c}^{\nu}}$ is guaranteed by A2 and A3.

In sum, **LEW** incorporates several key features of exploitation as the UE of labour, and it sets a weak restriction on the class of admissible definitions. Indeed, that all of the main definitions in the literature, suitably extended to economies with heterogeneous labour, satisfy **LEW**. Consider first Morishima's (1974) classic definition. According to Morishima, the labour value of a commodity vector c, denoted as $l.v.(c)$, is the minimum amount of (effective) labour necessary to produce c as net output. Formally:

$$l.v.(c) \equiv \min \left\{ \alpha_l \mid \alpha = (-\alpha_l, -\underline{\alpha}, \overline{\alpha}) \in \phi(c) \right\}.$$

It is easy to see that $\phi(c)$ is non-empty by A2 and that the set

$$\{\alpha_l \mid \alpha = (-\alpha_l; -\underline{\alpha}; \overline{\alpha}) \in \phi(c)\}$$

is bounded from below by 0, by the assumption $\mathbf{0} \in P$ and by A1. Hence, $l.v.(c)$ is well defined and, by A1, it is positive whenever $c \neq \mathbf{0}$. Then, the following definition of exploitation can be provided.

Definition 2 (Morishima, 1974)**:** A worker $\nu \in W$, who supplies Λ^ν and consumes $c^\nu \in \mathbb{R}^n_+$, is *exploited, i.e.* $\nu \in N^{ted}$, if and only if $\Lambda^\nu > l.v.(c^\nu)$.

In order to show that Definition 2 satisfies **LEW**, at any RS, let $\overline{c}^\nu \equiv c^\nu \in B(p, w\Lambda^\nu)$ and:

$$\alpha^{\overline{c}^\nu} \in \arg\min \{\alpha_l \mid \alpha = (-\alpha_l, -\underline{\alpha}, \overline{\alpha}) \in \phi(\overline{c}^\nu)\}.$$

Unlike Morishima's (1974) definition, Roemer's (1982) definition of labour value depends on prices. Given a price vector (p, w), let $\phi(c; p, w) \equiv \{\alpha \in P^\pi(p, w) \mid \widehat{\alpha} \geqq c\}$ be the set of profit-rate-maximising activities that produce at least c as net output. According to Roemer (1982), the labour value of vector c, denoted as $l.v.(c; p, w)$, is the minimum amount of (effective) labour necessary to produce c as net output among profit-rate-maximising activities. Formally:

$$l.v.(c; p, w) \equiv \min \{\alpha_l \mid \alpha = (-\alpha_l, -\underline{\alpha}, \overline{\alpha}) \in \phi(c; p, w)\}.$$

Again, it is immediately verified that $l.v.(c; p, w)$ is well defined and it is positive for all $c \neq \mathbf{0}$. Then the following definition can be stated.

Definition 3 (Roemer, 1982)**:** Consider an economy $E \in \mathcal{E}$. Let (p, w) be a RS for E. A worker $\nu \in W$, who supplies Λ^ν and consumes c^ν, is *exploited, i.e.* $\nu \in N^{ted}$, if and only if $\Lambda^\nu > l.v.(c^\nu; p, w)$.

In order to show that Definition 3 satisfies **LEW**, at any RS, let $\overline{c}^\nu \equiv c^\nu \in B(p, w\Lambda^\nu)$ and

$$\alpha^{\overline{c}^\nu} \in \arg\min \{\alpha_l \mid \alpha = (-\alpha_l, -\underline{\alpha}, \overline{\alpha}) \in \phi(\overline{c}^\nu; p, w)\}.$$

In addition to the above two classic definitions, in this paper, a new definition is analysed, which has been recently proposed by Yoshihara and Veneziani (2009, 2010) and Yoshihara (2010). For any $p \in \mathbb{R}^n_+$ and $c \in \mathbb{R}^n_+$, let $\mathcal{B}(p,c) \equiv \left\{x \in \mathbb{R}^n_+ \mid px = pc\right\}$: $\mathcal{B}(p,c)$ is the set of bundles that cost exactly as much as c at prices p.

Definition 4: Consider an economy $E \in \mathcal{E}$. Let (p,w) be a RS for E such that $\widehat{\alpha}^{p,w}$ is aggregate net output and $\alpha_l^{p,w}$ is aggregate (effective) labour expended. Let $\tau^c \in [0,1]$ be such that $\tau^c \widehat{\alpha}^{p,w} \in \mathcal{B}(p,c)$. The *labour embodied in* c at the social reproduction point $\alpha^{p,w}$ is $\tau^c \alpha_l^{p,w}$.

As in Roemer's (1982) approach, in Definition 4 the labour content of a commodity can be identified only if the price vector is known. Yet social relations play a more central role than in Roemer's theory, because the definition of labour content requires a prior knowledge of the social reproduction point and labour content is explicitly linked to the redistribution of total social labour (total labour employed), which corresponds to the total labour value of national income. Then, the following definition identifies the set of propertyless workers who are exploited.

Definition 5: Consider an economy $E \in \mathcal{E}$. Let (p,w) be a RS for E such that $\widehat{\alpha}^{p,w}$ is the social reproduction point. For any $\nu \in W$, who supplies Λ^ν and consumes c^ν, let τ^{c^ν} be defined as in Definition 4. Then, $\nu \in W$ is *exploited, i.e.* $\nu \in N^{ted}$, if and only if $\Lambda^\nu > \tau^{c^\nu} \alpha_l^{p,w}$.

Definition 5 is conceptually related to the 'New Interpretation' developed by Duménil (1980) and Foley (1982). In fact, for any $\nu \in W$, τ^{c^ν} represents ν's share of national income, and so $\tau^{c^\nu} \alpha_l^{p,w}$ represents the share of social labour which ν receives by earning income barely sufficient to buy pc^ν. Then, as in the New Interpretation, the notion of exploitation is related to the production and distribution of national income and social labour.

In order to show that Definition 5 satisfies **LEW**, given any (p,w) such that $\alpha^{p,w}$ is the social production point, let $\tau^{c^\nu} = \frac{pc^\nu}{p\widehat{\alpha}^{p,w}}$, $\overline{c}^\nu \equiv \tau^{c^\nu} \cdot \widehat{\alpha}^{p,w} \in B(p, w\Lambda^\nu)$ and $\alpha^{\overline{c}^\nu} \equiv \tau^{c^\nu} \alpha^{p,w}$.

The previous arguments provide strong support to the idea that **LEW** does represent an appropriate domain condition in exploitation theory. **LEW** is formally weak and it incorporates some arguably compelling and widely shared views on exploitation as the UE of labour. Thus, although it can be proved that the axiom is not trivial and not all definitions in the literature satisfy it,[11] all of the major approaches do.[12] The next question, then, is how to discriminate among the various definitions satisfying **LEW**.

A key tenet of the standard Marxist approach concerns the relation between profits and exploitation. This has been incorporated into FMT (see, e.g., Morishima, 1974; Roemer, 1981), according to which exploitation is synonymous with positive profits. The relevance of FMT, especially in standard approaches, is such that although it is proved as a result, its epistemological status is that of a postulate: the appropriate definition of exploitation is considered to be one which preserves FMT. Therefore it is theoretically appropriate to formalise FMT explicitly in axiomatic terms.

FMT: *Given an economy $E \in \mathcal{E}$ and a RS for E, (p, w), with aggregate production activity $\alpha^{p,w}$, it holds that:*

$$\left[p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} > 0 \Leftrightarrow N^{ted} \supseteq W_+\right]$$

where $W_+ \equiv \{\nu \in W : \Lambda^\nu > 0\}$.

A number of points are worth noting about **FMT**. First, the axiom is formulated without specifying any definition of labour exploitation: *whatever* the definition adopted, propertyless agents should be exploited if and only if profits are positive in equilibrium. Second, **FMT** is more general than in standard two-class models. This is because it both applies to advanced capitalist economies with a complex class structure, and allows for the possibility that propertyless workers in W_+ are a *strict* subset of the set of exploited agents, that is $W_+ \subset N^{ted}$. Note that the axiom focuses only on propertyless workers who perform at least some labour: this is a theoretically appropriate restriction, since the exploitation status of agents who do not engage in any economic activities is unclear. Finally, unlike in standard models, **FMT** is general also in the sense that it allows for very general assumptions on agents and technology, including heterogeneous preferences and skills, a convex cone technology, and so on. Thus, axiom **FMT** encompasses the standard formulations as special cases.

[11] For example, it can be proved that the subjectivist notion of labour exploitation based on workers' preferences recently proposed by Matsuo (2008) does not satisfy **LEW**. For a thorough discussion, see Yoshihara and Veneziani (2010).

[12] It is worth noting that based on Flaschel's (1983) definition of *additive labor values*, it is possible to derive another formulation of labor exploitation that satisfies **LEW**. Similarly, Definition 6 in Yoshihara (2010) also satisfies **LEW**.

Let $B_{++}(p, w\Lambda) \equiv \left\{c \in \mathbb{R}_+^n \mid pc > w\Lambda\right\}$: $B_{++}(p, w\Lambda)$ is the set of consumption bundles that an agent in W supplying Λ units of effective labour cannot afford. Let $\Gamma(p, w; k) \equiv \left\{\widehat{\alpha} \in \partial\widehat{P}(\alpha_l = k) \cap \mathbb{R}_+^n \mid \widehat{\alpha} \in B_{++}(p, wk)\right\}$: $\Gamma(p, w; k)$ is the set of net outputs that can be produced efficiently using k units of (effective) labour, which cannot be afforded by propertyless agents supplying k units of effective labour. The next theorem characterises the class of definitions of exploitation that satisfy **LEW** and such that **FMT** holds. Recall that if **LEW** holds, then for any $\nu \in W$, there is a $\overline{c}^\nu \in B(p, w\Lambda^\nu)$ and $\alpha^{\overline{c}^\nu} \in \phi(\overline{c}^\nu) \cap \partial P$ with $\widehat{\alpha}^{\overline{c}^\nu} \not> \overline{c}^\nu$ such that $\nu \in N^{ted} \Leftrightarrow \alpha_l^{\overline{c}^\nu} < \Lambda^\nu$.

Theorem 1: *For any definition of labour exploitation satisfying* **LEW**, *the following two statements are equivalent for any $E \in \mathcal{E}$ and for any RS (p, w) with associated aggregate production activity $\alpha^{p,w}$:*
(1) **FMT** *holds under this definition*;
(2) *for each $\nu \in W_+$, $\left[\text{there exists } \underline{c}^\nu \in \Gamma(p, w; \Lambda^\nu) \cup \left\{\frac{\Lambda^\nu}{\alpha_l^{p,w}}\widehat{\alpha}^{p,w}\right\} \text{ such that } \underline{c}^\nu > \widehat{\alpha}^{\overline{c}^\nu}\right]$ $\Leftrightarrow \pi^{\max} > 0$.*

Proof: (2)⇒(1): Consider any $E \in \mathcal{E}$ and any RS (p, w) with $\alpha^{p,w}$. Suppose that, for each $\nu \in W_+$, $\left[\text{there exists } \underline{c}^\nu \in \Gamma(p, w; \Lambda^\nu) \cup \left\{\frac{\Lambda^\nu}{\alpha_l^{p,w}}\widehat{\alpha}^{p,w}\right\} \text{ such that } \underline{c}^\nu > \widehat{\alpha}^{\overline{c}^\nu}\right] \Leftrightarrow$ $\pi^{\max} > 0$.

Let $\pi^{\max} > 0$, so that by Lemma 1, $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} > 0$. Note that, for any $\nu \in W_+$, if $\underline{c}^\nu = \frac{\Lambda^\nu}{\alpha_l^{p,w}}\widehat{\alpha}^{p,w}$ then $\underline{c}^\nu \in \partial\widehat{P}(\alpha_l = \Lambda^\nu)$. Then, for all $\nu \in W_+$, since $\underline{c}^\nu \in \partial\widehat{P}(\alpha_l = \Lambda^\nu)$ and $\underline{c}^\nu > \widehat{\alpha}^{\overline{c}^\nu}$, and noting that $\alpha^{\overline{c}^\nu} \in \phi(\overline{c}^\nu) \cap \partial P$, we have $\alpha_l^{\overline{c}^\nu} < \Lambda^\nu$. Thus, by **LEW**, $\nu \in N^{ted}$ holds for any $\nu \in W_+$.

Let $\pi^{\max} = 0$, so that by Lemma 1, $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} = 0$. First, note that by **A2**, $\pi^{\max} = 0$ implies $w > 0$. Next, for each $\nu \in W_+$, if $\pi^{\max} = 0$, then $\partial\widehat{P}(\alpha_l = \Lambda^\nu) \cap \mathbb{R}_+^n \subseteq B_-(p, w\Lambda^\nu) \equiv \left\{c \in \mathbb{R}_+^n \mid pc \leqq w\Lambda^\nu\right\}$, which implies that $\Gamma(p, w; \Lambda^\nu) = \varnothing$. Thus, (2) implies that for each $\nu \in W_+$, for $\underline{c}^\nu = \frac{\Lambda^\nu}{\alpha_l^{p,w}}\widehat{\alpha}^{p,w}$, $\underline{c}^\nu > \widehat{\alpha}^{\overline{c}^\nu}$ does not hold. Then, $\underline{c}^\nu = \frac{\Lambda^\nu}{\alpha_l^{p,w}}\widehat{\alpha}^{p,w}$ and $\overline{c}^\nu \in B(p, w\Lambda^\nu)$ imply that for any $\alpha^{\overline{c}^\nu} \in \phi(\overline{c}^\nu)$ with $\widehat{\alpha}^{\overline{c}^\nu} \not> \overline{c}^\nu$, $\alpha_l^{\overline{c}^\nu} \geqq \Lambda^\nu$. Thus, by **LEW**, $\nu \notin N^{ted}$ holds for any $\nu \in W_+$.

In sum, (2) implies that **FMT** holds under any definition of exploitation satisfying **LEW**.

(1)⇒(2): Consider any $E \in \mathcal{E}$ and any RS (p, w) with $\alpha^{p,w}$. Suppose that $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} > 0 \Leftrightarrow N^{ted} \supseteq W_+$. First of all, note that if either $W = \varnothing$ or $\Lambda^\nu = 0$ for all $\nu \in W$, then condition (2) is vacuously satisfied. Therefore suppose that $\Lambda^\nu > 0$ for at least some $\nu \in W$, and $W \neq \varnothing$.

Let $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} > 0$, so that $\pi^{\max} > 0$. By **LEW** and **FMT**, for each $\nu \in W_+$, there exist $\overline{c}^{\nu} \in \mathbb{R}^n_+$ and $\alpha^{\overline{c}^{\nu}} \in \phi(\overline{c}^{\nu}) \cap \partial P$ with $\alpha^{\overline{c}^{\nu}} \not> \overline{c}^{\nu}$ such that $p\overline{c}^{\nu} = w\Lambda^{\nu}$ and $\alpha_l^{\overline{c}^{\nu}} < \Lambda^{\nu}$. Then, for all $\nu \in W_+$, let $\delta^{\nu} > 1$ be such that $\delta^{\nu}\alpha_l^{\overline{c}^{\nu}} = \Lambda^{\nu}$. For each $\nu \in W_+$, let us first consider the case that $\widehat{\alpha}^{\overline{c}^{\nu}} > \mathbf{0}$. Then, let $\underline{c}^{\nu} \equiv \delta^{\nu}\widehat{\alpha}^{\overline{c}^{\nu}}$. Clearly $\underline{c}^{\nu} \in \Gamma(p, w; \Lambda^{\nu})$ and $\underline{c}^{\nu} > \widehat{\alpha}^{\overline{c}^{\nu}}$. Secondly, let us consider the case that $\widehat{\alpha}^{\overline{c}^{\nu}} \not> \mathbf{0}$. Note that, because of A2, $\partial\widehat{P}(\alpha_l = \Lambda^{\nu}) \cap \mathbb{R}^n_{++} \neq \varnothing$ holds. Since $\delta^{\nu}\widehat{\alpha}^{\overline{c}^{\nu}} \in \partial\widehat{P}(\alpha_l = \Lambda^{\nu}) \cap \mathbb{R}^n_+$, the convexity of $\widehat{P}(\alpha_l = \Lambda^{\nu})$ guarantees that any convex combination of $\delta^{\nu}\widehat{\alpha}^{\overline{c}^{\nu}}$ and any points in $\partial\widehat{P}(\alpha_l = \Lambda^{\nu}) \cap \mathbb{R}^n_{++}$ is feasible, and any point derived from this convex combination belongs to $\widehat{P}(\alpha_l = \Lambda^{\nu}) \cap \mathbb{R}^n_{++}$, even if such a point is very close to $\delta^{\nu}\widehat{\alpha}^{\overline{c}^{\nu}}$. Thus, for any open neighbourhood $\mathcal{V}$ of $\delta^{\nu}\widehat{\alpha}^{\overline{c}^{\nu}}$, $\mathcal{V} \cap \widehat{P}(\alpha_l = \Lambda^{\nu}) \cap \mathbb{R}^n_{++} \neq \varnothing$. Thus, for some sufficiently small neighbourhood $\mathcal{V}^*$ of $\delta^{\nu}\widehat{\alpha}^{\overline{c}^{\nu}}$, there is $x^{\nu} \in \mathcal{V}^* \cap \widehat{P}(\alpha_l = \Lambda^{\nu}) \cap \mathbb{R}^n_{++}$ which is sufficiently close to $\delta^{\nu}\widehat{\alpha}^{\overline{c}^{\nu}}$ and $x^{\nu} > \widehat{\alpha}^{\overline{c}^{\nu}}$ holds. Then, there is $\epsilon \geqq 1$ such that $\epsilon x^{\nu} \in \partial\widehat{P}(\alpha_l = \Lambda^{\nu}) \cap \mathbb{R}^n_{++}$. Take $\underline{c}^{\nu} \equiv \epsilon x^{\nu}$. Suppose $w > 0$. Then $\delta^{\nu}\widehat{\alpha}^{\overline{c}^{\nu}} \in B_{++}(p, w\Lambda^{\nu})$, and $\underline{c}^{\nu} \in B_{++}(p, w\Lambda^{\nu})$ follows from the fact that $\underline{c}^{\nu}$ is sufficiently close to $\delta^{\nu}\widehat{\alpha}^{\overline{c}^{\nu}}$ and $B_{++}(p, w\Lambda^{\nu})$ is open. Thus, $\underline{c}^{\nu} \in \Gamma(p, w; \Lambda^{\nu})$ and $\underline{c}^{\nu} > \widehat{\alpha}^{\overline{c}^{\nu}}$. Suppose $w = 0$. If $p\widehat{\alpha}^{\overline{c}^{\nu}} > 0$, the result follows in a similar manner. If $p\widehat{\alpha}^{\overline{c}^{\nu}} = 0$, the result follows from the fact that $\underline{c}^{\nu} \in \mathbb{R}^n_{++}$, noting that $\pi^{\max} > 0$ implies $p \geq \mathbf{0}$.

Let $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} = 0$, so that by Lemma 1, $\pi^{\max} = 0$. By **LEW** and **FMT**, for some $\nu \in W_+$, there exist $\overline{c}^{\nu} \in \mathbb{R}^n_+$ and $\alpha^{\overline{c}^{\nu}} \in \phi(\overline{c}^{\nu}) \cap \partial P$ with $\alpha^{\overline{c}^{\nu}} \not> \overline{c}^{\nu}$ such that $p\overline{c}^{\nu} = w\Lambda^{\nu}$ and $\alpha_l^{\overline{c}^{\nu}} \geqq \Lambda^{\nu}$. Actually, the latter property must hold for all $\nu \in W_+$. For suppose, to the contrary, that for some $\nu \in W_+$, there exist $\overline{c}^{\nu} \in \mathbb{R}^n_+$ and $\alpha^{\overline{c}^{\nu}} \in \phi(\overline{c}^{\nu}) \cap \partial P$ with $\alpha^{\overline{c}^{\nu}} \not> \overline{c}^{\nu}$ such that $p\overline{c}^{\nu} = w\Lambda^{\nu}$ and $\alpha_l^{\overline{c}^{\nu}} < \Lambda^{\nu}$. Then this implies $p\widehat{\alpha}^{\overline{c}^{\nu}} \geqq p\overline{c}^{\nu} = w\Lambda^{\nu} > w\alpha_l^{\overline{c}^{\nu}}$, which violates the assumption that $\pi^{\max} = 0$. Thus, for any $\nu \in W_+$, there exist $\overline{c}^{\nu} \in \mathbb{R}^n_+$ and $\alpha^{\overline{c}^{\nu}} \in \phi(\overline{c}^{\nu}) \cap \partial P$ with $\alpha^{\overline{c}^{\nu}} \not> \overline{c}^{\nu}$ such that $p\overline{c}^{\nu} = w\Lambda^{\nu}$ and $\alpha_l^{\overline{c}^{\nu}} \geqq \Lambda^{\nu}$. Then, for each $\nu \in W_+$, let $\underline{c}^{\nu} = \frac{\Lambda^{\nu}}{\alpha_l^{p,w}}\widehat{\alpha}^{p,w}$, since $\Gamma(p, w; \Lambda^{\nu})$ is empty when $\pi^{\max} = 0$. Since $p\underline{c}^{\nu} = w\Lambda^{\nu} = p\overline{c}^{\nu}$, it follows that $\underline{c}_i^{\nu} \not> \widehat{\alpha}_i^{\overline{c}^{\nu}}$ for at least some i with $\widehat{\alpha}_i^{\overline{c}^{\nu}} > 0$. Note that by **A2**, $\pi^{\max} = 0$ implies that $w > 0$.

In sum, if **FMT** holds, then (2) holds under any definition of exploitation satisfying **LEW**.■

Theorem 1 can be interpreted as follows. **FMT** states that propertyless workers are exploited if and only if equilibrium profits are positive. According to **LEW**, the exploitation status of propertyless workers is determined by identifying a set of reference bundles (call them the *exploitation-reference bundles*). By Theorem 1, in every convex economy, **FMT** holds if and only if the existence of positive profits in equilibrium is also determined by identifying a set of reference bundles (call them the *profit-reference bundles*). According to **LEW**, the exploitation-reference bundles must be affordable by the workers and must be producible with less than Λ^ν units of labour for all exploited workers. According to condition (2), instead, for all workers $\nu \in W_+$, the profit-reference bundles must be producible with a technically efficient process using Λ^ν units of labour, and must be such that they are not affordable by ν and dominate the exploitation-reference vector if and only if the maximum profit rate is positive. The relevance of Theorem 1, then, is not only in the identification of a general condition for the validity of the relation between exploitation and profits. Methodologically, Theorem 1 suggests that different views about exploitation, and the analysis of the key features of exploitation theory, should focus on the identification of the relevant reference bundles.

Theorem 1 does not identify a unique definition of exploitation that meets axiom **FMT**, but rather a class of definitions satisfying condition (2). Yet Theorem 1 has surprising implications concerning the main received approaches in exploitation theory. For there are economies in which no point in $\Gamma(p, w; \Lambda^\nu) \cup \left\{ \frac{\Lambda^\nu}{\alpha_l^{p,w}} \widehat{\alpha}^{p,w} \right\}$ satisfies condition (2), if $\widehat{\alpha}^{\overline{c}^\nu}$ is given either by Definition 2 or by Definition 3. In contrast, it can be proved that Definition 5 satisfies condition (2), and thus **FMT** holds in general convex economies with heterogeneous agents.[13]

Corollary 1: *There exists an economy $E \in \mathcal{E}$ and a* RS (p, w) with associated aggregate production point $\alpha^{p,w}$ *such that neither* Definition 2 *nor* Definition 3 *satisfy* **FMT**. *Instead,* Definition 5 *satisfies* **FMT** *for all $E \in \mathcal{E}$ and all* RS (p, w).

Proof: For the proof that neither Definition 2 nor Definition 3 satisfy **FMT**, see Appendix 2. We need to prove that Definition 5 satisfies condition (2) of Theorem 1. We consider two cases for any $E \in \mathcal{E}$ and any RS (p, w).

Case 1: $\widehat{\alpha}^{p,w} > \mathbf{0}$ or $\widehat{\alpha}^{p,w} = \mathbf{0}$. By setting $\underline{c}^\nu = \frac{\Lambda^\nu}{\alpha_l^{p,w}} \widehat{\alpha}^{p,w}$ for all $\nu \in W_+$, it is immediately seen that Definition 5 satisfies condition (2).

[13] An example similar to those analysed in Appendix 2 below is used in Yoshihara (2010; Corollary 2) to prove that the *Class-Exploitation Correspondence Principle* (Roemer, 1982) does not hold under Definition 3.

Case 2: $\widehat{\alpha}^{p,w} \geq \mathbf{0}$ and $\widehat{\alpha}^{p,w} \not> \mathbf{0}$. Firstly, let this RS (p,w) be associated to $\pi^{\max} = 0$. Then, only $\underline{c}^{\nu} = \frac{\Lambda^{\nu}}{\alpha_l^{p,w}} \widehat{\alpha}^{p,w}$ is available for all $\nu \in W_{+}$, since $\Gamma(p,w;\Lambda^{\nu}) = \varnothing$ as shown in the proof of Theorem 1. Then, it is immediately seen that Definition 5 does not meet $\underline{c}^{\nu} > \widehat{\alpha}^{\overline{c}^{\nu}}$. Secondly, let this RS (p,w) be associated to $\pi^{\max} > 0$. Then, for each $\nu \in W_{+}$, $\frac{\Lambda^{\nu}}{\alpha_l^{p,w}} \widehat{\alpha}^{p,w} \in \partial \widehat{P}(\alpha_l = \Lambda^{\nu}) \cap \partial \mathbb{R}^n_{+}$. Thus, using the same argument as in Theorem 1, it can be shown that there is $\epsilon x^{\nu} \in \partial \widehat{P}(\alpha_l = \Lambda^{\nu}) \cap \mathbb{R}^n_{++}$ for each $\nu \in W_{+}$, which is sufficiently close to $\frac{\Lambda^{\nu}}{\alpha_l^{p,w}} \widehat{\alpha}^{p,w}$. Let $\underline{c}^{\nu} \equiv \epsilon x^{\nu}$ for each $\nu \in W_{+}$. Since $\frac{\Lambda^{\nu}}{\alpha_l^{p,w}} \widehat{\alpha}^{p,w} \in B_{++}(p, w\Lambda^{\nu})$ by $p\frac{\Lambda^{\nu}}{\alpha_l^{p,w}} \widehat{\alpha}^{p,w} - w\Lambda^{\nu} = \frac{\Lambda^{\nu}}{\alpha_l^{p,w}} (p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w}) > 0$, $\underline{c}^{\nu} \in B_{++}(p, w\Lambda^{\nu})$ follows from the fact that $\underline{c}^{\nu}$ is sufficiently close to $\frac{\Lambda^{\nu}}{\alpha_l^{p,w}} \widehat{\alpha}^{p,w}$ and $B_{++}(p, w\Lambda^{\nu})$ is open. Thus, $\underline{c}^{\nu} \in \Gamma(p,w;\Lambda^{\nu})$. Noting $\tau^{c^{\nu}} = \frac{pc^{\nu}}{p\widehat{\alpha}^{p,w}} = \frac{w\Lambda^{\nu}}{w\alpha_l^{p,w}} \frac{w\alpha_l^{p,w}}{p\widehat{\alpha}^{p,w}} = \frac{\Lambda^{\nu}}{\alpha_l^{p,w}} \frac{w\alpha_l^{p,w}}{p\widehat{\alpha}^{p,w}}$ and $\frac{w\alpha_l^{p,w}}{p\widehat{\alpha}^{p,w}} < 1$ by $\pi^{\max} > 0$, it follows that $\tau^{c^{\nu}} < \frac{\Lambda^{\nu}}{\alpha_l^{p,w}}$. Thus, $\tau^{c^{\nu}} \widehat{\alpha}^{p,w} < \frac{\Lambda^{\nu}}{\alpha_l^{p,w}} \widehat{\alpha}^{p,w}$ holds, so that $\tau^{c^{\nu}} \widehat{\alpha}^{p,w} < \underline{c}^{\nu}$ since $\underline{c}^{\nu}$ is sufficiently close to $\frac{\Lambda^{\nu}}{\alpha_l^{p,w}} \widehat{\alpha}^{p,w}$. Finally, since $\widehat{\alpha}^{\overline{c}^{\nu}} = \tau^{c^{\nu}} \widehat{\alpha}^{p,w}$ under Definition 5, $\widehat{\alpha}^{\overline{c}^{\nu}} < \underline{c}^{\nu}$ holds for each $\nu \in W_{+}$.

In summary, condition (2) of Theorem 1 holds for any RS (p,w). ■

4 Exploitation and Profits: Two extensions

Given the theoretical relevance of FMT in the Marxian theory of exploitation, Theorem 1 and Corollary 1 provide strong support for Definition 5 as the appropriate notion of UE exploitation. In this section, two extensions of the analysis are presented, which provide further support to the 'New Interpretation'. The generality of the model is exploited to show that Definition 5 can be extended to analyse, first, the exploitation status of all agents and then the relation between exploitation and profits outside of equilibrium allocations, in economies with heterogeneous preferences and unequal endowments of physical and human capital. This suggests that, if the 'New Interpretation' is adopted, then exploitation theory can be extended to yield interesting insights on advanced capitalist economies. As a first step, Definition 5 is generalised to identify the exploitation status of all agents.

Definition 6: Consider any economy $E \in \mathcal{E}$. Let (p,w) be a RS for E with aggregate production activity $\alpha^{p,w}$. For any $\nu \in N$, who supplies Λ^{ν} and consumes $c^{\nu} \in \mathbb{R}^n_{+}$, let $\tau^{c^{\nu}}$ be defined as in Definition 4. Agent ν is: exploited if and only if $\Lambda^{\nu} > \tau^{c^{\nu}} \alpha_l^{p,w}$; an *exploiter* if and only if $\Lambda^{\nu} < \tau^{c^{\nu}} \alpha_l^{p,w}$;

and *neither exploited nor an exploiter* if and only if $\Lambda^{\nu} = \tau^{c^{\nu}} \alpha_l^{p,w}$.

Theorem 2 proves that, based on Definition 6, it is possible to characterise the exploitation status of all agents - and not only of the poorest segments of the working class - *and* to derive a more general relation between profits and exploitation, extending FMT beyond the set of propertyless agents. Recall that N^{ted} is the set of exploited agents and N^{ter} is the set of exploiters.

Theorem 2: *Consider an economy $E \in \mathcal{E}$. Let (p, w) be a RS for E with associated aggregate production activity $\alpha^{p,w}$. Under Definition 6*:
(1) *if* $\pi^{\max} > 0$, agent ν is: exploited *if and only if* $\frac{p\omega^{\nu}}{p\omega} < \frac{\Lambda^{\nu}}{\alpha_l^{p,w}}$; neither exploited nor an exploiter *if and only if* $\frac{p\omega^{\nu}}{p\omega} = \frac{\Lambda^{\nu}}{\alpha_l^{p,w}}$; and an exploiter *if and only if* $\frac{p\omega^{\nu}}{p\omega} > \frac{\Lambda^{\nu}}{\alpha_l^{p,w}}$.
(2) *if* $\pi^{\max} > 0$, *then* $\left\{\nu \in N \mid \frac{p\omega}{\alpha_l^{p,w}} < \frac{p\omega^{\nu}}{s^{\nu}}\right\} \subseteq N^{ter}$*. Furthermore, if there is a subsistence bundle* $b \in \mathbb{R}_+^n$ *such that* $c^{\nu} \geqq b$*, for all* $\nu \in N$*, then* $\left\{\nu \in N \mid \frac{p\omega^{\nu}}{p\omega} < \frac{pb}{p\widehat{\alpha}^{p,w}}\right\} \subseteq N^{ted}$.
(3) *if* $\pi^{\max} = 0$, $N^{ted} = N^{ter} = \varnothing$.

Proof: Part 1. Let (p, w) be a RS for $E \in \mathcal{E}$. Then by Definition 1-(i), it follows that $p\widehat{\alpha}^{\nu} + \left[p\widehat{\beta}^{\nu} - w\beta_l^{\nu}\right] + w\gamma^{\nu} = pc^{\nu}$ for all $\nu \in N$. Since $p\left(\underline{\alpha}^{\nu} + \underline{\beta}^{\nu}\right) = p\omega^{\nu}$, for all $\nu \in N$, and noting that only processes yielding the maximum rate of profit are going to be activated, the latter expression can be written as $\pi^{\max}p\omega^{\nu} + w\Lambda^{\nu} = pc^{\nu}$. Then, by Definition 1-(ii) and Definition 1-(iv), it follows that $\pi^{\max}p\omega + w\alpha_l^{p,w} = p\widehat{\alpha}^{p,w}$. Therefore $\Lambda^{\nu} = \tau^{c^{\nu}} \alpha_l^{p,w}$ if and only if $\Lambda^{\nu} = \frac{\pi^{\max}p\omega^{\nu} + w\Lambda^{\nu}}{\pi^{\max}p\omega + w\alpha_l^{p,w}} \alpha_l^{p,w}$, which yields the desired result. The other two inequalities are proved similarly.

Part 2. Let (p, w) be a RS for $E \in \mathcal{E}$. The first part of the statement follows immediately from part 1, noting that $\lambda^{\nu} \leqq 1$. In order to prove the second part of the statement, note that by Definition 1-(i), it follows that $p\widehat{\alpha}^{\nu} + \left[p\widehat{\beta}^{\nu} - w\beta_l^{\nu}\right] + w\gamma^{\nu} = pc^{\nu}$ for all $\nu \in N$. Since $p\left(\underline{\alpha}^{\nu} + \underline{\beta}^{\nu}\right) = p\omega^{\nu}$, for all $\nu \in N$, and noting that only processes yielding the maximum rate of profit are going to be activated, the latter expression can be written as $\pi^{\max}p\omega^{\nu} + w\Lambda^{\nu} = pc^{\nu}$. Therefore it follows that $\Lambda^{\nu} > \tau^{c^{\nu}} \alpha_l^{p,w}$ if and only if $\left[\frac{pc^{\nu} - \pi^{\max}p\omega^{\nu}}{w}\right] > \frac{pc^{\nu}}{p\widehat{\alpha}^{p,w}} \alpha_l^{p,w}$, which is in turn equivalent to $pc^{\nu}\left[1 - \frac{w\alpha_l^{p,w}}{p\widehat{\alpha}^{p,w}}\right] > \pi^{\max}p\omega^{\nu}$. Then, setting $c^{\nu} \geqq b$, for all $\nu \in N$, gives the desired result.

Part 3. If $\pi^{\max} = 0$, then it follows that $w\Lambda^{\nu} = pc^{\nu}$, for all $\nu \in N$, and $\frac{w\alpha_l^{p,w}}{p\widehat{\alpha}^{p,w}} = 1$, which yields the desired result. ■

Theorem 2-(1) completely characterises the exploitation structure of an economy in equilibrium: an agent is exploited (respectively, an exploiter) if and only if her share of social wealth is lower (respectively, higher) than her share of social labour. Theorem 2-(2) shows that at the two extremes of the wealth distribution, exploitation status can be determined independently of individual choices, an intuition of standard Marxist theory that is proved to be robust. Indeed, if a subsistence bundle exists, the set of agents that are exploited regardless of their individual choices will be larger than the set of propertyless agents (those who have 'nothing to lose but their chains'). This set can be sizable if b is not interpreted as a *physical* subsistence bundle, but rather as reflecting moral and social elements. Jointly with Theorem 2-(3), this result generalises FMT to a larger set of agents than the propertyless segment of the working class.

Theorem 2 completes the analysis of the relation between exploitation and profits in equilibrium and it extends the main insights of UE exploitation theory to all agents in the general economies considered in this paper, under Definition 6. This is crucial given the focal theoretical interest in equilibrium allocations, but one may argue that a robust theory of exploitation should provide insights also on out-of-equilibrium allocations. In the rest of this section, an extension of Definition 5 is proposed, and a general relation between exploitation and profits is derived, at *any feasible allocation*.

The key point to note is that there are various possible ways of conceptualising exploitation at general disequilibrium allocations and, consequently, there is no trivial way of extending Definition 5. For example, outside of a RS, it is unclear whether exploitation status should be determined relative to the *actual* features of the allocation. On the one hand, if individual plans are not realised, coordination failures arise, and perhaps even sheer mistakes are made, then by focusing on *actual* data one may be capturing only purely transient and ephemeral phenomena that do not tell much about the structural features of the economy. On the other hand, one may insist that, even outside of an RS, only the information contained in the *actual* allocation point is relevant to analyse exploitation. For, ultimately, the actual features of the allocation are what matters to the agents.

In the extension of Definition 5 to disequilibrium allocations proposed here, the actual features of the allocation, including the actual price vector, the aggregate production activity, and the individual work and consumption choices of all agents remain central in the definition of the labour content of a bundle of commodities and the exploitation status of propertyless agents. However, the effects of sheer individual mistakes in technical choices, or of purely temporary market imbalances leading to productive inefficiency are discounted. To be precise, given a price vector (p, w) and an associated aggregate production activity $\alpha^{p,w} \in P$, define

$$\phi\left(c; \alpha^{p,w}\right) \equiv \left\{\alpha' \in P \mid \exists t, \mu \in \mathbb{R}_+ : \left(\alpha'_l, \widehat{\alpha}'\right) = \left(t\alpha_l^{p,w}, t\mu\widehat{\alpha}^{p,w}\right), \mu\widehat{\alpha}^{p,w} \in \widehat{P}\left(\alpha_l = \alpha_l^{p,w}\right) \ \&\ \widehat{\alpha}' \geqq c\right\}$$

$\phi\left(c; \alpha^{p,w}\right)$ denotes the set of production activities which are along the ray defined by $\left(\alpha_l^{p,w}, \widehat{\alpha}^{p,w}\right)$ and produce at least c as net output. Then:

$$l.v.\left(c; \alpha^{p,w}\right) \equiv \min\left\{\alpha_l \mid \alpha = \left(-\alpha_l, -\underline{\alpha}, \overline{\alpha}\right) \in \phi\left(c; \alpha^{p,w}\right)\right\}.$$

Clearly, $l.v.\left(c; \alpha^{p,w}\right)$ is well-defined and bounded below by 0. The labour content of a bundle c at any given allocation can be defined as follows.

Definition 7: Consider an economy $E \in \mathcal{E}$. Let (p, w) be a price vector for E with associated aggregate production activity $\alpha^{p,w}$. Let $\tau^c \in [0, 1]$ be such that $\tau^c\widehat{\alpha}^{p,w} \in \mathcal{B}(p, c)$. The *labour embodied in* c at the social reproduction point $\alpha^{p,w}$ is $l.v.\left(\tau^c\widehat{\alpha}^{p,w}; \alpha^{p,w}\right)$.

The following definition identifies the set of propertyless workers who are exploited at any given allocation.

Definition 8: Consider an economy $E \in \mathcal{E}$. Let (p, w) be a price vector for E such that $\alpha^{p,w}$ is the associated production point. For any $\nu \in W$, who supplies Λ^ν and consumes c^ν, let τ^{c^ν} be defined as in Definition 7. Then, $\nu \in W$ is *exploited, i.e.* $\nu \in N^{ted}$, if and only if $\Lambda^\nu > l.v.\left(\tau^{c^\nu}\widehat{\alpha}^{p,w}; \alpha^{p,w}\right)$.

From a formal viewpoint, Definitions 7 and 8 generalise Definitions 4 and 5 and they reduce to the latter at a RS. In fact, if (p, w) is a RS for E, then $\alpha^{p,w} \in \partial P$ and $l.v.\left(\tau^c\widehat{\alpha}^{p,w}; \alpha^{p,w}\right) = \tau^c\alpha_l^{p,w}$ holds. From a theoretical viewpoint, in Definitions 7 and 8, the actual allocation of the economy plays a pivotal role. In order to define labour content and the exploitation status of propertyless agents, the *actual* price vector and the *actual* individual choices on work and consumption are central. The only possible deviation from actual data concerns the focus on technically efficient production activities in the definition of labour content, but the set of admissible efficient activities used in Definitions 7 and 8 is significantly constrained by the *actual* social production point $\alpha^{p,w}$ (unlike in Roemer's or Morishima's definitions).

The focus on efficient aggregate production vectors is theoretically reasonable. For technically inefficient activities in the interior of the production possibilities set are the product of transient contingencies and do not reveal much about the structural features of the economy.[14] Moreover, note that, given the nature of **LEW** as a domain condition, in section 3 a weak formulation of the axiom is adopted by restricting its application to RS's. It is straightforward, however, to extend **LEW** to all price vectors (p, w) with associated social production point $\alpha^{p,w}$ and, from a theoretical viewpoint, none of the arguments used to defend **LEW** in section 3 depends on the assumption that the allocation is an equilibrium. Therefore one may argue that **LEW** remains an appropriate domain condition to define UE exploitation even at disequilibrium allocations. From this perspective, it is worth noting that Definition 8 satisfies **LEW**, at any (p, w) with associated social production point $\alpha^{p,w}$. To see this, let $\tau^{c^\nu} = \frac{pc^\nu}{p\widehat{\alpha}^{p,w}}$, $\overline{c}^\nu \equiv \tau^{c^\nu} \cdot \widehat{\alpha}^{p,w} \in B\left(p, w\Lambda^\nu\right)$ and $\alpha^{\overline{c}^\nu} \equiv \arg\min\left\{\alpha_l \mid \alpha = (-\alpha_l, -\underline{\alpha}, \overline{\alpha}) \in \phi\left(\overline{c}^\nu; \alpha^{p,w}\right)\right\}$.

Let $C^W = \sum_{\nu \in W} c^\nu$ and $\Lambda^W = \sum_{\nu \in W} \Lambda^\nu$. Based on Definition 8, Theorem 3 establishes a general relation between exploitation and profits for any general convex cone economies and at any feasible allocations.

Theorem 3: *For any economy $E \in \mathcal{E}$, any $(p, w) \in \mathbb{R}_+^{n+1}$ and any allocation $\left(\alpha^\nu,\ \beta^\nu,\ \gamma^\nu, c^\nu\right)_{\nu \in N}$ with $pc^\nu = p\left(\widehat{\alpha}^\nu + \widehat{\beta}^\nu\right) - w\left(\beta_l^\nu - \gamma^\nu\right)$ $(\forall \nu \in N)$, the following statements are equivalent for any $\alpha^* \in \partial P\left(\alpha_l = \Lambda^W\right)$ with $\widehat{\alpha}^* \in \partial \widehat{P}\left(\alpha_l = \Lambda^W\right) \cap \mathbb{R}_+^n$:*
(1) $p\widehat{\alpha}^* - w\alpha_l^* > 0$ holds;
(2) for any $\nu \in W_+$, $\Lambda^\nu > l.v.\left(\tau^{c^\nu}\widehat{\alpha}^*; \alpha^*\right)$, where $l.v.\left(\tau^{c^\nu}\widehat{\alpha}^*; \alpha^*\right) = \tau^{c^\nu}\alpha_l^*$ for $\tau^{c^\nu} \in [0, 1)$ with $\tau^{c^\nu}\widehat{\alpha}^* \in \mathcal{B}\left(p, c^\nu\right)$.

Proof: Taking a point $\widehat{\alpha}^*$ from $\partial \widehat{P}\left(\alpha_l = \Lambda^W\right) \cap \mathbb{R}_+^n$. Let $\alpha^* \in \partial P\left(\alpha_l = \Lambda^W\right)$ be a production point corresponding to $\widehat{\alpha}^*$.

[14] Indeed, Marx's own notion of Socially Necessary Labour Time may be interpreted as ruling out inefficient technologies and involving a counterfactual analysis. See Sen (1978) for a discussion.

Suppose (1) holds. Then, $p\widehat{\alpha}^{*} - w\Lambda^{W} = p\left(\widehat{\alpha}^{*} - C^{W}\right) > 0$, since the budget constraint holds for all agents. Note that, for any $\nu \in W_{+}$, $pc^{\nu} = w\Lambda^{\nu} = w\Lambda^{W}\frac{\Lambda^{\nu}}{\Lambda^{W}} = pC^{W}\frac{\Lambda^{\nu}}{\Lambda^{W}}$, thus $\frac{pc^{\nu}}{pC^{W}} = \frac{\Lambda^{\nu}}{\Lambda^{W}}$. Then, let $\tau^{c^{\nu}} = \frac{pc^{\nu}}{p\widehat{\alpha}^{*}}$ for any $\nu \in W_{+}$. Clearly $\tau^{c^{\nu}} \in [0,1)$ with $\tau^{c^{\nu}}\widehat{\alpha}^{*} \in \mathcal{B}(p, c^{\nu})$. Moreover, for any $\nu \in W_{+}$, $\tau^{c^{\nu}}\alpha_{l}^{*} = \frac{pc^{\nu}}{p\widehat{\alpha}^{*}}\Lambda^{W} = \frac{pc^{\nu}}{pC^{W}}\frac{pC^{W}}{p\widehat{\alpha}^{*}}\Lambda^{W} = \frac{\Lambda^{\nu}}{\Lambda^{W}}\frac{pC^{W}}{p\widehat{\alpha}^{*}}\Lambda^{W} = \Lambda^{\nu}\frac{pC^{W}}{p\widehat{\alpha}^{*}} < \Lambda^{\nu}$, where the latter inequality follows from $p\left(\widehat{\alpha}^{*} - C^{W}\right) > 0$. Finally, since $\alpha^{*} \in \partial P\left(\alpha_{l} = \Lambda^{W}\right)$, $l.v.\left(\tau^{c^{\nu}}\widehat{\alpha}^{*}; \alpha^{*}\right) = \tau^{c^{\nu}}\alpha_{l}^{*}$ holds. Thus, (2) is obtained.

Suppose (2) holds. Then, for any $\nu \in W_{+}$, $\Lambda^{\nu} > l.v.\left(\tau^{c^{\nu}}\widehat{\alpha}^{*}; \alpha^{*}\right)$, where $l.v.\left(\tau^{c^{\nu}}\widehat{\alpha}^{*}; \alpha^{*}\right) = \tau^{c^{\nu}}\alpha_{l}^{*}$ holds for $\tau^{c^{\nu}} \in [0,1)$ with $\tau^{c^{\nu}}\widehat{\alpha}^{*} \in \mathcal{B}(p, c^{\nu})$. Thus, $\Lambda^{W} > \sum_{\nu \in W_{+}}\tau^{c^{\nu}}\alpha_{l}^{*}$ holds. Since $\tau^{c^{\nu}} = \frac{pc^{\nu}}{p\widehat{\alpha}^{*}}$ for any $\nu \in W_{+}$, the last inequality implies that $\Lambda^{W} > \frac{pC^{W}}{p\widehat{\alpha}^{*}}\Lambda^{W}$, thus $p\left(\widehat{\alpha}^{*} - C^{W}\right) > 0$ holds. Since $pC^{W} = w\Lambda^{W} = w\alpha_{l}^{*}$ by the budget constraint, $p\widehat{\alpha}^{*} - w\alpha_{l}^{*} > 0$ holds. ■

Theorem 3 states that a general relation between exploitation and profits holds, at any price vector and corresponding allocation, provided that productive inefficiencies and temporary disequilibrium phenomena are ruled out: at every technically efficient production vector α^{*} (which is feasible using actual, effective labour $\Lambda^{W} = \sum_{\nu \in W}\Lambda^{\nu}$) society realises positive profits if and only if every propertyless worker is exploited. In order to appreciate the full generality of Theorem 3, it is important to stress that no significant restriction is imposed on individual behaviour (except that the budget constraint holds for all agents) and on the actual allocation. As a result, Theorem 3 does not establish necessary and sufficient conditions for the existence of positive profits and the exploitation of propertyless workers at the *actual* allocation, and the social production point $\alpha^{p,w}$ may, or may not, coincide with one of the vectors α^{*}. For given the extremely weak restrictions on the set of admissible allocations, the link between profits and exploitation may be somewhat weakened. For instance, if $\frac{\Lambda^{W}}{\alpha_{l}+\beta_{l}}\left(\widehat{\alpha} + \widehat{\beta}\right) \in \widehat{P}\left(\alpha_{l} = \Lambda^{W}\right) \backslash \partial\widehat{P}\left(\alpha_{l} = \Lambda^{W}\right)$ and $C^{W} \in \widehat{P}\left(\alpha_{l} = \Lambda^{W}\right) \backslash \partial\widehat{P}\left(\alpha_{l} = \Lambda^{W}\right)$ hold at the actual allocation, then the corresponding profit rate may be non-positive while propertyless agents are exploited. However, Theorem 3 derives the general conditions under which the economy can generate positive profits and propertyless workers are exploited, starting from the actual individual consumption/leisure choices, price system, and aggregate production actitivity. In other words, if one abstracts from temporary disequilibrium phenomena, Theorem 3 does derive a fully general relation between the appropriation of surplus by capitalists and the exploitation of (propertyless) workers, which holds even if exchanges do not take place at equilibrium prices.

5 Conclusions

This paper provides a novel axiomatic analysis of the notion of exploitation as the unequal exchange of labour focusing on the relation between exploitation and profits. General convex economies with agents endowed with heterogeneous preferences and with different amounts of physical and human capital are considered. First, an axiomatic characterisation of the class of definitions that preserve the Fundamental Marxian Theorem (FMT) in equilibrium is derived (Theorem 1). Based on this characterisation, it is shown that under none of the main received definitions is the exploitation of the (propertyless segment of the) working class synonymous with positive profits in general. Instead, a definition related to the 'New Interpretation' is presented which preserves the link between the appropriation of surplus and the exploitation of (at least some) workers. This definition also allows one to generalise some key insights of exploitation theory in complex convex economies with heterogeneous agents: it is possible to characterise the exploitation status of all agents in equilibrium (Theorem 2) and to derive a general relation between exploitation and profits even outside of equilibrium allocations (Theorem 3).

Given the relevance of FMT in exploitation theory, the results presented in this paper provide strong support to the 'New Interpretation' as the appropriate notion of exploitation in advanced capitalist economies. Thus, they complement and strengthen the analysis developed by Yoshihara and Veneziani (2009) in the context of convex *subsistence* economies. In fact, as mentioned in section 2 above, the main results of the paper could be derived by assuming the function u^ν to be weakly monotone on $C \times [0,1]$ and strictly monotone in at least one argument, provided some additional technical conditions to ensure local nonsatiation are added.[15] This assumption encompasses the special case where there is a subsistence bundle $b \in \mathbb{R}^n_+$ such that $C \equiv \left\{c \in \mathbb{R}^n_+ \mid c \geqq b\right\}$, and $u^\nu(c,\lambda) = 1 - \lambda$, for all ν and for any $(c,\lambda) \in C \times [0,1]$. If $\mathbf{u}$ is given by a profile of functions of the latter type and $\mathbf{s} = (1,\ldots,1)$, then $E(P,N,\mathbf{u},\mathbf{s},\Omega)$ is a *subsistence economy* of

[15] For example, if agents minimise labour over $[0,1]$, subject to a subsistence constraint, then something like Roemer's (1982) 'Non Benevolent Capitalists' assumption should be made. For a thorough discussion, see Yoshihara and Veneziani (2009).

the type analysed by Roemer (1982) and Yoshihara and Veneziani (2009). But then, it is possible to conclude that the 'New Interpretation' provides the unique appropriate definition of exploitation because, as shown above, it preserves FMT in general, and, as shown by Yoshihara and Veneziani (2009), it is fully characterised by a small set of weak and intuitive axioms in the set of subsistence economies which is a subset of the general class of economies considered in this paper. Moreover, the set of axioms is satisfied by the 'New Interpretation' definition of exploitation even in the general class of convex economies discussed in this paper.

The results presented above, however, raise some interesting questions. First of all, Theorem 2-(2) confirms the standard Marxist analysis of exploitation at the two ends of the wealth distribution: propertyless agents are exploited and the very wealthy are exploiters. Yet, outside of the two extremes, the exploitation status of an agent is in general determined not only by her endowment of physical capital, but also by her choice of consumption and leisure, as well as her endowment of human capital - namely, her skills. This raises some interesting issues for exploitation theory, in particular from a normative viewpoint: except for the agents at the two extremes of the distribution of productive assets, it may well be the case that agents with nonnegligible amounts of physical assets, who do not work much appear as exploited because they have a large endowment of human capital, which increases their overall labour contribution to the economy.

Second, this paper focuses on exploitation, and on the key relation between profits and exploitation. Another interesting issue concerns the relation between class and exploitation: Roemer (1982), for example, maintains that the correspondence between class and exploitation status is a core insight of Marxian exploitation theory. Definition 6 above might provide interesting results on this issue, too. For example, Yoshihara and Veneziani (2009) and Yoshihara (2010) prove that, unlike in the standard approaches, if the New Interpretation is adopted, it is possible to derive the full class and exploitation structure, and a robust correspondence between class and exploitation status in convex economies with agents endowed with identical preferences and skills. To extend the latter results to general economies with heterogeneous agents is an interesting direction for further research.

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7 Appendix 1: The existence of a RS

This appendix proves the existence of an equilibrium for a theoretically relevant subset of the set of economies $\mathcal{E}$ set out in section 2 above. It focuses on the polar case where $C = \mathbb{R}^n_+$ and it generalises the proofs of existence in Roemer (1981, 1982). Yoshihara and Veneziani (2009) prove the existence of a RS for another polar case of convex economies in which $C = \left\{c \in \mathbb{R}^n_+ \mid c \geqq b\right\}$ for some subsistence vector $b \in \mathbb{R}^n_+ \setminus \{\mathbf{0}\}$, u^ν is not strictly increasing on C, and agents minimise labour.

It is assumed that u^ν is continuous, quasi-concave, and strictly increasing on C for all $\nu \in N$: these assumptions are standard in microeconomics and need no further comment. Further, the following *boundary condition of utility functions* is assumed, which is also standard in microeconomics: $u^\nu(c, \lambda) > u^\nu(\mathbf{0}, \lambda')$ for any $c \in \mathbb{R}^n_+ \setminus \{\mathbf{0}\}$, and any $\lambda, \lambda' \in [0, 1]$. This assumption implies that any propertyless agent $\nu \in W$ would rather participate in labour markets to earn some revenue and purchase *some* consumption goods, than drop out of labour markets consuming nothing. Thus, if some propertyless agents are unemployed, they are involuntarily unemployed.

Finally, A1 is slightly strengthened to require that *some* produced inputs be used in the production of commodities:

Assumption 1′ (A1′)**.** For all $\alpha \in P$ such that $\alpha_l \geq 0$ and $\underline{\alpha} \geqq \mathbf{0}$, if $\overline{\alpha} \geq \mathbf{0}$ then $\alpha_l > 0$ and $\underline{\alpha} \geq \mathbf{0}$.

A1′ is an essential property of a capitalist economy in the sense that if it is not satisfied, anyone can in principle hire anyone else, including propertyless agents. Given the twin role of agents as consumers and producers, A1′ also allows us to prove the boundedness of the aggregate demand correspondences.

Let a profile $(c^\nu, \gamma^\nu, \beta^\nu)_{\nu \in N}$ be a *feasible allocation* for $E \in \mathcal{E}$ if and only if $(c^\nu, \gamma^\nu, \beta^\nu)_{\nu \in N}$ satisfies Definition 1-(ii), 1-(iii), and 1-(iv), and $(c^\nu, \gamma^\nu, \beta^\nu) \in C \times [0, s^\nu] \times P$ holds for all $\nu \in N$. If the social endowment of capital ω of an economy $E \in \mathcal{E}$ only allows for feasible allocations with $\sum_{\nu \in N} c^\nu = \mathbf{0}$, then if a RS exists for this economy, it can only be a trivial RS. However, by A2, it is always possible to have a non-trivial feasible allocation with $\sum_{\nu \in N} c^\nu \neq \mathbf{0}$ if ω is placed appropriately. Thus, in order to guarantee the existence of non-trivial feasible allocations, the following assumption is made:

Assumption 4. The economy $E(P, N, \mathbf{u}, \mathbf{s}, \Omega)$ has the following property:

$$\omega \in \left\{ \underline{\alpha} \in \mathbb{R}^n_+ \mid \exists \alpha \in P \text{ s.t. } \alpha_l \leqq \sum_{\nu \in N} s^\nu \text{ and } \widehat{\alpha} \geq \mathbf{0} \right\}.$$

By A4, there exists $\alpha' \in P$ with $\alpha'_l \leqq \sum_{\nu \in N} s^\nu$ and $\underline{\alpha}' = \omega$ such that for any $p > \mathbf{0}$, $p(\overline{\alpha}' - \omega) > 0$. Thus, for a sufficiently small $w' > 0$, $p(\overline{\alpha}' - \omega) - w\alpha'_l \geqq 0$ holds for any $w \leqq w'$. This implies that for any $p > \mathbf{0}$, there is $w' > 0$ such that for any $w \leqq w'$, $\max_{\alpha \in P:\ p\underline{\alpha} = p\omega} p\widehat{\alpha} - w\alpha_l$ is non-negative.

For any vector (p, w), let $\Pi^\nu(p, w) \equiv p\widehat{\alpha}^\nu + \left[p\widehat{\beta}^\nu - w\beta^\nu_l\right] + w\gamma^\nu$ denote agent ν's net revenue. Note that, for any (p, w), the set of optimal solutions $\mathcal{O}^\nu(p, w)$ always contains vectors of the form $(\mathbf{0}, \beta^\nu, \gamma^\nu, c^\nu)$ such that $\Pi^\nu(p, w) = \pi^{\max} p\underline{\beta}^\nu + w\gamma^\nu = pc^\nu$ with $p\underline{\beta}^\nu = p\omega^\nu$ for all ν. Let $\triangle \equiv \left\{(p, w) \in \mathbb{R}^{n+1}_+ \mid \sum_{i=1}^n p_i + w = 1\right\}$ and $\triangle_+ \equiv \left\{(p, w) \in \triangle \mid p > \mathbf{0}\right\}$.

In order to analyse the existence of a RS, for all $(p, w) \in \triangle_+$, and for all $\nu \in N$, define the feasibility correspondence

$$B^\nu(p, w) \equiv \left\{(c^\nu, \beta^\nu, \gamma^\nu) \in C \times P \times [0, s^\nu] \mid pc^\nu \leqq \Pi^\nu(p, w);\ p\underline{\beta}^\nu \leqq p\omega^\nu\right\}.$$

The next result establishes some basic properties of $B^\nu(p, w)$.

Lemma A1.1: *For each $\nu \in N$, the correspondence B^ν is non-empty, closed-valued and convex-valued, and continuous on $\triangle_+$. Moreover, every (c^ν, γ^ν) in $B^\nu(p, w)$ is bounded for each $(p, w) \in \triangle_+$.*

Proof: It is obvious that B^ν is non-empty, closed-valued, and convex-valued. Since $pc^\nu \leqq \Pi^\nu(p, w) \leqq \max\{\pi^{\max} p\omega^\nu, 0\} + ws^\nu$, the boundedness of (c^ν, γ^ν) in $B^\nu(p, w)$ follows from A1′, for all $(p, w) \in \triangle_+$.

Finally, we prove the continuity of B^ν. First, we show that B^ν is lower hemi-continuous. Let $\left\{\left(p^k, w^k\right)\right\} \subseteq \Delta_+$ be a sequence such that $\left(p^k, w^k\right) \to (p, w)$ and $(c^\nu, \beta^\nu, \gamma^\nu) \in B^\nu(p, w)$. For each $\left(p^k, w^k\right)$, let $\beta^{k\nu} \equiv \mu^{k\nu}\beta^\nu$ where $\mu^{k\nu} = 0$ if $\left(p^k\widehat{\beta}^\nu - w^k\beta_l^\nu\right) < 0$ and if $\left(p^k\widehat{\beta}^\nu - w^k\beta_l^\nu\right) \geqq 0$ then

$$\mu^{k\nu} \equiv \begin{cases} \frac{p^k\omega^\nu}{p\omega^\nu} & \text{if } p^k\underline{\beta}^\nu \leqq p^k\omega^\nu, \\ \frac{p^k\omega^\nu}{p^k\underline{\beta}^\nu} & \text{otherwise.} \end{cases}$$

Then, let $\gamma^{k\nu} = \gamma^\nu$. Moreover, if $c^\nu \neq \mathbf{0}$, then let $\sigma^{k\nu} \equiv \min\left\{\frac{\mu^{k\nu}\left(p^k\widehat{\beta}^\nu - w^k\beta_l^\nu\right) + w^k\gamma^{k\nu}}{p^kc^\nu}, 1\right\}$ and $c^{k\nu} \equiv \sigma^{k\nu}c^\nu$, whereas if $c^\nu = \mathbf{0}$, then let $c^{k\nu} \equiv c^\nu$. Then, $\left(c^{k\nu}, \beta^{k\nu}, \gamma^{k\nu}\right) \in B^\nu(p^k, w^k)$ with $\left(c^{k\nu}, \beta^{k\nu}, \gamma^{k\nu}\right) \to (c^\nu, \beta^\nu, \gamma^\nu)$ as $\left(p^k, w^k\right) \to (p, w)$. Thus, B^ν is lower hemi-continuous.

To prove that B^ν is upper hemi-continuous, suppose that $\left\{\left(p^k, w^k\right)\right\} \subseteq \Delta_+$ is a sequence such that $\left(p^k, w^k\right) \to (p, w)$ and $\left(c^{k\nu}, \beta^{k\nu}, \gamma^{k\nu}\right) \in B^\nu(p^k, w^k)$ with $\left(c^{k\nu}, \beta^{k\nu}, \gamma^{k\nu}\right) \to (c^\nu, \beta^\nu, \gamma^\nu)$ as $\left(p^k, w^k\right) \to (p, w)$, and $(c^\nu, \beta^\nu, \gamma^\nu) \notin B^\nu(p, w)$. Then, either $(c^\nu, \beta^\nu, \gamma^\nu) \notin C \times P \times [0, s^\nu]$, or $pc^\nu > \Pi^\nu(p, w)$, or $p\underline{\beta}^\nu > p\omega^\nu$. Since $C \times P \times [0, s^\nu]$ is closed, $\left(c^{k\nu}, \beta^{k\nu}, \gamma^{k\nu}\right) \to (c^\nu, \beta^\nu, \gamma^\nu)$ implies that $(c^\nu, \beta^\nu, \gamma^\nu) \in C \times P \times [0, s^\nu]$. Thus, either $pc^\nu > \Pi^\nu(p, w)$ or $p\underline{\beta}^\nu > p\omega^\nu$. Suppose $p\underline{\beta}^\nu > p\omega^\nu$. Then, for some $\left(p^k, w^k\right)$ close enough to (p, w), its corresponding $\left(c^{k\nu}, \beta^{k\nu}, \gamma^{k\nu}\right)$ is also sufficiently close to $(c^\nu, \beta^\nu, \gamma^\nu)$, which implies $p^k\underline{\beta}^{k\nu} > p^k\omega^\nu$, which yields a contradiction. This implies that $(c^\nu, \beta^\nu, \gamma^\nu) \in B^\nu(p, w)$. A similar argument holds if $pc^\nu > \Pi^\nu(p, w)$ and therefore B^ν is upper hemi-continuous. ■

Lemma A1.2 establishes some properties of optimal choice correspondences.

Lemma A1.2: *For each ν, the correspondence $\mathcal{O}^\nu$ is non-empty, closed-valued, convex-valued, and upper hemi-continuous on Δ_+. Moreover, every (c^ν, γ^ν) in $\mathcal{O}^\nu(p, w)$ is bounded for each $(p, w) \in \Delta_+$.*

Proof: Non-emptiness, closed-valuedness, and convexity can be proved in the standard manner. Since every (c^ν, γ^ν) in $B^\nu(p, w)$ is bounded by Lemma A1.1, every (c^ν, γ^ν) in $\mathcal{O}^\nu(p, w)$ is bounded for any $(p, w) \in \Delta_+$.

We only need to show that $\mathcal{O}^\nu$ is upper hemi-continuous. Let $\left\{\left(p^k, w^k\right)\right\} \subseteq \Delta_+$ be a sequence such that $\left(p^k, w^k\right) \to (p, w)$ and $\left(c^{k\nu}, \beta^{k\nu}, \gamma^{k\nu}\right) \in \mathcal{O}^\nu(p^k, w^k)$ with $\left(c^{k\nu}, \beta^{k\nu}, \gamma^{k\nu}\right) \to (c^\nu, \beta^\nu, \gamma^\nu)$ as $\left(p^k, w^k\right) \to (p, w)$. Suppose $(c^\nu, \beta^\nu, \gamma^\nu) \notin$

$\mathcal{O}^{\nu}(p,w)$. This implies that (c^{ν},γ^{ν}) is not a maximizer of u^{ν} over $B^{\nu}(p,w)$ and $(c^{\nu},\beta^{\nu},\gamma^{\nu}) \in B^{\nu}(p,w)$ by the upper hemi-continuity of B^{ν}. Then, there exists $(c'^{\nu},\beta'^{\nu},\gamma'^{\nu}) \in B^{\nu}(p,w)$ such that $u^{\nu}\left(c'^{\nu},\frac{\gamma'^{\nu}}{s^{\nu}}\right) > u^{\nu}\left(c^{\nu},\frac{\gamma^{\nu}}{s^{\nu}}\right)$. Since B^{ν} is lower hemi-continuous, there exists a sequence $\left\{\left(c'^{k\nu},\beta'^{k\nu},\gamma'^{k\nu}\right)\right\}$ such that for each $\left(p^{k},w^{k}\right) \in\Delta_{+}$, $\left(c'^{k\nu},\beta'^{k\nu},\gamma'^{k\nu}\right) \in B^{\nu}(p^{k},w^{k})$ with $\left(c'^{k\nu},\beta'^{k\nu},\gamma'^{k\nu}\right) \to (c'^{\nu},\beta'^{\nu},\gamma'^{\nu})$ as $\left(p^{k},w^{k}\right) \to (p,w)$. Then, for $\left(p^{k},w^{k}\right)$ which is sufficiently close to (p,w), $u^{\nu}\left(c'^{k\nu},\frac{\gamma'^{k\nu}}{s^{\nu}}\right) > u^{\nu}\left(c^{k\nu},\frac{\gamma^{k\nu}}{s^{\nu}}\right)$ holds. However, since $\left(c^{k\nu},\beta^{k\nu},\gamma^{k\nu}\right) \in \mathcal{O}^{\nu}(p^{k},w^{k})$, this is a contradiction. Thus, $(c^{\nu},\beta^{\nu},\gamma^{\nu}) \in \mathcal{O}^{\nu}(p,w)$, and so $\mathcal{O}^{\nu}$ is upper hemi-continuous. ■

Note that for any $\nu \in N$, if $(p,w) \in\Delta_{+}$ is associated with $p\widehat{\alpha}-w\alpha_{l} < 0$ for all $\alpha \in P\setminus\{\mathbf{0}\}$, then $(c^{\nu},\beta^{\nu},\gamma^{\nu}) \in \mathcal{O}^{\nu}(p,w)$ implies $\beta^{\nu} = \mathbf{0}$. However, by A4, for any $p > \mathbf{0}$, there is $w' > 0$ such that for any $w \leqq w'$, $\max_{\alpha\in P:\, p\underline{\alpha}=p\omega} p\widehat{\alpha} - w\alpha_{l}$ is non-negative, so that there exists $(c^{\nu},\beta^{\nu},\gamma^{\nu})_{\nu\in N} \in \times_{\nu\in N}\mathcal{O}^{\nu}(p,w)$ with $\sum_{\nu\in N}\beta^{\nu} \neq \mathbf{0}$.

For each $(p,w) \in\Delta_{+}$, let $P(p,w;\omega) \equiv \left\{\alpha \in \arg\max_{\alpha'\in P:\, p\underline{\alpha}\leqq p\omega} p\widehat{\alpha}' - w\alpha'_{l}\right\}$. Suppose that for any $\alpha \in P(p,w;\omega)$, $\alpha_{l} > \sum_{\nu\in N}s^{\nu}$. Then, there exists $\alpha^{*} \in P(p,w;\omega)$ such that $\alpha^{*} \in \arg\min\{\alpha_{l} \mid \alpha \in P(p,w;\omega)\}$. Denote the set of such α^{*} by $\overline{P}(p,w;\omega)$. Suppose that there exists $\alpha \in P(p,w;\omega)$ such that $\alpha_{l} \leqq \sum_{\nu\in N}s^{\nu}$. Then, define $\overline{\overline{P}}(p,w;\omega) \equiv \left\{\alpha \in P(p,w;\omega) \mid \alpha_{l} \leqq \sum_{\nu\in N}s^{\nu}\right\}$. Note that, for each $(p,w) \in\Delta_{+}$, if $\overline{P}(p,w;\omega) \neq \varnothing$, then $\overline{\overline{P}}(p,w;\omega) = \varnothing$. Finally, let $P^{*}(p,w;\omega) \equiv \overline{P}(p,w;\omega)\cup\overline{\overline{P}}(p,w;\omega)$. By this definition, $P^{*}(p,w;\omega)$ is non-empty, closed, and compact.

For each $(p,w) \in\Delta_{+}$, define the aggregate excess demand correspondence:

$$
\begin{aligned}
Z(p,w) \;\equiv\; & \left\{\left(\sum_{\nu\in N}c^{\nu} - \sum_{\nu\in N}\widehat{\beta}^{\nu}, \sum_{\nu\in N}\beta_{l}^{\nu} - \sum_{\nu\in N}\gamma^{\nu}\right) \mid \sum_{\nu\in N}\beta^{\nu} \in P^{*}(p,w;\omega)\right.\\
& \left.\ \&\ (c^{\nu},\beta^{\nu},\gamma^{\nu}) \in \mathcal{O}^{\nu}(p,w)\ \ (\forall\nu \in N)\right\}.
\end{aligned}
$$

Given the above Lemmas and the definition of $P^{*}(p,w;\omega)$, it follows that Z is compact-valued, convex-valued, and upper hemi-continuous on Δ_{+}. To see that it is non-empty, firstly suppose that $(p,w) \in\Delta_{+}$ is such that $p\widehat{\alpha}-w\alpha_{l} < 0$ for all $\alpha \in P\setminus\{\mathbf{0}\}$. Then, $P(p,w;\omega) = \{\mathbf{0}\} = \overline{\overline{P}}(p,w;\omega) = P^{*}(p,w;\omega)$. If $p\widehat{\alpha}-w\alpha_{l} \geqq 0$ for some $\alpha \in P\setminus\{\mathbf{0}\}$, $P(p,w;\omega) \supseteq \{\alpha \in \arg\max_{\alpha'\in P:\, p\underline{\alpha}'=p\omega} p\widehat{\alpha}' - w\alpha'_{l}\}$ holds by A1′, so that $P^{*}(p,w;\omega) \cap (P\setminus\{\mathbf{0}\}) \neq \varnothing$. In the former case, $P^{*}(p,w;\omega) = \{\mathbf{0}\}$ holds, so that there exists $(\beta^{\nu})_{\nu\in N}$ such that $\beta^{\nu} = \mathbf{0}$ for all ν. In the latter case, if $\alpha \in P^*(p, w; \omega) \setminus \{\mathbf{0}\}$, then there exists $(\beta^\nu)_{\nu \in N}$ such that $\sum_{\nu \in N} \beta^\nu = \alpha$, and $p\underline{\beta}^\nu = p\omega^\nu$ for all ν. Then, in either case, for (c^ν, γ^ν) in $\mathcal{O}^\nu(p, w)$, it follows that $(c^\nu, \beta^\nu, \gamma^\nu) \in \mathcal{O}^\nu(p, w)$ for each $\nu \in N$. By definition, since $\sum_{\nu \in N} \beta^\nu \in P^*(p, w; \omega)$, $Z(p, w)$ is non-empty. Then:

Lemma A1.3: *There exists a price vector* $(\overline{p}, \overline{w}) \in \Delta_+$ *such that* $\mathbf{0} \in Z(\overline{p}, \overline{w})$.

Proof: 1. First, we prove that Z satisfies the Strong Walras Law (SWL), namely for each $(p, w) \in \Delta_+$, and each $(z_1, z_2) \in Z(p, w)$, $pz_1 + wz_2 = 0$. In fact, for each $(p, w) \in \Delta_+$, and each $(z_1, z_2) \in Z(p, w)$,

$$
\begin{aligned}
pz_1 + wz_2 &= p\left(\sum_{\nu \in N} c^\nu - \sum_{\nu \in N} \widehat{\beta}^\nu\right) + w\left(\sum_{\nu \in N} \beta_l^\nu - \sum_{\nu \in N} \gamma^\nu\right) \\
&= \sum_{\nu \in N} \left[pc^\nu - \left\{\left(p\widehat{\beta}^\nu - w\beta_l^\nu\right) + w\gamma^\nu\right\}\right] = 0,
\end{aligned}
$$

since $pc^\nu = \left(p\widehat{\beta}^\nu - w\beta_l^\nu\right) + w\gamma^\nu$ for every ν, by the strict motonicity of u^ν.

2. Next, we prove that Z satisfies the following Boundary condition: there is a $(\widetilde{p}, \widetilde{w}) \in \Delta_+$ such that for every sequence $\left\{\left(p^k, w^k\right)\right\} \subseteq \Delta_+$ with $\left(p^k, w^k\right) \to (p, w) \in \Delta \setminus \Delta_+$, there is an M such that for every $k \geqq M$, $(\widetilde{p}, \widetilde{w}) \cdot \left(z_1^k, z_2^k\right) > 0$ holds for every $\left(z_1^k, z_2^k\right) \in Z\left(p^k, w^k\right)$. Take a sufficiently small but positive real number ε, and define $(\widetilde{p}, \widetilde{w}) \in \Delta_+$ as $\widetilde{w} = \varepsilon > 0$, and for all j, $\widetilde{p}_j = \frac{1-\varepsilon}{n} > 0$. Then, consider any price vector $(p, w) \in \Delta \setminus \Delta_+$, such that $p_i = 0$ for one i. Firstly, note that because $\left\{\left(p^k, w^k\right)\right\} \subseteq \Delta_+$, it is possible that $w^k = 0$ for sufficiently large k. Thus, in this case, $c^{k\nu} = \mathbf{0}$ for any $\nu \in W$. However, in this case, the corresponding $\pi^{\max k}$ is strictly positive by A4, and so $\Pi^\nu\left(p^k, w^k\right) > 0$ for any $\nu \in O$. Hence, by the strict monotonicity of utility functions, $c^{k\nu} \geq \mathbf{0}$ for any $\nu \in O$, and in particular, $c_i^{k\nu}$ is sufficiently large at p^k. Secondly, $\left\{\left(p^k, w^k\right)\right\} \subseteq \Delta_+$ may also contain the case that $w^k > 0$ but $\pi^{\max k}$ is zero for sufficiently large k. In this case, because of the boundary condition for utility functions, any $\nu \in W$ optimally supplies a positive amount of labour, so that $\Pi^\nu\left(p^k, w^k\right) > 0$. Thus, by the strict monotonicity of utility functions, $c^{k\nu} \geq \mathbf{0}$ for any $\nu \in W$, and in particular, $c_i^{k\nu}$ is sufficiently large at p^k. In sum, noting that $\beta^k \in P^*(p, w; \omega)$ is bounded above, it follows that $z_{1i}^k > 0$ is sufficiently large for p^k sufficiently close to p. Then, even if $\widetilde{w} > 0$, $\widetilde{w}z_2^k$ will never compensate for $\widetilde{p}z_1^k > 0$, since z_2^k is bounded below by $-\sum_{\nu \in N} s^\nu$ whereas $\widetilde{p}z_1^k$ grows infinitely large due to

a sufficiently large $z_{1i}^k > 0$. Thus, there is a neighbourhood $\mathcal{N}((p,w),\delta)$ of (p,w) such that $(\widetilde{p},\widetilde{w})\cdot\left(z_1^k,z_2^k\right) > 0$ for all $\left(p^k,w^k\right)\in\mathcal{N}((p,w),\delta)\cap\Delta_+$. A similar argument holds if $(p,w)\in\Delta\setminus\Delta_+$, with $p_i=0$, for more than one i.

3. Set $K_m \equiv co\left\{(q,w)\in\Delta_+ | \ dist\left((q,w),\Delta\setminus\Delta_+\right) \geqq \frac{1}{m}\right\}$. Then, $\{K_m\}$ is an increasing family of compact convex sets and $\Delta_+ = \cup_m K_m$. Then, as in Border (1985, Theorem 18.13, p. 85), it follows that there exists $(\overline{p},\overline{w})\in\Delta_+$ and $\overline{z}\in Z(\overline{p},\overline{w})$ such that $\overline{z}\leqq\mathbf{0}$. This fact together with (SWL) imply that $\overline{z}=\mathbf{0}$. In fact, since $\overline{p}>\mathbf{0}$, (SWL) and $\overline{z}\leqq\mathbf{0}$ imply that $\overline{z}_1=\mathbf{0}$. Second, if $\overline{w}>0$, then $\overline{z}_2=0$ holds by (SWL) and $\overline{z}\leqq\mathbf{0}$. Thus, suppose $\overline{w}=0$ and $\overline{z}_2\equiv\sum_{\nu\in N}\beta_l^{*\nu}-\sum_{\nu\in N}\gamma^{*\nu}<0$. In this case, given that every agent's utility function u^ν is strictly monotonic on C, a corresponding real-valued function $V^\nu\left(\Pi^\nu(p,w),\gamma^\nu\right)\equiv\max_{(c^\nu,\beta^\nu,\gamma^\nu)\in B^\nu(p,w)}u^\nu\left(c^\nu,\gamma^\nu\right)$ is strictly monotonic on $\Pi^\nu(p,w)$, for all ν.

Since $\Pi^\nu(\overline{p},\overline{w})=\pi^{\max}\overline{p}\underline{\beta}^{*\nu}+\overline{w}\gamma^{*\nu}=\pi^{\max}\overline{p}\underline{\beta}^{*\nu}$, then $V^\nu\left(\Pi^\nu(\overline{p},\overline{w}),\gamma^{*\nu}\right)=V^\nu\left(\Pi^\nu(\overline{p},\overline{w}),0\right)$ because $\overline{u^\nu}$ is (weakly) decreasing in γ^ν on $[0,1]$. Thus, whenever $(c^{*\nu},\beta^{*\nu},\gamma^{*\nu})\in\mathcal{O}^\nu(\overline{p},\overline{w})$ for all $\nu\in N$, then for any $\gamma^{**\nu}\in[0,\gamma^{*\nu}]$, we have $(c^{*\nu},\beta^{*\nu},\gamma^{**\nu})\in\mathcal{O}^\nu(\overline{p},\overline{w})$, which implies that, for any $\left(\gamma^{**\nu}\right)_{\nu\in N}\in\times_{\nu\in N}[0,\gamma^{*\nu}]$ with $\sum_{\nu\in N}\gamma^{**\nu}=\sum_{\nu\in N}\beta_l^{*\nu}$, $(c^{*\nu},\beta^{*\nu},\gamma^{**\nu})\in\mathcal{O}^\nu(\overline{p},\overline{w})$ holds for any $\nu\in N$. Let $\overline{z}_2'\equiv\sum_{\nu\in N}\beta_l^{*\nu}-\sum_{\nu\in N}\gamma^{**\nu}=0$. Then, $(\overline{z}_1,\overline{z}_2')\in Z(\overline{p},\overline{w})$, which yields the desired result. ■

Lemma A1.3 proves the existence of a fixed point for the aggregate excess demand correspondences, and therefore the existence of a price vector $(\overline{p},\overline{w})\in\Delta_+$ such that conditions (i), (ii) and (iv) of Definition 1 are satisfied. In order to complete the proof of existence of a RS, it is necessary to show that condition (iii) also holds. Theorem A1.1 provides a condition on aggregate social endowments under which the capital constraint (iii) is satisfied.

Theorem A1.1: *Let* A1′ ˜A3 *hold and let u^ν be continuous, quasi-concave, strictly increasing on C , and satisfying the boundary condition for all $\nu\in N$. Then, for any given profile $\Omega=(\omega^\nu)_{\nu\in N}$ with $\sum_{\nu\in N}\omega^\nu=\omega\geq\mathbf{0}$ which satisfies* A4, *there exists a distribution $\Omega'=(\omega'^\nu)_{\nu\in N}$ with $\sum_{\nu\in N}\omega'^\nu=\omega'$ and a RS $(p,w)\in\Delta_+$ for the economy $E(P,N,\mathbf{u},\mathbf{s},\Omega')$ with $p\omega'=p\omega$.*

Proof: Let $P,N,\mathbf{s}$, and $\Omega=(\omega^\nu)_{\nu\in N}$ satisfy A1′ ˜A4, and let $\mathbf{u}$ be such that for all $\nu\in N$, u^ν is continuous, quasi-concave, strictly increasing on C , and it satisfies the boundary condition. Then, we can apply Lemmas A1.1-A1.3, to prove that there exists $(p^*, w^*) \in \Delta_+$ such that $\left(\sum_{\nu\in N} c^{*\nu} - \sum_{\nu\in N} \widehat{\beta}^{*\nu}\right) = \mathbf{0}$ and $\left(\sum_{\nu\in N} \beta_l^{*\nu} - \sum_{\nu\in N} \gamma^{*\nu}\right) = 0$.

Thus, (p^*, w^*) is associated with $p^*\widehat{\alpha} - w^*\alpha_l \geqq 0$ for some $\alpha \in P\backslash\{\mathbf{0}\}$. In fact, if (p^*, w^*) is such that $p^*\widehat{\alpha} - w^*\alpha_l < 0$ for all $\alpha \in P\backslash\{\mathbf{0}\}$, then $\beta^{*\nu} = \mathbf{0}$ for all $\nu \in N$, but $\gamma^{*\nu} > 0$ and $c^{*\nu} \neq \mathbf{0}$ follow from $w^* > 0$ and the boundary condition for utility functions. (Note that if $p^*\widehat{\alpha} - w^*\alpha_l < 0$ for all $\alpha \in P\backslash\{\mathbf{0}\}$, then $w^* > 0$.) Hence, $\left(\sum_{\nu\in N} c^{*\nu} - \sum_{\nu\in N} \widehat{\beta}^{*\nu}\right) \geq \mathbf{0}$ and $\left(\sum_{\nu\in N} \beta_l^{*\nu} - \sum_{\nu\in N} \gamma^{*\nu}\right) < 0$ follow if $p^*\widehat{\alpha} - w^*\alpha_l < 0$ for all $\alpha \in P\backslash\{\mathbf{0}\}$, which is a contradiction. Thus, $p^*\widehat{\alpha} - w^*\alpha_l \geqq 0$ for some $\alpha \in P\backslash\{\mathbf{0}\}$.

Note that, since $p^*\widehat{\alpha} - w^*\alpha_l \geqq 0$ for some $\alpha \in P\backslash\{\mathbf{0}\}$, $(\mathbf{0}, \beta^{*\nu}, \gamma^{*\nu}, c^{*\nu})_{\nu\in N}$ is a profile of optimal solutions of all MP^ν with $p^*\underline{\beta}^{*\nu} = p^*\omega^\nu$ for all $\nu \in N$, thus $p^*\underline{\beta}^* = p^*\omega$ at (p^*, w^*). By A4, the existence of such a profile is guaranteed.

Let us define $\Omega' = (\omega'^\nu)_{\nu\in N}$ as $\omega'^\nu = \underline{\beta}^{*\nu}$ for each $\nu \in N$. Then, since $p^*\omega'^\nu = p^*\omega^\nu$ holds for each $\nu \in N$, it follows that $(\mathbf{0}, \beta^{*\nu}, \gamma^{*\nu}, c^{*\nu})_{\nu\in N}$ remains a profile of optimal solutions of all MP^ν such that $\left(\sum_{\nu\in N} c^{*\nu} - \sum_{\nu\in N} \widehat{\beta}^{*\nu}\right) = \mathbf{0}$ and $\left(\sum_{\nu\in N} \beta_l^{*\nu} - \sum_{\nu\in N} \gamma^{*\nu}\right) = 0$. Moreover $\underline{\beta}^* = \omega$, and therefore condition (iii) of Definition 1 is also satisfied. Hence, for the economy $E(P, N, \mathbf{u}, \mathbf{s}, \Omega')$, (p^*, w^*) is a RS with associated profile $(\mathbf{0}, \beta^{*\nu}, \gamma^{*\nu}, c^{*\nu})_{\nu\in N}$. ■

8 Appendix 2: Definitions 2 and 3

Lemma A2.1: *There exists an economy* $E \in \mathcal{E}$ *and a* RS (p, w) with associated aggregate production point $\alpha^{p,w}$ *such that neither* Definition 2 *nor* Definition 3 *satisfy* condition (2) *of* Theorem 1.

Proof: Consider the following von Neumann technology:

$$B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4.5 & 5.25 \end{bmatrix}, A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 3.5 \end{bmatrix}, L = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix},$$

where A is the input matrix; B is the output matrix; and L is the vector of labour coefficients. Define the production possibility set $P_{(A,B,L)}$ by

$$P_{(A,B,L)} \equiv \left\{\alpha \in \mathbb{R}_- \times \mathbb{R}^2_- \times \mathbb{R}^2_+ \mid \exists x \in \mathbb{R}^3_+ : \alpha \leqq (-Lx, -Ax, Bx)\right\}.$$

$P_{(A,B,L)}$ is a closed convex cone in $\mathbb{R}_- \times \mathbb{R}^m_- \times \mathbb{R}^m_+$ with $\mathbf{0} \in P_{(A,B,L)}$ and it satisfies A1∼A3. Let $\mathbf{e}_j \in \mathbb{R}^m_+$ be a unit column vector with 1 in the j-th component and 0 in any other component. Let $\alpha^1 \equiv (-L\mathbf{e}_1, -A\mathbf{e}_1, B\mathbf{e}_1)$, $\alpha^2 \equiv (-L\mathbf{e}_2, -A\mathbf{e}_2, B\mathbf{e}_2)$, and $\alpha^3 \equiv (-L\mathbf{e}_3, -A\mathbf{e}_3, B\mathbf{e}_3)$. Then,

$$\begin{aligned} \widehat{\alpha}^1 &\equiv (B-A)\,\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \widehat{\alpha}^2 \equiv (B-A)\,\mathbf{e}_2 = \begin{pmatrix} 1 \\ 1.5 \end{pmatrix}, \\ \widehat{\alpha}^3 &\equiv (B-A)\,\mathbf{e}_3 = \begin{pmatrix} 0 \\ 1.75 \end{pmatrix}. \end{aligned}$$

Also, we have $\widehat{P}(\alpha_l = 1) = co\{(1,0), (1,1.5), (0,1.75), \mathbf{0}\}$.

Let $W \neq \varnothing$ and let N be such that $|N| > |W|$. Let $c^* = (1,1)$ and let the social endowment of capital be given by $\omega = (2|N|, 3|N|)$. Let $\mathbf{u} \equiv (u, \ldots, u)$ with $u(c, \lambda) \equiv c_1 + c_2$, and $\mathbf{s} \equiv (1, \ldots, 1)$. Finally, let $\omega^\nu = \left(\frac{2|N|}{|N|-|W|}, \frac{3|N|}{|N|-|W|}\right)$ for all $\nu \in N \backslash W$, so that $\sum_{\nu \in N} \omega^\nu = \omega$. This completely defines the economy $E\left(N, P_{(A,B,L)}, \mathbf{u}, \mathbf{s}, \Omega\right)$. Then, a pair $(p, 1)$ with $p = (0.5, 0.5)$ constitutes a RS for $E\left(N, P_{(A,B,L)}, \mathbf{u}, \mathbf{s}, \Omega\right)$ associated with a social production point $|N|\,\alpha^2$. To see this, note first that

$$\begin{aligned} \frac{[p(B-A)-L]\,\mathbf{e}_1}{pA\mathbf{e}_1} &= -\frac{1}{2}; \frac{[p(B-A)-L]\,\mathbf{e}_2}{pA\mathbf{e}_2} = \frac{1}{10}; \\ \frac{[p(B-A)-L]\,\mathbf{e}_3}{pA\mathbf{e}_3} &= -\frac{1}{14}. \end{aligned}$$

Thus, for all $\nu \in N \backslash W$, $\beta^\nu = \frac{|N|}{|N|-|W|}\alpha^2$, $c^\nu = \left(1, \frac{1.5|N|-|W|}{|N|-|W|}\right)$, and $\lambda^\nu = 1$ is an optimal solution to MP^ν. Further, for every $\nu \in W$, $(c^\nu, \lambda^\nu) = (c^*, 1)$ is an optimal solution to MP^ν, so that at this RS, $W_+ = W$. Then, it is immediate to check that conditions (ii)-(iv) of Definition 1 are all satisfied.

Since $c^\nu = c^*$, then in both Definition 2 and Definition 3, $\overline{c}^\nu = c^*$ and $\widehat{\alpha}^{\overline{c}^\nu} = c^*$ hold for every $\nu \in W$. Then it is immediate to show that for all $\nu \in W$, there exists no $\underline{c}^\nu \in \Gamma(p, w; \Lambda^\nu) \cup \left\{\frac{\Lambda^\nu}{\alpha_l^{p,w}}\widehat{\alpha}^{p,w}\right\}$ such that $\underline{c}^\nu > (1,1) = \widehat{\alpha}^{\overline{c}^\nu}$ even though $\pi^{\max} = \frac{1}{10} > 0$, which implies that in this economy, neither Definition 2 nor Definition 3 satisfy condition (2) of Theorem 1.■[16]

[16] The constructed economy in this proof does not satisfy 'independence of production,' which was introduced by Roemer (1981; Ch2) as the necessary and sufficient condition for preserving FMT under Definition 2 in convex cone economies with homogeneous agents. Note, however, that in convex cone economies with heterogeneous agents, independence of production is no longer necessary nor sufficient, since it is easy to find an economy

and a RS with heterogenous consumption bundles $(c^{\nu})_{\nu \in W}$ among propertyless workers, in which the production set satisfies independence of production while condition (2) of Theorem 1 does not hold for some propertyless worker.

In contrast, FMT does not hold in general under Definition 3 even if the production set satisfies independence of production and the economy has homogeneous agents. This can be seen by checking Figure 2 in Yoshihara (2010; footnote 13).