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The Paradox of Thrift and Crowding-In of Private Investment in a Simple IS-LM Model

by

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Abstract

This paper derives conditions for two key Keynesian propositions in a simple IS-LM model: (a) the paradox of thrift, and (b) the crowding-in of private investment expenditures by government expenditures. A linear specification of the model is then presented as a special case that can be used for empirical analysis. Using data for the US economy for the period $1959-2009$, time series estimation of the linear model using instrumental variables regression shows that the paradox of thrift and crowding-in are real possibilities, especially in the sub-period, $1974-2009$, that excludes the Golden Age of capitalism.

JEL Codes: E12, E20.

Keywords: paradox of thrift, crowding-in, instrumental variables.

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1 Introduction

The "paradox of thrift" and the "crowding-in" of private investment expenditure by government expenditure are two key propositions within the Keynesian tradition. Both these propositions, among others, have been points of debate between Keynesian and Monetarist (mark I and mark II) economists over the last five decades. This paper throws some light on these long standing debates within macroeconomics in two ways. First, it derives the conditions for the occurrence of both the paradox of thrift and crowding in of private investment within a simple IS-LM model of short-run fluctuations of a capitalist economy; and second, it estimates the parameters of a linear specification of the model using instrumental variables regression and uses them to devise a novel test to investigate whether there is any empirical evidence in support of the paradox of thrift and the crowding-in of private investment expenditure.

The paradox of thrift refers to a situation where consumers' efforts to increase savings by reducing autonomous consumption expenditures can, in fact, lead to either no change or a decrease in aggregate savings in the short run. For it is possible that when consumers' reduce autonomous consumption expenditures *ceteris paribus*, the ensuing fall in aggregate demand leads to a much lower level of equilibrium output. Thus, though consumers's save more out of every unit of output, aggregate savings might be lower than, or equal to, that obtaining in the original situation because aggregate output has itself fallen relative to its original level. The paradox of thrift demonstrates, in a straightforward manner, the operation of the fallacy of composition, the proposition that what might be true for the individual need not necessarily be true for the aggregate.

The paradox of thrift has a long intellectual history and finds mention in several places in Keynes' *General Theory*. Discussing the apparent conflict between the savings-investment equality and the decisions of individuals to save as they choose, Keynes points out that

savings, like spending, is a two-sided affair.

> "For although the amount of his own saving is unlikely to have any significant influence on his own income, the reactions of the amount of his consumption on the incomes of others makes it impossible for all individuals simultaneously to save any given sums. Every such attempt to save more by reducing consumption will so affect incomes that the attempt necessarily defeats itself." (p. 84, Keynes, 1964)

In the chapter devoted to "notes on mercantilism", Keynes gives a brief historical sketch of the life of the related idea that under-consumption can lead to unemployment, starting with Bernard Mandeville's *Fable of the Bees* and running all the way up to the work of the English socialist J. A. Hobson (pp. 358-71, Keynes, 1964). Though the paradox of thrift has gradually percolated even to mainstream macroeconomics textbooks (Blanchard, 2008), to the best of my knowledge it has not been studied in any great detail, especially from an empirical angle. This paper attempts to partially fill in that lacuna.

The question of whether government expenditures crowd in or crowd out private investment expenditure has, on the other hand, been extensively studied. Theoretically, both the standard IS-LM framework and the basic real business cycle (RBC) model allows for a positive effect of government expenditure on private investment expenditure. Within a standard RBC model, an increase in government expenditure leads to an increase in employment. If the increase in employment is sufficiently persistent, then this can lead to an increase in the expected return on capital, which, in turn, may lead to an increase in current private investment expenditures. Within a standard IS-LM model, an increase in government expenditure leads to an increase in aggregate output through the multiplier effect of the increase in aggregate demand. If the monetary authorities accommodate the increased demand by increasing the supply of money and thereby keeping the interest rate constant, the increased

output can lead to increased private investment expenditure through an investment function that depends on aggregate output.

The theoretical literature is matched, even surpassed, by a large body of literature that has tried to empirically investigate the issue, ranging from time series to cross sectional to VAR to panel data studies (Spencer and Yohe, 1970; Easterly and Rebelo, 1993; Argimon, et al., 1997; Edelberg, et al., 1999; Furceri and Sousa, 2009). As Table 1 in Furceri and Sousa (2009) show, there is no consensus on the issue of crowding-out. Some studies report results in favour of crowding-out Barro, 1991; Mountford and Uhlig, 2004), some present evidence in favour of crowding-in (Easterly and Rebelo, 1993; Edelberg et al., 1999; Blanchard and Perotti, 2002) and others report the absence of any effect of government expenditures on private investment expenditures (Fatas and Mihov, 2001; Perotti, 2004). This paper adds to this large and growing literature by investigating the issue of crowding-in within a simple IS-LM model and using a novel strategy to confront the results from a linear specification of the model with time series data for the US.

The rest of the paper is organized as follows: the next section sets up the basic model and poses the two questions addressed in this paper; the next section works out the conditions for the operation of the paradox of thrift in the basic model; the following section derives the conditions for the possibility of crowding-in; the next section provides the same results with linear specifications for the key behavioural relationships in the model; the following section is devoted to the empirical analysis; the final section concludes the paper with indications of some possible extensions. Details of the data used in the empirical analysis is provided in an appendix.

2 The Set-up

Consider a simple short-run aggregate model of a typical capitalist economy which is closed to international trade in goods, services and capital. To keep the analysis simple and to understand the essential results, we make two standard assumptions: (1) we assume that the economy produces only one good, thereby allowing us to talk about "the" good market, and (2) we assume that the financial market has only one bond, thereby allowing us to talk about "the" interest rate. Since we are confining ourselves to a short-run model, we will assume that the price of the single good is given.

2.1 The Good Market

The aggregate demand for the single good produced in this economy arises from consumption, investment and government expenditures:

$$Z \equiv C + I + G. \tag{1}$$

Aggregate consumption expenditure, C, depends on the level of disposable income, $Y_D = Y - T$, where Y is the value of output and T represents the tax revenue of the government; note that by definition, output is equal to income and hence Y stands for the aggregate level of income. Aggregate consumption expenditure has an autonomous part too, represented by c_0, and thus we have

$$C = C(Y_D, c_0), \tag{2}$$

where $C_1 \equiv (\partial C/\partial Y_D) > 0$ and $C_2 \equiv (\partial C/\partial c_0) > 0$. Aggregate investment expenditure, I, depends on two factors, the level of sales and the interest rate. The level of sales is proxied by the value of aggregate output, Y, and the interest rate, i, stands for the cost of borrowing

funds necessary to finance investment expenditures; thus, we have

$$I = I(Y, i), \tag{3}$$

where $I_1 \equiv (\partial I/\partial Y) > 0$ and $I_2 \equiv (\partial I/\partial i) < 0$. Government expenditure is assumed to be exogenous while the tax revenue of the government is assumed to be a function of the level of aggregate income; thus, we have

$$G = \bar{G}, \tag{4}$$

and

$$T = T(Y), \tag{5}$$

where $T'(Y) \equiv (dT/dY) > 0$. Equilibrium in the good market is attained when aggregate supply, Y, is equal to aggregate demand, Z, giving us the following condition:

$$Y = C((Y - T(Y)), c_0) + I(Y, i) + \bar{G}. \tag{6}$$

The equilibrium relationship can also be written as the familiar equality of savings and investment:

$$I(Y, i) = S + (T(Y) - \bar{G}), \tag{7}$$

where $S \equiv Y - T - C$ is the private savings and $(T(Y) - G)$ represents public savings.

2.2 The Financial Market

There are two types of assets in which wealth can be stored in this simple economy: money and bonds. By Walras Law, equilibrium in one of the asset markets implies equilibrium in the other; hence we can focus our attention on only one of the asset markets. The working of the asset markets determines the interest rate, i, which feeds into aggregate demand through it's effect on aggregate investment expenditures. There are two different ways to characterize the determination of the interest rate, one where the supply of money is considered exogenous and another where the supply of money is considered endogenous.

Exogenous money supply is the familiar building block of the traditional IS/LM model, where the Central Bank is assumed to supply some exogenously determined quantity of real money, M/P, where M is the supply of money and P is the price of the single good; the supply curve for money can be understood as a vertical line in $i - Y$ space. The demand for money arises from both the needs to finance transactions and from considerations of foregone interest income from holding bonds; a simple characterization of the demand for money, M^d is

$$M^d = M^d(Y, i). \tag{8}$$

Hence, equilibrium interest rate is the value of i that ensures

$$\frac{M}{P} = M^d(Y, i). \tag{9}$$

The alternative formulation of the determination of the interest rate relies on the claim that money supply is not exogenous as has been assumed in traditional IS/LM models. In an advanced capitalist economy with a complex financial architecture, the Central Bank cannot control the supply of money in any meaningful sense. What it can instead do is to control

some key interest rates to influence the flow of money and credit in the economy; this is more or less how the Federal Reserve, the Central Bank of the United States, seems to function.[1] Thus, the supply of money adjusts endogenously to the fluctuations in the demand for money at the level of the interest rate that the Central Bank wants to maintain; the supply of money can be understood as a horizontal line in $i - Y$ space. With this formulation, the interest rate becomes exogenous in the short run, determined and defended by the open market operations of the Central bank:

$$i = \bar{i}. \tag{10}$$

It must be noted that this is only one way to operationalize the idea of endogenous money supply, an issue that has been extensively studied by economists in the post-Keynesian tradition. Summarizing the large literature on endogenous money, Palley (2002), points out the various senses in which money supply can be understood as being endogenous: central bank endogeniety, fiscal endogeneity, portfolio endogeneity, endogeneity of credit money, among others. According to the debates within the post-Keynesian tradition, the operationalization of endogenous money that is captured by (10) can be called the accommodationist view of money supply endogeneity.

2.3 Basic Model and Questions

The simple short-run aggregate model of the economy that we are working with, thus, has two endogenous variables, Y (aggregate output) and i (the interest rate) and two equilibrium

[1] See the website of the Federal Open Market Committee for a succinct statement of what the Fed means by "monetary policy" and how it affects the federal funds rate, a key short-term interest rate; for instance see `http://www.federalreserve.gov/monetarypolicy/fomc.htm`

relationships:

$$Y = C((Y - T(Y)), c_0) + I(Y, i) + \bar{G}$$

and

$$\frac{M}{P} = M^d(Y, i)$$

or

$$i = \bar{i},$$

where the crucial exogenous variables are c_0 (autonomous part of aggregate consumption expenditure) and $\bar{G}$ (aggregate government expenditure). The first gives the familiar IS relation and the second gives the LM relation; the third is an alternative formulation for the determination of the interest rate and rests on a simple operationalization of the idea of endogenous money supply. We want to pose two questions in this simple model.

The first question that we wish to pose is to understand the conditions under which the "paradox of thrift" would be in operation. The paradox of thrift, as pointed out earlier, is the phenomenon whereby attempts by consumers to reduce aggregate consumption expenditure and increase aggregate savings might, in fact, lead to the opposite effect: reduction of savings. The simple reason is that when consumers reduce consumption expenditures that reduces aggregate output and income through the multiplier and thus consumers are left with a lower level of aggregate income to save from; the net result might very well be a fall in aggregate savings negating the original intentions of the consumers.

The second question we wish to address is to understand the conditions under which "crowding-in" can become operational. Crowding-in, as discussed above, is the claim that

government expenditures can, under certain conditions, increase the level of aggregate private investment expenditures rather than crowding them out as is often claimed by economists of Monetarist persuasion. The logic is simple. If investment expenditures by firms respond to the level of sales and if aggregate output can be a proxy for aggregate sales, then exogenous increases in government expenditures can increase investment expenditures due to the boost to aggregate demand that it gives, leading to a higher level of equilibrium aggregate output and income.

3 The Paradox of Thrift

The paradox of thrift refers to a situation where aggregate savings in an economy declines or remains unchanged despite, or precisely because, consumers try to reduce autonomous consumption expenditures and increase savings. To understand the conditions under which this result might arise in our simple aggregate short-run model, let us recall the savings-investment equality, which is one way to characterise equilibrium in the good market:

$$I(Y, i) = S + T(Y) - \bar{G}.$$

One way to investigate the paradox of thrift is to understand how a change in autonomous consumption expenditure affects the level of aggregate savings, i.e., to determine the sign of the derivative dS/dc_0. When the sign of the derivative is positive, the paradox of thrift is in operation because autonomous consumption expenditure and aggregate savings move in the same direction: an decrease in autonomous consumption expenditure reduces aggregate savings. Since $S = I(Y, i) - T(Y) + \bar{G}$, we have

$$\frac{dS}{dc_0} = I_1 \frac{dY}{dc_0} + I_2 \frac{di}{dc_0} - T'(Y) \frac{dY}{dc_0}, \tag{11}$$

where $I_1 = (\partial I/\partial Y)$, $I_2 = (\partial I/\partial i)$ and $T'(Y) = (dT/dY)$. We wish to investigate conditions under which the expression in (11) is non-negative.

3.1 Exogenous Money

When the supply of money is assumed to be exogenous, equilibrium levels of aggregate output, Y, and interest rate, i are determined by the following two equations:

$$Y = C((Y - T(Y)), c_0) + I(Y, i) + \bar{G}$$

and

$$\frac{M}{P} = M^d(Y, i).$$

Thus, in this case, we have

$$\frac{dS}{dc_0} = \frac{\left\{I_1 - I_2\left(\frac{M_1^d}{M_2^d}\right) - T'(Y)\right\} C_2}{\left\{1 - C_1(1 - T'(Y)) - I_1 + I_2(\frac{M_1^d}{M_2^d})\right\}} \tag{12}$$

where $C_1 = (\partial C/\partial Y_D)$, $C_2 = (\partial C/\partial c_0)$, $M_1^d = (\partial M^d/\partial Y)$ and $M_2^d = (\partial M^d/\partial i)$. This gives us the first result as

Proposition 1 *If*

$$T'(Y) \leq I_1 - I_2\left(\frac{M_1^d}{M_2^d}\right) - T'(Y) \leq 1 - C_1(1 - T'(Y))$$

then $dS/dc_0 \geq 0$ *and the paradox of thrift is in operation.*

Remarks: The proof follows easily by investigating the conditions under which the ratio on the RHS of (12) is non-negative.

To interpret the condition in the above proposition, note that the expected signs of the terms are as follows: $T'(Y) > 0, I_1 > 0, I_2 < 0, M_1^d > 0, M_2^d < 0, 0 < C_1 < 1$. Since $0 < T'(Y) < 1$, we have $(1 - C_1)T'(Y) < (1 - C_1)$, and so $T'(Y) < 1 - C_1(1 - T'(Y))$; thus, as long as $I_1 - I_2(M_1^d/M_2^d) - T'(Y)$ lies between these two quantities the paradox of thrift will be in operation. On the one hand, this means that $I_1 - I_2(M_1^d/M_2^d)$ must be greater than $2T'(Y)$; on the other it means that $I_1 - I_2(M_1^d/M_2^d)$ must be less than $1 - C_1 + C_1T'(Y) + T'(Y)$.

Now, $I_1 - I_2(M_1^d/M_2^d) = M_1^d(I_1/M_1^d - I_2/M_2^d)$. Thus, the condition boils down to the fact that the difference of the response of investment expenditure to changes in aggregate output and the interest rate, both scaled by corresponding responses of the demand for money should be greater than twice the response of tax revenues to aggregate output scaled by the response of the demand for money to aggregate output but smaller than the sum of the marginal propensity to save, $1 - C_1$, and a positive term involving the marginal propensity to consume and the effect of aggregate output on the tax revenue, but again scaled by the response of the demand for money to aggregate output. The condition can, therefore, be written as:

$$\frac{T'(Y)}{M_1^d} \leq \frac{I_1}{M_1^d} - \frac{I_2}{M_2^d} \leq \frac{1 - C_1 + C_1T'(Y) + T'(Y)}{M_1^d};$$

the condition, therefore, requires that the difference of the response of investment to aggregate output scaled by the response of the money demand function to aggregate output and the response of investment to the interest rate scaled by the response of the money demand function to the interest rate be bounded by two quantities: below by the response of tax revenues to aggregate output scaled by the response of the money demand function to aggregate output, and above by the marginal propensity to save scaled by the response of the money demand function to aggregate output.

3.2 Endogenous Money

When the Central Bank is assumed, more realistically, to set and defend an interest rate by allowing the money supply to be endogenously determined by the demand for money, equilibrium levels of aggregate output, Y, and interest rate, i are determined by the following two equations:

$$Y = C((Y - T(Y)), c_0) + I(Y, i) + \bar{G}$$

and

$$i = \bar{i}.$$

Thus, in this scenario, we have

$$\frac{dS}{dc_0} = \frac{\{I_1 - T'(Y)\} C_2}{\{1 - C_1(1 - T'(Y)) - I_1\}}.$$

This gives us the next result as

Proposition 2 *If*

$$T'(Y) \leq I_1 \leq 1 - C_1(1 - T'(Y))$$

then $dS/dc_0 \geq 0$ *and the paradox of thrift is in operation.*

Remark: Since the interest rate is now exogenous and fixed by the Central Bank, the effect of the interest on aggregate investment expenditure, I_2, is zero; hence the term involving I_2 drops out.

4 Crowding-In

The next issue that we wish to address is whether, and under what conditions, government expenditures can crowd-in, i.e., increase, private investment expenditure. Since, in our simple model, aggregate investment expenditure is influenced by aggregate output (as a proxy for aggregate sales) and the interest rate (as a proxy for the cost of financing investment expenditures), we have

$$\frac{dI}{dG} = I_1 \frac{dY}{dG} + I_2 \frac{dY}{di}. \tag{13}$$

Now we wish to investigate the conditions under which the expression in (13) is positive, because a positive value of the derivative implies that government expenditures and private investment expenditures move together: when government expenditures increase, that leads to an increase in aggregate private expenditure.

4.1 Exogenous Money

In the case when money is considered to be exogenous, we have, as before, equilibrium determined by the following two equations:

$$Y = C((Y - T(Y)), c_0) + I(Y, i) + \bar{G}$$

and

$$\frac{M}{P} = M^d(Y, i).$$

Thus, we have

$$\frac{dI}{dG} = \frac{\left\{I_1 - I_2\left(\frac{M_1^d}{M_2^d}\right)\right\}}{\left\{1 - C_1(1 - T'(Y)) - I_1 + I_2(\frac{M_1^d}{M_2^d})\right\}},$$

which gives us the next result as

Proposition 3 *If*

$$0 \leq I_1 - I_2\left(\frac{M_1^d}{M_2^d}\right) \leq 1 - C_1(1 - T'(Y))$$

then $dI/dG \geq 0$ *and government expenditures crowd-in private investment expenditures.*

Remark: Since $1 - C_1(1 - T'(Y)) > 0$, crowding-in occurs whenever $I_1 - I_2\left(\frac{M_1^d}{M_2^d}\right)$ is positive but less than $1 - C_1(1 - T'(Y))$. To interpret the condition, we can re-write it as

$$0 \leq \frac{I_1}{M_1^d} - \frac{I_2}{M_2^d} \leq \frac{1 - C_1 + C_1 T'(Y)}{M_1^d}.$$

Thus, as long as the difference of the response of investment to aggregate output scaled by the response of the money demand function to aggregate output and the response of investment to the interest rate scaled by the response of money demand to the interest rate is positive but less than the marginal propensity to save, $(1 - C_1)$ scaled by the response of money demand with respect to aggregate output, the economy will experience crowding-in of private investment expenditure.

4.2 Endogenous Money

With the interest rate exogenous and the supply of money endogenous, equilibrium is determined by

$$Y = C((Y - T(Y)), c_0) + I(Y, i) + \bar{G}$$

and

$$i = \bar{i}.$$

In this case,

$$\frac{dI}{dG} = \frac{I_1}{\{1 - C_1(1 - T'(Y)) - I_1\}},$$

which gives us the next result as

Proposition 4 *If*

$$0 \leq I_1 \leq 1 - C_1(1 - T'(Y))$$

then $dI/dG \geq 0$ *and government expenditures crowd-in private investment expenditures.*

5 Results under Linear Specification

To test the key results of this paper, i.e., the four propositions, we will work with linear specifications of the behavioural equations of the model: the aggregate consumption function, the investment function, the tax revenue function and the money demand function. Since aggregate consumption expenditure is a function of c_0 and disposable income, Y_D, a linear

specification of the consumption function that is commonly used is the following:

$$C = c_0 + c_1 Y_D, \tag{14}$$

where $0 < c_1 < 1$ is the marginal propensity to consume. Using the fact that aggregate investment expenditure is a function of Y and i, we will use the following linear investment function:

$$I = b_0 + b_1 Y + b_3 i, \tag{15}$$

where $b_1 > 0$ and $b_3 < 0$. A linear specification of the tax revenue function that we will use is:

$$T = t_0 + t_1 Y, \tag{16}$$

where $0 < t_1 < 1$ is the marginal tax rate. A linear specification of the money demand function that we will use is the following:

$$M^d = a_0 + a_1 Y + a_2 i, \tag{17}$$

where $a_1 > 0$ and $a_2 < 0$. Since the demand for money is known to display non-linearities, a better specification of the money demand function might be:

$$\ln(M^d) = a_0 + a_1 \ln(Y) + a_2 \ln(i), \tag{18}$$

where $a_1 > 0$ and $a_2 < 0$. With these linear specifications, the main results of this paper can be re-written in terms of a few crucial parameters, which can then be estimated using macroeconomic data.

5.1 Paradox of Thrift

Here we re-state the two results about the paradox of thrift for the linear specification of the behavioural equations.

Proposition 5 *(Exogenous Money) If (14), (15), (16) and (17) are used as representations of the behavioural equations of the model and*

$$t_1 \leq b_1 - b_2\frac{a_1}{a_2} - t_1 \leq 1 - c_1 + c_1 t_1$$

then $(dS/dc_0 \geq 0)$ *and the paradox of thrift is in operation. If (18) is instead used as a specification for the money demand function and*

$$t_1 \leq b_1 - b_2\frac{a_1}{a_2}\frac{i}{Y} - t_1 \leq 1 - c_1 + c_1 t_1,$$

where i *and* Y *refer to some suitable average values of the interest rate and aggregate output, then* $(dS/dc_0 \geq 0)$ *and the paradox of thrift is in operation.*

Proposition 6 *(Endogenous Money) If (14), (15), (16) are used as representations of the behavioural equations of the model and money supply is endogenous, i.e.,* $i = \bar{i}$ *and we also have*

$$t_1 \leq b_1 \leq 1 - c_1 + c_1 t_1$$

then $(dS/dc_0 \geq 0)$ *and the paradox of thrift is in operation.*

5.2 Crowding-In

Now we re-state the key results on crowding-in of private investment expenditure by government expenditure for a linear specification of the behavioural equations of the model.

Proposition 7 *(Exogenous Money) If (14), (15), (16) and (17) are used as representations of the behavioural equations of the model and*

$$0 \leq b_1 - b_2 \frac{a_1}{a_2} \leq 1 - c_1 + c_1 t_1$$

then $(dI/dG \geq 0)$ *and the crowding-in effect of government expenditures is in operation. If (18) is instead used as a specification for the money demand function and*

$$0 \leq b_1 - b_2 \frac{a_1}{a_2} \frac{i}{Y} \leq 1 - c_1 + c_1 t_1$$

where i *and* Y *refer to some suitable average values of the interest rate and aggregate output, then* $(dI/dG \geq 0)$ *and private investment expenditure is crowded-in by government expenditure.*

Proposition 8 *(Endogenous Money) If (14), (15), (16) are used as representations of the behavioural equations of the model and money supply is endogenous, i.e.,* $i = \bar{i}$ *and we also have*

$$0 \leq b_1 \leq 1 - c_1 + c_1 t_1$$

then $(dI/dG \geq 0)$ *and government expenditures crowd-in private investment expenditure.*

6 Empirical Analysis

What does the empirical evidence from the US economy show about the possibility of the occurrence of the paradox of thrift and crowding in? To answer this question, and link up with the results from the simple model in this paper, we will estimate the parameters that appear in Proposition 5, 6, 7 and 8: the marginal propensity to consume (c_1), the

marginal effect of aggregate output on tax revenues (t_1), the marginal effect of aggregate output on investment (b_1) and the marginal effect of interest rates on investment (b_2). Using estimates for these four crucial parameters, we will use devise statistical tests to check whether Proposition 5, 6, 7 and 8 are confirmed or rejected by the data.

6.1 Empirical Models

To motivate estimation of the consumption function, let us refer to Figure 1, which is scatter plot of quarterly change of consumption and the quarterly change of disposable income. As can be seen from the figure, there appears to be a strong positive relation between changes in consumption expenditure and changes in disposable income. Since we are interested in the marginal effect of disposable income on aggregate consumption expenditure, this suggests that we estimate the following linear form of the relationship between aggregate consumption and disposable income:

$$C_t = c_0 + c_1 Y_{D,t} + u_{1,t}$$

which, on differencing, gives

$$\Delta C_t = c_1 \Delta Y_{D,t} + \varepsilon_{1t},$$

where $\Delta C_t = C_t - C_{t-1}$ is the first difference of real aggregate consumption expenditure, $\Delta Y_{D,t} = Y_{D,t} - Y_{D,t-1}$ is the first difference of real disposable income, $\varepsilon_{1t} = u_{1,t} - u_{1,t-1}$, and the parameter of interest is c_1, the marginal propensity to consume.[2] Since some effect of past changes in disposable income on current consumption expenditure cannot be ruled out, we include a lagged independent variable to capture this possibility; hence, the equation we

[2]A difference specification has been used in all the regression equation because of the presence of unit roots in all the variables that this paper deals with; for details of unit root tests see the data appendix.

estimate is

$$\Delta C_t = c_1 \Delta Y_{D,t} + c_2 \Delta Y_{D,t-1} + \varepsilon_{1t}. \tag{19}$$

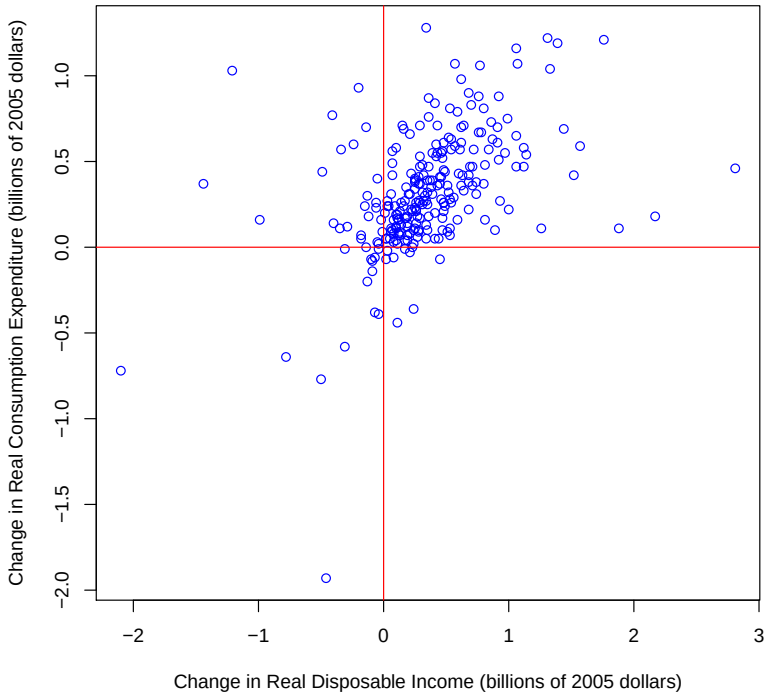

Figure 1: *Change in Consumption versus Change in Disposable Income*

Estimating (19) by ordinary least squares (OLS) is problematic because of endogeneity problems. Not only do changes in disposable income affect aggregate consumption expenditure, but changes in aggregate consumption expenditure also affects disposable income through the multiplier effect of changes in aggregate consumption on aggregate output; thus, there is problem of simultaneous causation.[3] One way to deal with this potential problem of endogeneity is use instrumental variables (IV) regression. With a proper instrument for

[3]Note that the lagged independent variable is exogenous because current changes in consumption expenditure cannot affect past changes in disposable income.

changes in disposable income, one would be able to capture the effect of exogenous changes in disposable income on aggregate consumption expenditure; that would, then, provide a consistent estimate of the marginal propensity to consume, c_1.

A good instrument must satisfy two criterion: (a) it must be correlated with the regressor, in this case changes in disposable income, and (b) it must affect the dependent variable, in this case changes in aggregate consumption expenditure, only *through* the endogenous regressor and not directly. It is only then that the part of the variation in the endogenous regressor that is not explained by the instrument can be used as a source of exogenous variation to consistently estimate the effect of the regressor on the dependent variable. These considerations suggest that changes in non-residential fixed investment expenditures can act as a legitimate instrument for changes in disposable income. Changes in non-residential fixed investment spending would affect aggregate output through the multiplier and would hence affect disposable income but there is no reason to believe that it would directly affect aggregate consumption expenditure. Hence, we will use non-residential fixed investment spending as an instrument for the estimation of (19).

The next relationship that we need to estimate is the tax revenue function. To motivate the estimation, let us look at Figure 2, which is a scatter plot of changes in real tax revenue versus changes in real output. The figure suggests that there is a positive relationship between the two variables and so we will estimate the following simple linear tax revenue function:

$$T_t = t_0 + t_1 Y_t + u_{2,t},$$

which, in first difference form, is

$$\Delta T_t = t_1 \Delta Y_t + \varepsilon_{2t},$$

where $\Delta T_t = T_t - T_{t-1}$ is the first difference of real tax revenues, $\Delta Y_t = Y_t - Y_{t-1}$ is the first difference of real GDP, $\varepsilon_{2t} = u_{2,t} - u_{2,t-1}$ and the parameter of interest is t_1, the marginal tax rate. To take account of possible delays in the effect of aggregate output on tax revenue, we also include a lagged independent variable in the regression equation; hence, we estimate the following:

$$\Delta T_t = t_1 \Delta Y_t + t_2 \Delta Y_{t-1} + \varepsilon_{2,t}. \tag{20}$$

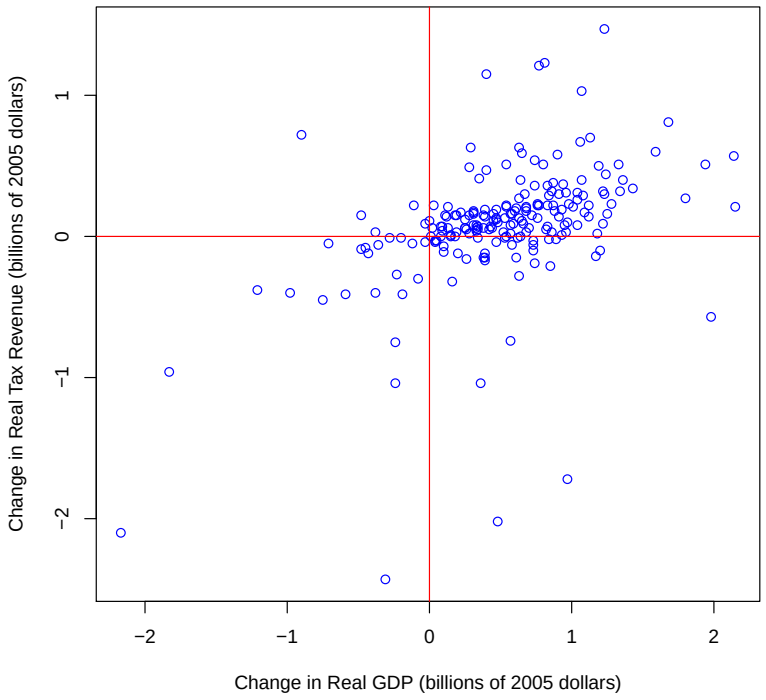

Figure 2: *Change in Tax Revenue versus Change in Output*

Estimating (20) by OLS runs into the same problems as the consumption function: simultaneous causation leading to endogeneity. While it is obvious that changes in aggregate output will affect the tax revenue, it is equally true that changes in taxes will affect aggregate output through its effect on aggregate demand via changes in disposable income. We can,

as before, deal with the problem of consistent estimation of t_1 by using IV regression. We can use changes in government defence expenditure as an instrument for changes in aggregate output: while changes in defence expenditures of the government will affect aggregate output through its effect on aggregate demand, it can be reasonably expected that changes in defence expenditures of the government will not directly impact changes in aggregate tax revenues.

Next, we will estimate the following linear specification of an aggregate investment function:

$$I_t = b_0 + b_1 Y_t + b_2 i_t + u_{3,t},$$

which, in first difference form, is

$$\Delta I_t = b_1 \Delta Y_t + b_2 \Delta i_t + \varepsilon_{3t}, \tag{21}$$

where $\Delta I_t = I_t - I_{t-1}$ is the first difference of real aggregate private investment expenditure, $\Delta Y_t = Y_t - Y_{t-1}$ is the first difference of real GDP, $\Delta i_t = i_t - i_{t-1}$ is the first difference of a long-term interest rate, $\varepsilon_{3t} = u_{3,t} - u_{3,t-1}$ and the parameters of interest are b_1 and b_2.

Just as in the case of the consumption function, estimating the parameters in (21) cannot use OLS because of problems of endogeneity; both the regressors are likely to be endogenous because of two-way causation. While it is obvious that changes in aggregate output (acting as a proxy for aggregate sales) and changes in the interest rate (acting as a measure for the cost of finance) affects aggregate investment expenditure, it is also the case that causation might run the other way too. Changes in aggregate investment expenditures are likely to impact aggregate output through the multiplier; similarly, changes in aggregate investment expenditure might affect market interest rates when large parts of investment expenditures are debt-financed, as is typical in capitalist countries. Hence, we need to use instruments for

both the regressors in (21).

Two instruments suggest themselves naturally, change in consumption expenditure as an instrument for change in aggregate output and change in narrow money ($M1$) as an instrument for the change in interest rate. Changes in aggregate consumption expenditure will affect the aggregate output through the multiplier but can be expected not to have any direct impact on aggregate investment expenditure. Similarly, changes in $M1$ will affect the interest rate by affecting the tightness of the money market, but it can be expected not to directly affect aggregate investment expenditure, i.e., changes in narrow money will affect aggregate investment expenditure only through the channel of market interest rates.

6.2 Data and Estimates

Estimation of (19), (21) and (20) uses macroeconomic data for the US economy at quarterly frequency obtained from the Bureau of Economic Analysis and the Federal Reserve Bank of St. Louis; more details of the data are provided in an appendix. Table 1 summarises estimation results using both OLS and the method of two-stage least squares (2SLS) for two separate, but overlapping, time periods: $1959-2009$ and $1974-2009$. The separate focus on these two time periods is motivated by the well-known fact that the US economy entered a slow-growth regime when compared to the 25 years following the second world war, often referred to as the Golden Age of capitalism. Since the paradox of thrift might be expected to operate more strongly during periods of stagnation of capitalist economies, it will be instructive to study the sub-period, $1974-2009$, separately.

All the estimates have expected signs except the one for b_2, but the estimate of b_2, using IV regression, is not statistically significantly different from zero. The statistically significant 2SLS estimates of the parameters have straightforward interpretations. Since for the period $1959-2009$, $\hat{c}_1 = 0.607$, it means that every dollar increase in disposable income leads to an increase in aggregate consumption expenditure by 61 cents; a similar scenario obtains in

the sub-period, $1974-2009$ too. Moving to the tax revenue function we see that $\hat{t}_1 = 0.238$ for the period $1959-2009$ and $\hat{t}_1 = 0.227$ in the sub-period; this means that every dollar increase in aggregate output leads to about 24 cents in increased tax revenues for both periods. We see that the investment function displays the main change in the estimates for the two periods. Since $\hat{b}_1 = 0.209$ for the period $1959-2009$, this implies that for every dollar increase in aggregate output, aggregate investment expenditures increase by about 21 cents. The corresponding estimate for the sub-period $1974-2009$ is $\hat{b}_1 = 0.254$; hence, aggregate investment expenditure is about 20% more responsive to changes in aggregate output in the sub-period of slow growth. The fact that $\hat{b}_2$ is not statistically significant from zero in both the time periods implies that the interest elasticity of investment expenditures is negligible.

6.3 Testing for the Paradox of Thrift

Using 2SLS estimates of the crucial parameters reported in Table 1, we can devise statistical tests of Propositions 5 and Proposition 6. But we will restrict ourselves to only testing Proposition 6; this is because of two reasons. One, endogenous money captured by the Central Bank setting interest rates is more realistic and in line with evidence about how the Fed actually operates in practice. Second, note that the main difference in the conditions for Propositions 5 and 6 arise from the term involving b_2, the effect of interest rates on investment expenditure; but Table 1 shows that the estimate of b_2 is not statistically significantly different from zero. Hence, we can restrict ourselves to Proposition 6.

Since the estimate of b_2 is not statistically different from zero, terms involving b_2 can be safely ignored. This implies that the parameters in the money demand function, which give an idea of the elasticity of money demand both with respect to income and interest rates, are not required for our analysis; that is why we did not estimate the money demand function. Of course, there is a large literature which deals with the estimation of money demand functions and we could have drawn on the results emerging from that literature if

we had needed to; for a recent and innovative method of estimation see Bae and de Jong (2007), and for a review of the literature see Sriram (2001).

Note that the condition in Proposition 6 is the following: $t_1 \leq b_1 \leq 1 - c_1 + c_1 t_1$. Since economic theory suggests that $c_1 > 0$ and $t_1 > 0$, the condition will be satisfied as long as we have: $t_1 \leq b_1 \leq 1 - c_1$. We can break this into two inequalities: $0 \leq b_1 - t_1$ and $b_1 + c_1 - 1 \leq 0$. Hence, the first hypothesis test emerges as:

$$H_0 : b_1 - t_1 = 0; H_1 : b_1 - t_1 > 0; \tag{22}$$

and the second hypothesis test becomes

$$H_0 : b_1 + c_1 - 1 = 0; H_1 : b_1 + c_1 - 1 < 0. \tag{23}$$

The test statistic for the hypothesis test in (22) is

$$T_1 = \frac{\hat{b}_1 - \hat{t}_1}{\sqrt{SE(\hat{b}_1)^2 + SE(\hat{t}_1)^2}},$$

and under the null hypothesis $H_0 : b_1 - t_1 = 0$, it becomes

$$T_1 = \frac{\hat{b}_1 - \hat{t}_1 - b_1 + t_1}{\sqrt{SE(\hat{b}_1)^2 + SE(\hat{t}_1)^2}},$$

which, in large samples, is distributed as a standard normal random variable.

From the first panel in Table 1, we see that $\hat{b}_1 < \hat{t}_1$; hence, there is no evidence, even weak, in favour of the alternative. Since we cannot reject the null in (22), we need not even test the null in (23). Thus, the data seems to suggest that the paradox of thrift is not in operation in the linear representation of the US economy for the period from 1959 to 2009.

Basic economic theory suggests that the paradox of thrift has a higher probability to

be in operation when a capitalist economy is in a downturn. Since the period from 1974 onwards has been widely recognized as period of relative stagnation in the US economy, we would now like to carry out the same test for this sub-period. From the second panel Table 1 for the period $1974-2009$, we see that $\hat{b}_1 > \hat{t}_1$; so, there seems to be some evidence for the operation of the paradox of thrift. The actual value of T_1, in his case, is

$$T_1^{act} = \frac{0.254 - 0.227}{\sqrt{0.099^2 + 0.080^2}} = 0.212.$$

The test statistic has a positive value though it is not large enough to reject the null in favour of the alternative; compared to the situation in the period $1959-2009$, of course this provides stronger support for the possibility that $0 \leq b_1 - t_1$. We can at least claim that the evidence does not lend support to the hypothesis that $\hat{b}_1 < \hat{t}_1$; since $\hat{b}_1 = \hat{t}_1$ is compatible with the operation of the paradox of thrift, we will now test the other half of the condition, namely (23). The test statistic is

$$T_2 = \frac{\hat{b}_1 + \hat{c}_1 - 1}{\sqrt{SE(\hat{b}_1)^2 + SE(\hat{c}_1)^2}},$$

and under the null hypothesis $H_0 : b_1 - t_1 = 0$, it becomes

$$T_1 = \frac{\hat{b}_1 + \hat{c}_1 - 1 - b_1 - c_1 + 1}{\sqrt{SE(\hat{b}_1)^2 + SE(\hat{c}_1)^2}},$$

which, in large samples, is distributed as a standard normal random variable. The actual value of the test statistic for the period $1974-2009$ is

$$T_4^{act} = \frac{0.254 + 0.604 - 1}{\sqrt{0.099^2 + 0.060^2}} = -1.226,$$

which corresponds to a p-value of 0.11. Together with the fact that we have evidence for

$\hat{b}_1 = \hat{t}_1$, this suggests that the paradox of thrift is in operation during the period $1974-2009$.

6.4 Testing for Crowding-in of Private Investment

We can devise a statistical test for Proposition 7 and 8 in an exactly similar manner. For the same two reasons that we stated earlier, we can restrict ourselves to Proposition 8 only.

Note that the condition in Proposition8 is the following: $0 < b_1 < 1 - c_1 + c_1 t_1$. Since, as before, economic theory suggests that $c_1 > 0$ and $t_1 > 0$, the condition will be satisfied as long as we have: $0 < b_1 < 1 - c_1$. We can break this into two inequalities: $0 < b_1$ and $b_1 + c_1 - 1 < 0$. Hence, the first hypothesis test emerges as:

$$H_0 : b_1 = 0; H_1 : b_1 > 0; \tag{24}$$

and the second hypothesis test becomes

$$H_0 : b_1 + c_1 - 1 = 0; H_1 : b_1 + c_1 - 1 < 0. \tag{25}$$

The test statistic for the hypothesis test in (22) is

$$T_3 = \frac{\hat{b}_1 - b_1}{SE(\hat{b}_1)},$$

and under the null hypothesis $H_0 : b_1 = 0$ and in large samples, it is distributed as a standard normal random variable. From Table 1, we see that under the null hypothesis, the actual value of T_3 is

$$T_3^{act} = \frac{0.209}{0.043} = 4.86,$$

which gives a p-value of 0.000; thus we can easily reject the null in favour of the alternative

in (24). In a similar manner we can see that the test statistic for (25) is

$$T_4 = \frac{\hat{b}_1 + \hat{c}_1 - 1}{\sqrt{SE(\hat{b}_1)^2 + SE(\hat{c}_1)^2}},$$

and under the null, $H_0 : b_1 + c_1 - 1 = 0$, it becomes

$$T_4 = \frac{\hat{b}_1 + \hat{c}_1 - 1 - b_1 - c_1 + 1}{\sqrt{SE(\hat{b}_1)^2 + SE(\hat{c}_1)^2}},$$

which is distributed as a normal random variable in large samples. Using parameter and standard error estimates from Table 1, we see that under the null hypothesis, the actual value of T_4 is

$$T_4^{act} = \frac{0.209 + 0.607 - 1}{\sqrt{0.043^2 + 0.052^2}} = -2.727,$$

which gives a p-value of 0.003. This implies that we can reject the null hypothesis in both (24) and (25). Hence, if linear specifications of the consumption, investment and tax revenue functions are good approximations of aggregate macroeconomic reality, then the US economy provides strong evidence for the existence of crowding-in of private investment expenditure by government expenditure during the period $1959 - 2009$.

7 Conclusion

The paradox of thrift refers to the phenomenon where aggregate savings in a capitalist economy declines despite efforts by consumers to reduce their consumption expenditures; crowding-in of private investment expenditure by government expenditures refer to the phenomenon whereby increases in government expenditures increases, rather than decrease,

Table 1: *Estimation Results*

	1959-2009			
	OLS		**2SLS**	
CONSUMPTION FUNCTION[a]	$\hat{c}_1 = 0.525$[d]		$\hat{c}_1 = 0.607$	
$(\Delta C_t = c_1 \Delta Y_{D,t} + c_2 \Delta Y_{D,t-1} + \varepsilon_{1t})$	(0.041)		(0.052)	
TAX REVENUE FUNCTION[b]	$\hat{t}_1 = 0.263$		$\hat{t}_1 = 0.238$	
$(\Delta T_t = t_1 \Delta Y_t + t_2 \Delta Y_{t-1} + \varepsilon_{2,t})$	(0.037)		(0.062)	
INVESTMENT FUNCTION[c]	$\hat{b}_1 = 0.381$	$\hat{b}_2 = 18.304$	$\hat{b}_1 = 0.209$	$\hat{b}_2 = 62.751$
$(\Delta I_t = b_1 \Delta Y_t + b_2 \Delta i_t + \varepsilon_{3,t})$	(0.029)	(5.957)	(0.043)	(50.173)
	1974-2009			
	OLS		**2SLS**	
CONSUMPTION FUNCTION[a]	$\hat{c}_1 = 0.512$		$\hat{c}_1 = 0.604$	
$(\Delta C_t = c_1 \Delta Y_{D,t} + c_2 \Delta Y_{D,t-1} + \varepsilon_{1t})$	(0.049)		(0.060)	
TAX REVENUE FUNCTION[b]	$\hat{t}_1 = 0.272$		$\hat{t}_1 = 0.227$	
$(\Delta T_t = t_1 \Delta Y_t + t_2 \Delta Y_{t-1} + \varepsilon_{2,t})$	(0.046)		(0.080)	
INVESTMENT FUNCTION[c]	$\hat{b}_1 = 0.390$	$\hat{b}_2 = 20.711$	$\hat{b}_1 = 0.254$	$\hat{b}_2 = 167.931$
$(\Delta I_t = b_1 \Delta Y_t + b_2 \Delta i_t + \varepsilon_{3,t})$	(0.036)	(7.189)	(0.099)	(139.930)

[a] Quarterly frequency; variables measured in billions of 2005 dollars; instrument is change in nonresidential fixed investment expenditure; summary statistics of the variables relevant for this regression is given in Table 2. The instrument relevance condition is checked using the F-statistic of the first stage regression.

[b] Quarterly frequency; variables measured in billions of 2005 dollars; instrument is change in total government defence expenditure and change in aggregate investment expenditure; summary statistics of the variables relevant for this regression is given in Table 3. The instrument relevance condition is checked using the F-statistic of the first stage regression.

[c] Quarterly frequency; variables measured in billions of 2005 chained dollars; instruments are changes in personal consumption expenditure and changes in narrow money; summary statistics of the variables relevant for this regression is given in Table 4.

[d] Hat quantities are estimates of the coefficients; standard errors appear in parentheses below the estimates.

private investment expenditures. Both these propositions are key results in the Keynesian analysis of short-run fluctuations of capitalist economies, and contrast the Keynesian from Monetarist and New Classical approaches. This paper has derived the conditions within a simple IS/LM model under which the paradox of thrift and crowding-in of private investment expenditures by government expenditures can occur; this exercise has been carried out both in the context of exogenous and endogenous money.

Using macroeconomic data from the US economy, this paper has also estimated the parameters of a set of linear behavioural equations - the consumption, investment and tax revenue function - using instrumental variables regression. These estimates were then used to test for the existence of the paradox of thrift and possibility of crowding-in of private investment expenditures. If the money supply is taken to be endogenous, as seems realistic, the estimates show that during the period $1974-2009$, there is evidence for operation of the paradox of thrift; the evidence for the paradox of thrift is not there for the period $1959-2009$. This is more or less what would be expected, given that the period since 1974 has been a period of relative stagnation of US capitalism and that the paradox of thrift is more likely to be in effect during periods of economic stagnation. When we understand the money supply process as being endogenous, the evidence also points to the strong crowding-in effect of private investment expenditure by government expenditure for the period $1959-2009$.

This analysis can be extended in two separate directions. In this paper, we have ignored the role of expectations in the determination of short-run equilibrium; this is unrealistic and one possible direction to extend the analysis in this paper is to explicitly incorporate expectations into the picture. The other direction in which the analysis could be extended is to bring in open economy considerations into the story.

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Appendix

The basic data for this analysis has been taken from two sources: the US Bureau of Economic Analysis (BEA) and the Federal Reserve Board of St. Louis.

- **Consumption function:** For estimation of the marginal propensity to consume in (19), the following three data series, at quarterly frequency, have been used: aggregate

real consumption expenditure, real disposable income and aggregate real investment expenditure; the data series runs from $1959 : Q1$ to $2009 : Q2$. The nominal consumption expenditure and nominal investment expenditure series is taken from NIPA Table 1.1.5 from the website of the U.S. Bureau of Economic Analysis (BEA); the data for the GDP deflator is taken from NIPA Table 1.1.9 of the BEA. Real quantities are obtained by dividing corresponding nominal quantities by the GDP deflator. Summary statistics and results of the augmented Dickey-Fuller unit root tests have been reported in Table 2. Results of the unit root tests show that the presence of unit roots cannot be rejected; hence the difference specification of the regression equation is justified.

- **Tax revenue function:** For estimating the marginal tax change due to a change in aggregate output in (20), the following five data series, at quarterly frequency, have been used: real tax revenue, real GDP, real investment expenditure and real defence expenditure of the government, the GDP deflator; the data series runs from $1959 : Q1$ to $2009 : Q2$. Nominal tax revenue data is taken from NIPA Table 3.1 of the BEA; nominal GDP, investment and defence expenditure data is taken from NIPA Table 1.1.5; the data for the GDP deflator is taken from NIPA Table 1.1.9 of the BEA. Real quantities are obtained by dividing corresponding nominal quantities by the GDP deflator. Summary statistics and results of the augmented Dickey-Fuller unit root tests have been reported in Table 3. Results of the unit root tests show that the presence of unit roots cannot be rejected at the 1 percent significance level for any of the variables; hence the difference specification of the regression equation is justified.

- **Investment function:** For estimation of the parameters of the investment function in (21), the following five data series, again at quarterly frequency, have been used: real

Table 2: *Consumption Function: Summary Statistics and ADF Unit Root Test (*$1959 - 2009$*)*

	DF-Stat[a]	Median	Std Dev	N
CONSUMPTION[b]	-1.751	41.39	23.33	202
	(0.681)			
DISPOSABLE INCOME	-1.145	47.67	24.43	202
	(0.913)			
INVESTMENT	-1.489	2.53	0.88	202
	(0.789)			

[a] Null hypothesis: time series has unit root; p-values in parentheses.
[b] All variables are measured in 2005 dollars.

GDP, real consumption expenditure, real investment expenditure, the AAA corporate bond yield, and the stock of narrow money (M1); the data series runs from $1959 : Q1$ to $2009 : Q2$. Data for real GDP, consumption, and investment are taken from NIPA Table 1.1.6 and are measured in 2005 chained dollars; the AAA corporate bond yield and the stock of narrow money is taken from the website of the Federal Reserve Bank of St. Louis. Summary statistics and results of the augmented Dickey-Fuller unit root tests have been reported in Table 4. Results of the unit root tests show that the presence of unit roots cannot be rejected; hence the difference specification of the regression equation is justified.

Table 3: *Tax Function: Summary Statistics and ADF Unit Root Tests (*1959 − 2009*)*

	DF-Stat[a]	Median	Std Dev	N
REAL TAX REVENUE[b]	−3.690	12.78	6.28	202
	(0.026)			
REAL GDP	−1.784	65.04	31.97	202
	(0.667)			
REAL DEFENCE EXPENDITURE	−2.297	4.09	1.03	202
	(0.452)			
REAL INVESTMENT	−2.559	11.19	5.25	202
	(0.342)			

[a] Null hypothesis: time series has unit root; p-values in parentheses.
[b] All variables measured in 2005 dollars.

Table 4: *Investment Function: Summary Statistics*[a] *and ADF Unit Root Tests (*$1959 - 2009$*)*

	DF-Stat[b]	Median	Std Dev	N
REAL GDP	-1.781	6503.95	3197.02	202
	(0.668)			
REAL CONSUMPTION	-1.721	4269.55	2297.83	202
	(0.693)			
REAL INVESTMENT	-2.069	893.95	585.60	202
	(0.547)			
AAA BOND YEILD	-1.474	7.41	2.48	202
	(0.796)			
NARROW MONEY (M1)	-1.861	532.65	448.32	202
	(0.634)			

[a] GDP, consumption and investment measured in chained 2005 dollars; M1 measured in current dollars, and bond yield measured in percentages.

[b] Null hypothesis: time series has unit root.