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Growth, instability and cycles: Harrodian and Kaleckian models of accumulation and income distribution

by

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Abstract

This paper compares Kaleckian and Harrodian models of accumulation. The simplicity of the canonical Kaleckian model is appealing but more complex Harrodian specifications are preferable from a behavioral perspective. The local instability of Harrodian-inspired specifications, moreover, offers a unified understanding of both trend and cycles.

JEL classification: E12, E32, O41

Key words: Kalecki, Harrod, Kaldor, Robinson, Steindl, investment function, stability, growth cycle, reserve army, multiple equilibria.

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1 Introduction

Post-Keynesian theory is sometimes seen as encompassing almost anything 'non-mainstream'. Following the seminal contributions by Rowthorn (1981), Dutt (1984) and Taylor (1985), however, Kaleckian models with stable steady-growth paths have come to dominate post-Keynesian and structuralist macroeconomics. These models are characterized by a low sensitivity of accumulation to variations in utilization, and with a given markup, the utilization rate becomes an accommodating variable in both the short and the long run. Thus, the steady-growth value of the utilization rate is not, as in Harrodian or Robinsonian models, tied to a structurally determined desired rate. Instead, shocks to demand (changes in saving rates, for instance) can have large, permanent effects on utilization.

A substantial literature discusses the long-run relation between actual and desired utilization rates. Kurz (1986), Committeri (1986), Dumenil and Levy (1993), and Auerbach and Skott (1988) are among those who have faulted Kaleckian models for their failure to ensure that actual utilization and desired utilization coincide in steady growth.[1] A Kaleckian response has been articulated by Lavoie (1995, 1996), Amadeo (1986), Dutt (1997), and Lavoie et al. (2004). I find the Kaleckian response unconvincing (see Skott 2008 for details), and in this paper I shall argue that an alternative Harrodian approach is both promising and analytically tractable. The paper goes over some of the same ground as Lavoie's interesting and influential 1995-article, but the conclusions are rather different.

Harrodian models are more complex than the standard Kaleckian formulation. They require a distinction between short-run and long-run accumulation functions and may generate unstable 'warranted growth paths'. Despite these complexities, the analysis remains tractable and the complexities bring significant rewards. The Harrodian assumptions, first, can be given clear behavioral justifications. The Kaleckian stability condition, by contrast, is usually introduced for instrumental reasons to ensure stability, stability being seen (implicitly but mistakenly) as imperative for the real-world relevance of the model. Harrodian investment functions, second, can be compatible with multiple steady-growth solutions, some of which may be stable, and the existence of multiple solutions carries interesting implications. The (local) instability of a warranted growth path, third, quite naturally leads to an integration of growth and cycles. As emphasized by Frisch, Slutsky and Kalecki in the 1930s and 1940s as well as by most contemporary theories of the business cycle, stochastic shocks may play a role in the generation of cyclical movements. But the presence of shocks does not exclude endogenous mechanisms, and Harrodian instability provides a powerful foundation for endogenous cycles.[2]

Section 2 outlines a basic Kaleckian model. A Harrodian perspective is presented in section 3. Drawing on Skott (1989, 1989a) and Nakatani and Skott (2007), section 4 analyses a Kaldor/Marshall version of the Harrodian model. Two different cases are

[1] The desired rate of utilization is sometimes referred to as the 'normal' rate or the 'target' rate.

[2] Other mechanisms may play a role as well. An example is endogenous, Minsky-type changes in financial behavior.

considered: a 'dual-economy' case in which the labor supply is perfectly elastic and the growth of the economy can be determined without any reference to the labor market, and a 'mature' economy in which the labor supply limits the long-run rate of growth. The relaxation of the standard Kaleckian assumption of a fixed markup is a key element in the analysis of both dual-economy and mature cases. In the Kaldor/Marshall version, the fixed markup is replaced by fast, demand-determined adjustments in the profit share and sluggish movements of output. An alternative Robinson/Steindl version assumes sluggish adjustments in prices and the profit share but fast output adjustments. This version is considered in Section 5 which draws on Skott (2005) and Flaschel and Skott (2006). Section 6 contains a few concluding comments.

2 A Kaleckian benchmark model

Kaleckian models have been extended and modified in many ways. Some extensions have introduced a government sector and an explicit analysis of policy issues (e.g. Lima and Setterfield (2008)); others add financial variables or open-economy complications (e.g. Blecker 1989, 1999; Lavoie and Godley 2001-02, Dos Santos and Zezza (2008), Hein and van Treeck 2007). For present purposes, however, a stripped-down model of a closed economy without public sector and without financial constraints on investment will suffice.

Algebraically, the canonical Kaleckian model is exceedingly simple:

$$\frac{I}{K} = \alpha + \beta u + \gamma r \tag{1}$$

$$\frac{S}{K} = s(\pi) u \sigma \tag{2}$$

$$\frac{I}{K} = \frac{S}{K} \tag{3}$$

$$r = \pi u \sigma \tag{4}$$

$$\pi = \bar{\pi} \tag{5}$$

$$g = \hat{K} = \frac{I}{K} - \delta \tag{6}$$

Using standard notation, equations (1)-(2) are the investment and saving functions. Investment is increasing in utilization (u) and the profit rate (r), and the saving rate out of income ($s(\pi)$) is an increasing function of the profit share (π); σ denotes the technical output-capital ratio. Equation (3) is the equilibrium condition for the product market; equation (4) defines the profit rate as the product of the profit share, the utilization rate and the technical output-capital ratio. Equation (5) is the pricing equation with the profit share fixed by a markup on marginal cost, the latter assumed constant and equal to unit labor cost. Equation (6) sets the growth rate of the capital stock ($g = \hat{K}$) equal to gross accumulation minus the rate of depreciation, δ. All parameters are assumed positive and the Keynesian stability condition is supposed to hold,

$$\frac{\partial(I/K)}{\partial u} = \beta + \gamma\bar{\pi}\sigma < s(\bar{\pi})\sigma = \frac{\partial(S/K)}{\partial u} \tag{7}$$

Simple manipulations of equations (1)-(6) imply that

$$u^* = \frac{\alpha}{s(\bar{\pi})\sigma - \beta - \gamma\bar{\pi}\sigma} \tag{8}$$

$$g^* = \frac{\alpha s(\bar{\pi})\sigma}{s(\bar{\pi})\sigma - \beta - \gamma\bar{\pi}\sigma} - \delta \tag{9}$$

It is readily seen that if the saving function is linear ($s(\pi) = s\pi$), the stability condition (7) implies that

$$\frac{\partial u^*}{\partial \bar{\pi}} < 0 \tag{10}$$

$$\frac{\partial g^*}{\partial \bar{\pi}} < 0 \tag{11}$$

Thus, the economy is both 'stagnationist' (equation (10)) and 'wage led' (equation (11)) in the terminology of Marglin and Bhaduri (1990).[3]

Marglin and Bhaduri challenged these implications of the model and suggested that the investment function be recast with accumulation as a function of utilization and the profit share, rather than utilization and the profit rate,

$$\frac{I}{K} = \alpha + \beta u + \gamma\pi \tag{12}$$

Using this alternative specification of the investment function, they showed that the Keynesian stability condition need not produce stagnationist and wage-led regimes. The utilization rate remains an accommodating variable, however, and the main difference between the investment functions (1) and (12) is that the sensitivity of investment to changes in utilization has been *reduced,* relative to the sensitivity with respect to the profit share. The non-stagnationist outcomes become possible precisely because, using (12) instead of (1), we may have $\frac{\partial(I/K)}{\partial\bar{\pi}} > \frac{\partial(S/K)}{\partial\bar{\pi}}$, even when the Keynesian stability condition is satisfied, something that cannot occur when the investment function is given by (1) and the saving function is linear ($s(\pi) = s\pi$). Equivalently, equation (12) does not exclude the possibility that, holding constant the rate of profit, an increase in utilization may reduce accumulation. This is in sharp contrast to Harrodian formulations. Thus, although both the Marglin-Bhaduri formulation and the Harrodian models below may produce profit-led

[3] The canonical model need not be stagnationist if the saving function is nonlinear (or just affine, $s(\pi) = s_0 + s\pi$ with $s_0 > 0$) since in this case the 'Robinsonian stability' condition ($\frac{\partial I}{\partial \pi} < \frac{\partial S}{\partial \pi}$) can be violated even if the 'Keynesian stability' condition ($\frac{\partial I}{\partial u} < \frac{\partial S}{\partial u}$) is met. This point, which may have been noted in the literature, was made by Ben Zipperer in comments on an early draft of this paper.

outcomes, the behavioral assumptions are very different, and from a Harrodian perspective the Marglin-Bhaduri specification suffers from the same problems as the original Kaleckian model.

To simplify the exposition I shall set γ equal to zero. In this special case, the two investment functions (1) and (12) coincide, the Keynesian stability condition can be written $s(\bar{\pi})\sigma > \beta$, and the equilibrium solutions for u^* and g^* take the form

$$u^* = \frac{\alpha}{s(\bar{\pi})\sigma - \beta} \tag{13}$$

$$g^* = \frac{\alpha s(\bar{\pi})\sigma}{s(\bar{\pi})\sigma - \beta} - \delta \tag{14}$$

The model is illustrated graphically in Figure 1. Unlike most illustrations, which focus on the qualitative properties, figure 1 is based on Kaleckian benchmark values. Empirically, the gross saving rate $s(\pi)$ typically falls in the range 0.15-0.3 and the technical output-capital ratio in the range 1-3. Figure 1 uses $s(\pi)\sigma = 0.12, b = 0.08$ and $a = 0.03$, yielding an equilibrium utilization rate of $u^* = a/(s(\bar{\pi})\sigma - b) = 0.75$.

Figure 1 about here

Figure 1 and the numerical example illustrate one of the main weaknesses of the Kaleckian analysis. Assume that the saving rate drops slightly, with $s(\bar{\pi})\sigma$ falling from 0.12 to 0.11. As a result, the growth rate increases by 2 percentage points while the utilization rate jumps from 75% to 100%. This strong sensitivity of utilization to variations in parameters is an intrinsic property of the Kaleckian model. For any reasonable specification of the saving function, the Kaleckian stability condition puts a very low ceiling on the maximum value of b (about 0.1). Shocks to the saving function therefore give rise to fluctuations in utilization rates that are at least about ten times larger than those in accumulation. Shocks to the accumulation function (changes in a) produce movements along the saving function and (given the stability condition) the ratio of variations in utilization to variations in the growth rate is slightly larger, but still unlikely to be much below ten. These implications do not fit the data. Utilization rates are difficult to measure, but existing data suggest modest long-run variations. As shown in figure 2a, utilization rates for US manufacturing industry fluctuate significantly in the short run (as one would expect) but the long-run trend is quite flat, and the ratio of long-run variations in utilization to long-run variations in growth is nowhere near the values suggested by the Kaleckian model (figure 2b gives growth rates of capital capacity in US manufacturing).

Figures 2a-2b about here

From a theoretical perspective the problems with the Kaleckian specifications arise from the combination of an exogenous markup with the extension to the long run of a standard, Keynesian short-run stability condition: the relative insensitivity of investment to variations in aggregate demand.[4] A Harrodian approach addresses these issues.

3 A Harrodian alternative

A Harrodian specification of the investment function makes a distinction between the short-run and the long-run sensitivity of investment to changes in aggregate demand. The insensitivity of investment is plausible in the short run, but changes in aggregate demand have lagged effects on investment, and a weak impact effect (which is required for the stability of the short-run Keynesian equilibrium) does not guarantee that the long-term effects of a sustained increase in aggregate demand and utilization will be weak as well.

In a discrete-time framework (and still assuming, for simplicity, that only utilization matters for investment), the presence of lags can be captured by a general specification,

$$(\frac{I}{K})_t = f(u_t, u_{t-1}, ..., u_{t-m}, (\frac{I}{K})_{t-1}, (\frac{I}{K})_{t-2}, ..., (\frac{I}{K})_{t-n}) \tag{15}$$

The short-run effect of utilization on accumulation is given by the partial derivative $\partial f/\partial u_t$, and the Keynesian stability condition can be written

$$s(\bar{\pi})\sigma > \frac{\partial f}{\partial u_t} \tag{16}$$

The long-run effect of changes in utilization, on the other hand, is given by

$$\hat{K} = \frac{I}{K} - \delta = \phi(u) \tag{17}$$

with

$$\phi'(u) = \frac{d\frac{I}{K}}{du}_{|u_t = u_{t-j}, \frac{I}{K}_t = \frac{I}{K}_{t-k}} = \frac{\sum_{i=0}^{m} \frac{\partial f_{u_{t-i}}}{\partial u_{t-i}}}{1 - \sum_{j=1}^{n} \frac{\partial f_{\frac{I}{K}_{t-j}}}{\partial \frac{I}{K}_{t-j}}} \tag{18}$$

The short-run condition (16) carries no implications for the relation between the long-run sensitivity, ϕ', and $s(\pi)\sigma$.

The significance of the distinction between short-run and long-run specifications depends on the magnitude of the lagged effects. According to Harrod the lagged effects are large and $\phi'(u) >> s(\bar{\pi})\sigma$. This condition is satisfied by the following special case of (15):

$$(\frac{I}{K})_t = \lambda(u_t - u^d) + (\frac{I}{K})_{t-1} \tag{19}$$

[4] Skott (2008) discusses the theoretical and empirical case against the Kaleckian investment function in greater detail.

or, in continuous time,

$$\dot{g} = \frac{d}{dt}\hat{K} = \lambda(u - u^d) \tag{20}$$

where u^d is the desired rate of utilization. The standard Harrodian specification in equation (20) implies that the accumulation rate becomes a state variable and that there is no immediate impact of changes in utilization on investment. In the long run, by contrast, accumulation is perfectly elastic: utilization must be at the desired rate in steady growth, but as long as this condition is satisfied, the accumulation function imposes no constraints on the growth rate. Thus, the specification (20) implies a particularly simple (even if unconventional) steady-growth accumulation function:

$$u = u^d \tag{21}$$

Equation (21) is a special case of (17) with $\phi' = \infty$ at $u = u^d$.

The behavioral story behind the Harrodian specification is quite straightforward. Firms have a well-defined objective (to maximize profits) and this objective implies a desired utilization rate. Since capital stocks adjust slowly and demand expectations are not always met, actual utilization may deviate from desired rates in the short run. It would be unreasonable, however, to assume that demand expectations can be persistently and systematically falsified in steady growth. Consequently, it is hard to conceive of a steady-growth scenario in which firms are content to accumulate at a constant rate despite having significantly more (or less) excess capacity than they desire. From a behavioral perspective the only real question concerns the determination of the desired rate of utilization.[5]

The desired utilization rate may deviate from unity. A firm may want to hold excess capacity to deter entry or to enable the firm to respond quickly to variations in demand; or excess capacity may exist simply as a result of indivisibilities of investment (non-convexities in adjustment costs). The desired degree of excess capacity, second, need not be constant over time; changes in the degree of product market competition or in the volatility of demand, for instance, could affect desired utilization rates. Managerial constraints or other bottlenecks, third, may make it difficult or costly to expand capacity at a rapid pace, and the desired utilization rate, consequently, may depend, inter alia, on the rate of accumulation. This case can be represented by equation (17) which specifies a long-run relation between accumulation and desired utilization. If the long-run accumulation function is given by (17) with $0 < \phi' < \infty$, the counterpart to (20) is

$$\dot{g} = \lambda(u - \phi^{-1}(g)) \tag{22}$$

Using (22) instead of the Kaleckian investment function (1), the steady growth solutions for u and g are determined by

$$g^* = \phi(u^*) = s(\bar{\pi})u^*\sigma - \delta \tag{23}$$

[5] Chick and Caserta (1997) suggest that although the utilization rate must be at (or near) the desired rate in long-run steady growth, deviations could last for significant periods of time. Long-lasting deviations, however, do not justify a depiction of this medium-run scenario as a self-sustaining equilibrium without internal forces for change.

and the economy is profit led in the long run: by assumption $\phi' > s(\bar{\pi})\sigma$ and hence

$$\frac{du^*}{d\bar{\pi}} = \frac{s'(\bar{\pi})\sigma u^*}{\phi'(u^*) - s(\bar{\pi})\sigma} > 0 \tag{24}$$

A Harrodian steady-growth path, however, may be unstable. This, indeed, is the case with the simple model based on (2)-(3) and (22). The accumulation rate is predetermined at any moment and the short-run Keynesian equilibrium is stable, but the trajectory of Keynesian equilibria does not converge to the steady-growth path. Combining (2)-(3) and (22), we get a one-dimensional differential equation with an unstable stationary solution:[6]

$$\dot{g} = \lambda[\frac{g+\delta}{s(\bar{\pi})\sigma} - \phi^{-1}(g)] \tag{25}$$

and (since$\phi' > s(\bar{\pi})\sigma$)

$$\frac{d\dot{g}}{dg} = \lambda[\frac{1}{s(\bar{\pi})\sigma} - \frac{1}{\phi'(g)}] > 0 \tag{26}$$

The instability of a Harrodian warranted growth path has been viewed as a powerful argument against this approach. The argument may not be spelled out in any detail but it is suggested, implicitly, that stability is needed for the model to make sense and/or for the properties of the steady-growth path to be empirically relevant (e.g. Lavoie 1995, p. 794). There are several possible answers to these implicit claims. As argued in sections 4.1 and 5.1, stability may be achieved without abandoning a Harrodian investment function if the fixed markup is abandoned. More importantly, perhaps, the steady growth path may be relevant even in the absence of asymptotic stability. Local instability is consistent with endogenously generated, bounded fluctuations around a steady-growth solution, and an unstable steady-growth path may provide a good approximation to average outcomes in the medium to long run.[7] Sections 4.2 and 5.2 consider how boundedness may be generated by a Marxian employment effect, but the general argument clearly does not depend on this particular mechanism.

4 Harrodian instability: a Kaldor/Marshall analysis

Kaldorian models from the 1950s and early 1960s include endogenous adjustments in the profit share. Since the profit share is determined by the pricing equation, this calls for a reconsideration of firms' price and output decisions.

[6] The instability of the 'warranted growth path' was emphasized by Harrod himself although he rejected the knife-edge metaphor (Harrod 1973, p. 33).

[7] Using the simple Harrodian specification in (20), it is readily seen that if the fluctuations in $\hat{K}$ are bounded, the time-average of the utilization rate ratio u must be approximately equal to u^d when the average is taken over a long period. To show this, integrate (20) to get $\bar{u} - u^d = \frac{\hat{K}_{t_1} - \hat{K}_{t_0}}{\lambda(t_1 - t_0)}$ where $\bar{u}$ is the average utilization rate over the interval $[t_0, t_1]$. If $\mid \hat{K}_{t_1} - \hat{K}_{t_0} \mid$ is bounded below some constant for all (t_0, t_1), it follows that $\bar{u}$ converges to u^* for $t_1 - t_0$ going to infinity.

In the Keynesian literature - both old and new - it is often assumed that firms set prices and that output adjusts instantaneously and costlessly to match demand. The empirical evidence in favour of significant price rigidity is quite weak, however,[8] and output does not adjust instantaneously. Production is subject to a production lag, and increases in production and employment typically give rise to substantial search, hiring and training costs; firing or layoffs also involve costs, both explicit costs like redundancy payments and hidden costs in the form of deteriorating industrial relations and morale. Based on these considerations, a Kaldor/Marshall approach assumes fast price adjustments and sluggish output movements: shocks to aggregate demand are accommodated initially by movements in prices and profit shares, rather than in output and utilization.

In a continuous-time setting the effects of lags and adjustment costs for output can be approximated by assuming that output is predetermined at each moment and that firms choose the rate of growth of output, rather than the level of output. If firms maximize profits (or pursue some other well-defined objectives), the growth of output is chosen so as to balance the costs of changes against the benefits of moving toward a preferred level of output and employment; (expected) costs and benefits, in turn, are determined by the demand and cost signals that firms receive from product and labor markets.

4.1 A dual economy

Consider first a dual economy in which there is a perfectly elastic supply of labor to the capitalist sector. Endogenous changes in the cost signal from input markets may be ignored in this kind of economy. A perfectly elastic labor supply, to be sure, does not rule out shifts in the perceived costs of changes in output. Exogenous shifts in worker militancy, for instance, may affect these perceived costs, but the dual-economy assumption implies that labor market conditions do not change endogenously as a result of firms' output and investment decisions.

The demand signal from product markets, by contrast, is endogenously determined. If prices are fully flexible, this signal can be captured by the prevailing profit share. By assumption the level of output is predetermined, and a rise in demand leads to an increase in the price of output. Wage contracts are cast in terms of money wages, and there is neither perfect foresight nor instantaneous feedbacks from output prices to money-wage rates. The real wage rate and the share of profits in income therefore respond to unanticipated movements in prices: a positive demand shock generates a rise in the profit

[8] The study by Levy et al. (1997) of menu costs in five supermarkets, for instance, is often cited in support of menu costs and price stickiness (e.g. Romer 2001, pp. 315-316). This study found that on average 16 percent of all prices were changed each *week*. These frequent changes in prices were not costless but the finding that menu costs constitute a significant proportion of net profits is largely irrelevant for an evaluation of price flexibility. With prohibitively high menu costs, for instance, there would be no price changes and the share of menu cost in revenue would be zero; negligible menu costs on the other hand may allow firms to change prices frequently as part of their marketing strategies, and the observed share of menu costs in net profits could be very high in this case.

share, and firms respond to this rise by increasing the growth rate of output.[9]

Algebraically, we get a generic growth function[10]

$$\hat{Y} = h(\pi); h_\pi > 0 \tag{27}$$

The growth function (27) replaces the pricing equation (5) and may, as the pricing equation, be influenced by the sectoral composition of the economy and the degree of competition in the product markets. In general, the function is likely to be highly non-linear. It seems reasonable to suppose that the adjustment costs for output are convex as a function of $\hat{Y}$, and there may also be upper and lower limits on the rate of growth, $g^{\min} \leq \hat{Y} \leq g^{\max}$. Thus, the growth rate will be more sensitive to variations in the profit share for intermediate values of the profit share than for very high or very low values.

In a Kaldor/Marshall model it must be possible to accommodate aggregate demand shocks through variations in prices and the profit share. This condition is satisfied here since a rise in the profit share raises aggregate saving and reduces excess demand, as in Keynes (1930) and Kaldor (1956). Using a linear version of equation (2),

$$\frac{S}{K} = s\pi u\sigma \tag{28}$$

the equilibrium condition for the product market yields the following solution for the profit share

$$\pi = \frac{g + \delta}{su\sigma} \tag{29}$$

where both $g = \hat{K}$ and u are predetermined, given a Harrodian investment function and sluggish output adjustment.

In order to close the model, equations (28)-(29) need to be combined with a specification of the accumulation function. Consider first the standard specification in equation (20). Using the saving function (28), the steady-growth condition $u = u^d$ (implied by (20)), and the equilibrium condition for the product market, the set of steady-growth solutions for (π, g) is characterized by

$$\begin{aligned} h(\pi^*) &= su^d\sigma\pi^* - \delta & (30) \\ g^* &= h(\pi^*) & (31) \end{aligned}$$

The non-linearity of the h-function implies that there may be multiple steady-growth solutions, as in Figure 3b. Outcomes with a unique solution are also possible (Figures 3a and 3c), and a case without steady-growth solutions can be obtained when the lower limit

[9] Demand signals could also be reflected in inventories. For the aggregate economy, however, changes in inventories tend to amplify fluctuations in other demand components over the cycle. Thus, the need for price adjustments would remain, even if inventories were included.

[10] The behavioral foundations of the function are discussed in greater detail by Skott (1989, chapter 4), who used the term 'output expansion function'.

on $\hat{Y}$ is abandoned ($g^{\min} = -\infty$); this case is illustrated in Figure 3d. Essentially, the cases in 3a and 3d are identical since with negative growth rates, there can be capitalist development in neither case.

Figures 3a-3d about here

Figure 3b represents the most interesting case. At the two extreme equilibria we have $h' < s\sigma u^d$; at the intermediate equilibrium this inequality is reversed. Not surprisingly, the inequality is closely related to stability conditions. The profit share at any moment is given by equation (29), and substituting (29) into the growth function (27), we get an equation of motion for the utilization rate

$$\hat{u} = \hat{Y} - \hat{K} = h(\frac{g+\delta}{su\sigma}) - g \tag{32}$$

Equations (20) and (32) define a two-dimensional system of differential equations. Evaluated at a stationary point, the Jacobian of the system is given by

$$J(g,u) = \begin{bmatrix} 0 & \lambda \\ u(\frac{h'}{s\sigma u} - 1) & -uh'\frac{g+\delta}{s\sigma u^2} \end{bmatrix} \tag{33}$$

and

$$\begin{aligned} \mathrm{tr}(J) &= -uh'\frac{g+\delta}{s\sigma u} < 0 \\ \det(J) &= -\lambda u(\frac{h'}{s\sigma u} - 1) > 0 \text{ iff } h' < s\sigma \end{aligned}$$

It follows that a steady-growth path is locally asymptotically stable if and only if $h'(\pi^*) < su^d\sigma$. The stability condition is satisfied at the two extreme solutions in figure 3b; the intermediate solution on the other hand will be unstable.

Similar results can be obtained if investment is described by the static equation (17). At first sight, this may seem a peculiar accumulation function in a Harrodian analysis but since utilization is treated as a state variable, the seemingly static specification (17) embodies the main Harrodian principle.[11] By assumption the impact effect of changes in aggregate demand falls entirely on prices and the profit share, and the insensitivity of investment to short-run fluctuations in demand is satisfied by (17); a strong long-run sensitivity follows if $\phi'(u)$ is 'large'. Using (17) and (27)-(28), the steady-growth conditions are given by

$$g^* = h(\pi^*) = s\sigma u^*\pi^* - \delta = \phi(u^*) \tag{34}$$

[11] The distinction between short- and long-run effects is observed as long as accumulation depends on a state variable. In the Robinsonian model below, utilization adjusts instantaneously but the profit share becomes a state variable.

These equations can be described using a modified figure 3; the only difference is that the IS-curve (the solutions to the last equation in (34) for given π) will now be non-linear in a (π, g)-space; see figure 4 which corresponds to 3b.[12] This specification of the model produces a one-dimensional dynamic system

$$\hat{u} = h(\frac{\phi(u) + \delta}{su\sigma}) - \phi(u) \tag{35}$$

and local stability, again, is achieved at the two extreme solutions.[13]

Figure 4 about here

The above analysis of a dual economy has several noteworthy implications. The existence of multiple steady-growth paths, first, implies that countries that initially seem quite similar may follow very different growth trajectories and that temporary aggregate demand policy may raise the long-run rate of growth. Suppose, for instance, that initially an economy is at the low growth path in figure 3b (an analogous argument applies to the specifications underlying figure 4). Using a trivial extension of the model to include a government sector, expansionary policy can reduce the average saving rate. The result is a rise in the profit share for any given growth rate or, equivalently, a downward shift in the IS-curve (the $g = s\pi u^d\sigma - \delta$ line in figure 3b). If the shift is large enough, the new configuration will be as in figure 3c, and a move to the high steady-growth equilibrium may get under way. Once at the high-growth path, the expansionary policy is no longer needed. Following a return to the old saving rate, the economy may now grow at the rate associated with the high solution.

Shifts in the h- or s-functions or in the desired utilization rate u^d (more generally, in the accumulation function ϕ), second, have permanent growth effects. An increase in animal spirits, for instance, may be reflected in an upward shift in the h-function (an increase in the growth of output for any given profit share) and/or a fall in the desired utilization rate (corresponding to an upward shift in the investment function). At a stable

[12] Assuming a linear accumulation function,

$$g = \phi(u) = \mu(u - u_0)$$

the equilibrium condition $I = S$ implies

$$g = s\sigma\pi\frac{\mu u_0 - \delta}{\mu - s\sigma\pi} - \delta$$

[13] Local stability requires

$$h'\ [\frac{\phi' - \pi s\sigma}{su\sigma}] - \phi' < 0$$

or, equivalently,

$$h' < \frac{dg^*}{d\pi}$$

where g^* is the growth rate that clears the product market for a given profit share, π.

growth path, not surprisingly, these shifts are unambiguously expansionary. A downward shift in the s-function also raises the steady-growth solutions for both π and $\hat{Y}$ if the initial position is at a stable steady-growth path. Since the profit share is endogenous, there is no direct counterpart to the stagnationist Kaleckian 'paradox of cost' but an increase in the concentration rate and decline in competition will be associated with a downward shift in the growth function and, starting from a stable growth path, a decline in the growth rate.

The high steady-growth solution may have empirical counterparts in the experience of successful developing countries, including Japan, Korea and China during their years of miracle growth (it should be noted in this context that the average growth rate for a successful developing economy with a large reserve of hidden unemployment understates the growth of the modern, capitalist sector). Empirical counterparts to the low-growth trap are not hard to find either, and the Japanese stagnation since about 1990 and its relation to the present framework are discussed in Nakatani and Skott (2007). But established industrialized countries without significant reserves of hidden unemployment and with relatively stable growth rates in the 1-5% range fit neither the low nor the high equilibrium. The intermediate solution might seem more promising, but the dual-economy assumption is questionable for these economies and the model needs to be modified.

4.2 The reserve army of labor

Many writers (including Steindl (1952), Kaldor (1966, 1978) and Marglin (1984)) have regarded capitalist accumulation as essentially unconstrained by the growth of the labor force, a position that is reflected also in the canonical Kaleckian model. This dual-economy assumption is reasonable for many LDCs and NICs where the existence of hidden unemployment makes the rate of open unemployment largely irrelevant as an indicator of conditions in the labor market. In most OECD countries, however, measured employment provides important information about the state of the labor market, and the growth function in section 4.1 needs to be extended: the cost of output variations can no longer be taken as independent of the employment rate.[14]

The employment rate influences the costs of changing output through its effects on the availability of labor with the desired qualifications. Labor markets are not perfectly competitive and it is harder for a firm to attract and retain workers when unemployment is low. Thus, high employment rates increase the costs of recruitment and since the quit rate tends to rise when labor markets are tight, the gross recruitment needs associated with

[14] A dual-economy scenario fits the OECD countries at an earlier stage of their development. Kaldor's rejection in the mid 1960s of his own labor-constrained models should be seen in the context of agricultural employment shares that were still above 25 percent in countries like Japan and Italy and at or above 20 percent in France; West Germany had a smaller share (just over 10 percent) but had been experiencing massive immigration in the 1950s (Kuznets (1971).

Arguably, the assumption still applies to the world economy as a whole, but a one-sector model of the world economy without spatial disaggregation has obvious limitations.

any given rate of expansion increase at a time when low unemployment makes it difficult to attract new workers. A high turnover of the labor force, on the other hand, allows firms to reduce production and employment more rapidly without large adjustment costs when the employment rate is high. These standard microeconomic effects may be reinforced by broader Marxian effects on the social relations of production. A high rate of employment strengthens workers vis-a-vis management. This shift in the balance of power may lead to increased worker militancy, and increased monitoring and additional managerial input may also be needed in order to maintain discipline and prevent shirking. As noted by Kalecki (1943), high employment is bad for business because "the self assurance and class consciousness of the working class" will grow and "the social position of the boss" will be undermined (quoted from Kalecki (1971, p. 140-1). Overall, one would expect the general deterioration of the business climate associated with high employment rates to put a damper on firms' expansion plans.

These considerations suggest a reformulation of the growth function for a 'mature economy': the growth of production now responds to signals from both goods and labor markets. Other input or cost signals could play a role but for simplicity intermediate inputs are left out and firms typically maintain excess capital capacity. As far as production decisions are concerned, the labor market therefore provides the relevant signal, and the employment rate is used as the indicator of the state of the labor market. Thus, the growth function for a mature economy includes two arguments, the profit share (π) and the employment rate (e):[15]

$$\hat{Y} = h(\pi, e); h_\pi > 0, h_e < 0. \tag{36}$$

As argued above, the key element in the Harrodian approach is the distinction between a small short-run and large long-run sensitivity of investment to variations in aggregate demand and with utilization as a state variable, this distinction can be captured by a static relation between the accumulation rate and the rate of utilization:

$$\hat{K} = \phi(u) \tag{37}$$

where ϕ describes the relation between accumulation and desired utilization, and $\phi' >> s\sigma\pi$.

Using (36)-(37) we have the following two-dimensional system:

$$\hat{u} = \hat{Y} - \hat{K} = h(\pi, e) - \phi(u) \tag{38}$$

$$\hat{e} = h(\pi, e) - n \tag{39}$$

[15] A static counterpart to this equation can be obtained by setting $\hat{Y} = 0$. The equation then defines the profit share as an increasing function of the employment rate. A short-run equilibrium relation of this kind could be derived from profit maximization if firms have monopsony power and the (perceived) elasticity of labor supply to the individual firm is decreasing as a function of the aggregate rate of employment. Manning (2003) provides an extended analysis of monopsonistic features of the labor market.

where n is the growth rate of the labor force. For simplicity I take n as exogenous; a straightforward extension allows n to depend positively on the employment rate e.[16] Retaining the linear saving function (28) and using a Kaldor/Marshall approach, the profit share is still determined by the equilibrium condition for the product market, as in (29)

$$\pi = \frac{\phi(u) + \delta}{s\sigma u} = \psi(u) \tag{40}$$

The strong long-run sensitivity of accumulation to variations in utilization ($\phi' > s\sigma\pi$) implies that $\psi' > 0$.

A stationary solution satisfies $\hat{u} = \hat{e} = 0$, and it follows that $\phi(u) = n$. With $\phi(u) = n$, equation (40) determines a unique value of π,

$$\pi^* = \frac{n + \delta}{s\sigma\phi^{-1}(n)} \tag{41}$$

Substituting this value into the growth function, there is at most one steady-growth solution for e. A solution in the admissible range ($0 \leq e \leq 1$) exists if and only if

$$h(\pi^*, 0) \geq n \geq h(\pi^*, 1) \tag{42}$$

The second inequality in (41) must be satisfied: as e increases it becomes progressively more difficult to expand employment, and if $e = 1$ it is logically impossible for the rate of growth of employment to exceed the rate of growth of the labor force. The first inequality, however, need not be satisfied: firms may be insufficiently dynamic and, as a result, a capitalist economy may not be capable of growth at the natural rate. The likelihood of this outcome increases if π^* is small, that is, for low values of the natural rate and high saving rates. As argued by Nakatani and Skott (2007), Japan's stagnation since about 1990 may be related to structural demand problems of this kind: with the exhaustion of hidden unemployment, the growth rate had to come down, but a high saving rate and low natural growth rate precluded a smooth transition to a path with minor fluctuations around a new steady-growth solution with $g = n$.

Assuming the existence of a steady-growth solution, the local stability is determined by the Jacobian,

$$J(u, e) = \begin{bmatrix} u[h_\pi\psi' - \phi'] & uh_e \\ eh_\pi\psi' & eh_e \end{bmatrix} \tag{43}$$

[16] High employment rates may stimulate the growth of the labor force in several ways. Immigration is an obvious mechanism in open economies; for a closed economy, changes in participation rates may affect the growth of the labor force in the medium run, and high employment and incipient labor shortages may serve as incentives for labor saving innovation in the long run. The argument could be formalized by assuming that $n = n(e), n'(e) \geq 0$.

with

$$\begin{aligned}\det(J) &= -ue\phi' h_e > 0 \\ \mathrm{tr}(J) &= u[h_\pi \psi' - \phi'] + eh_e\end{aligned}$$

The determinant is unambiguously positive and the trace must become negative if the employment effect is sufficiently strong. An outcome with a negative trace may require employment effects that are implausibly strong. A weaker employment effect, however, is sufficient to generate a stable limit cycle and bounded fluctuations around the locally unstable, stationary solution (see Skott (1989, 1989a)). The negative feedback effect from employment to the growth rate of output mirrors the homeostatic mechanism in Goodwin's (1967) formalization of a Marxian growth cycle. Goodwin's model excludes Keynesian effective demand problems, but the same basic feedback effects tend to stabilize the Harrodian system.

The phase diagram in figure 5 illustrates the dynamics. The model produces clockwise movements in an (e, u)-space (or equivalently, since $\pi = \psi(u)$, in (e, π)-space). The predicted movements in employment, utilization and profitability are broadly consistent with the stylized facts, and the marriage of destabilizing Harrodian effects with stabilizing Marxian mechanisms provides a unified explanation of growth and cycles.

Figure 5 about here

The boundedness of the fluctuations implies that the (locally) unstable steady-growth solution becomes relevant for the long-run effects of changes in parameters and exogenous variables. The average values of e, u and π in the long run need not be exactly equal to the steady-growth solutions, but the comparative statics of the steady-growth solution will give a good approximation to changes in the average values.[17] Using the steady-growth conditions, it is readily seen that improved animal spirits (an upward shift in the accumulation and /or growth function) will be expansionary. But since the growth rate is pinned down by the growth of the labor force, there is only a level effect: the employment rate goes up following a rise in animal spirits, as does the profit share if the accumulation function shifts up.[18] Analogously, a decline in the saving rate raises both the profit share and the rate of employment. An increase in labor militancy will be reflected in a downward shift in the growth function and, as in the Goodwin model, the result is a decline in the steady-growth value of the employment rate.

[17] The results will be biased only insofar as changes in a parameter affects the magnitude of the deviation between steady-growth solution and time-average. The existence of an unchanged deviation between the two generates no errors.

[18] The absence of a well-defined *NAIRU* is standard in post Keynesian and structuralist theory. My own take on this issue is discussed in Skott (1999, 2005a).

5 A Robinson/Steindl approach

Essentially, the Harrodian instability is curtailed in section 4 by abandoning the instantaneous output adjustments at a given markup and, in the mature economy, by variations in the reserve army of labor. I have referred to the models as Kaldorian or Marshallian since demand-determined variations in prices and income distribution are at the heart of the analysis, but the analysis in section 4 also has affinities with the work of Robinson (1956, 1962) and Steindl (1952).[19]

5.1 Dual economies

Robinson set up models with multiple steady-growth paths. The utilization rate is at the desired rate in these models but the mechanism is different than the one in section 4. Accumulation is a non-linear function of profitability while price competition, she suggested, keeps utilization at the desired rate.[20]

Her verbal argument (1962, p. 47) implies that the accumulation function takes the form

$$g = \frac{I}{K} = f(r^e) \tag{44}$$

where r^e is the expected future rate of profit on new investment and $f' > 0$ Retaining the linear saving function (28), the current rate of profit is determined by the market-clearing condition for the product market,

$$s\sigma u\pi = sr = g \tag{45}$$

In steady growth we have $r^e = r$, and assuming that the investment function f is strictly concave, the well-known 'banana diagram' emerges with two steady-growth solutions.

The stability properties of these steady-growth solutions depend on the formation of profit expectations, and most of Robinson's analysis seems to rely on static expectations. Under conditions of imperfect competition, however, firms' expected profit rate, r^e, cannot be independent of their investment decisions. Thus, implicitly, the specification in equation (44) seems to assume perfect competition. This assumption is logically consistent but unattractive, both theoretically and empirically, and Robinson acknowledges as much. She notes that "in reality, of course, markets for manufacturers are highly imperfect, prices are fairly sticky and changes in investment are generally accompanied by changes in output and employment"(Robinson, 1962, p. 65). The sluggish adjustment in prices can be formalized by letting the profit share, π, adjust to the difference between actual and desired capacity utilization

$$\dot{\pi} = \nu(u - u^d) \tag{46}$$

[19] Flaschel and Skott (2006) discuss Steindl's analysis.

[20] She assumes that "competition (in the short-period sense) is sufficiently keen to keep prices at the level at which normal capacity output can be sold" (Robinson, 1962, p. 46).

where $\nu > 0$ is the adjustment speed. With slow price adjustment it is now instantaneous movements in the utilization rate u that ensure the equalization of saving and investment in the short run. The saving-investment balance and the definition of the profit rate, $r = \pi u \sigma$, imply that

$$u = \frac{g + \delta}{s\pi\sigma} = \xi(g, \pi) \tag{47}$$

where $\xi_g = \frac{1}{s\sigma\pi} = \frac{u}{g+\delta} > 0$ and $\xi_\pi = -\frac{g+\delta}{s\sigma\pi^2} = -s\sigma\frac{u^2}{g+\delta} < 0$.

Turning to the specification of the investment function outside steady growth, the distinction between expected and actual profitability in Robinson's argument essentially serves to introduce sluggish adjustments in accumulation. In a continuous-time setting, this can be achieved by a dynamic version of the investment function (44),[21]

$$\dot{g} = \lambda[f(u, \pi) - g] \tag{48}$$

where $\lambda > 0$ and $f_u > 0, f_\pi > 0$, and where the ill-defined variable r^e has been replaced by the current values of the utilization rate and the profit share.

Equations (48) and (46) yield a two-dimensional dynamic system in the growth rate of the capital stock and the profit share

$$\dot{g} = \lambda\{f[\xi(g, \pi), \pi] - g\} \tag{49}$$

$$\dot{\pi} = \nu[\xi(g, \pi) - u^d] \tag{50}$$

Stationary solutions satisfy $u = u^d$ (using (46)) and $g = f(u^d, \pi) = f(u^d, \frac{g}{s\sigma u^d})$ (using (47)-(48) and $u = u^d$). Turning to local stability, the Jacobian is given by

$$J(g, \pi) = \begin{bmatrix} \lambda(f_u\xi_g - 1) & \lambda(f_\pi + f_u\xi_\pi) \\ \nu\xi_g & \nu\xi_\pi \end{bmatrix} \tag{51}$$

and, evaluated at the stationary point, we have

$$\det(J) = -\lambda\nu(\xi_\pi + f_\pi\xi_g) = \lambda\nu\frac{u^d}{g^* + \delta}(s\sigma u^d - f_\pi) \tag{52}$$

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[21] Mathematically this formulation is closely related to Robinson's own analysis. The equilibrium condition for the product market implies that $r = g/s$. If $g = f(r^e)$ and $\frac{d}{dt}r^e = \dot{r}^e = \lambda(r - r^e)$, it follows that

$$\dot{r}^e = \lambda(\frac{f(r^e)}{s} - r^e)$$

and hence,

$$\dot{g} = f\,'(r^e)\dot{r}^e = f\,'(r^e)\lambda[\frac{g}{s} - f^{-1}(g)]$$

Since $f\,' > 0$, this equation has the same stability properties as the equation

$$\dot{g} = \lambda[f(\frac{g}{s}) - g] = \lambda[f(r) - g]$$

The latter equation, in turn, is a special case of equation (48).

$$\mathrm{tr}(J) = \left[\lambda\left(f_u \frac{u^d}{g^* + \delta} - 1\right)\right] - \left[\nu \frac{u^d}{g^* + \delta} s\sigma u^d\right] \tag{53}$$

The Robinsonian stability condition – desired investment being less sensitive than saving to changes in the profit share – ensures that $\det(J)$ is positive. This condition is satisfied at the high equilibrium in the banana diagram, ruling out saddlepoint instability. Local asymptotic stability of the high solution depends on the sign of the trace. In the expression for the trace, the first term in square brackets may be either positive or negative. The second term, however, is negative and local stability is assured if the adjustment speed for prices is fast (relative to the adjustment speed of investment). Thus, the explicit introduction of pricing dynamics confirms Robinson's main conclusion in a setting without perfect competition.

5.2 Mature economies

The high and stable solution in the banana diagram satisfies the 'Robinsonian stability condition': investment is less sensitive than saving to variations in profitability. This condition (as the corresponding condition with respect to the growth function in section 4) may be plausible at growth rates that are empirically relevant for successful developing countries, but the model and the high solution seem less promising for mature economies with modest growth rates. As in section 4, variations in the reserve army can be included explicitly in the analysis of these mature economies: employment effects may stabilize the otherwise unstable low solution in the banana diagram.

The size of the reserve army could influence accumulation and/or pricing. As an example consider the following extension of the dual-economy model:[22]

$$\dot{g} = \lambda[f(u, \pi, e) - g]; \qquad f_u > 0, f_\pi > 0, f_e < 0 \tag{54}$$
$$\dot{\pi} = \nu(u - u^d) \tag{55}$$
$$\hat{k} = g - n \tag{56}$$

where the new state variable k describes the ratio of the capital stock to the labor force. The ratio k is definitionally related to employment and utilization, and - normalizing units so that labor productivity is equal to one - we have

$$e = uk \tag{57}$$

The pricing equation (55) is unchanged (but re-stated for convenience). The innovation compared to the dual economy is the introduction of the employment rate e as a determinant of the long-run accumulation function f in (54). The utilization rate adjusts to clear the product market and is still given by (47).

A stationary solution satisfies

$$u = u^d \tag{58}$$
$$g = n \tag{59}$$
$$\pi = \frac{n + \delta}{s\sigma u^d} \tag{60}$$
$$f(u^d, \pi, k) = n \tag{61}$$

[22] This example retains the 'dynamic' specification of the investment function in equation (48). It is straightforward to set up a two-dimensional analogue to the model in section 4.2. Having employment enter negatively in the growth function (36) corresponds to letting the change in the profit share depend positively on employment. Thus, let

$$\begin{aligned}
\dot{\pi} &= H(u, e) = \nu(u - u^d) = \nu(u - \theta(e)), \quad \theta' > 0 \\
\hat{k} &= \hat{K} - n = f(\pi) - n \\
u &= \frac{f(\pi) + \delta}{s\sigma\pi}
\end{aligned}$$

The accumulation function $f(\pi)$ conforms to the Harrodian principle since the profit share is now a state variable. The Jacobian for this two-dimensional system is given by

$$J(\pi, k) = \begin{bmatrix} \nu(u_\pi - \theta'\sigma k u_\pi) & -\nu\theta' u\sigma \\ kf' & 0 \end{bmatrix}$$

and

$$\begin{aligned}
\det(J) &= kf'\nu\theta' u\sigma > 0 \\
\mathrm{tr}(J) &= \nu u_\pi(1 - \theta'\sigma k)
\end{aligned}$$

The derivative u_π is positive at the low, unstable solution in the banana diagram, and stability requires that the employment effect on 'desired utilization' in the equation for $\dot{\pi}$ be sufficiently strong.

Equations (58)-(60) give explicit and unique solutions for u, g and π, and substituting these solutions into (61) we get a unique solution for k and thereby (using (57)) for e.

Local stability is determined by the Jacobian

$$J(g,\pi,k) = \begin{bmatrix} \lambda[(f_u + \sigma k f_e)\frac{1}{s\sigma\pi} - 1] & -\lambda[(f_u + \sigma k f_e)\frac{g+\delta}{s\sigma\pi^2} - f_\pi] & \lambda f_e \frac{g+\delta}{s\sigma\pi}\sigma \\ \nu\frac{1}{s\sigma\pi} & -\nu\frac{g+\delta}{s\sigma\pi^2} & 0 \\ k & 0 & 0 \end{bmatrix} \quad (62)$$

The necessary and sufficient Routh-Hurwitz conditions for local stability are that, evaluated at the equilibrium,

1. $\mathrm{tr}(J) = \lambda[(f_u + \sigma k f_e)\frac{1}{s\sigma\pi} - 1] - \nu\frac{g+\delta}{s\sigma\pi^2} < 0$
2. $\det(J_1) + \det(J_2) + \det(J_3) = [\lambda\nu\frac{g+\delta}{s\sigma\pi^2} - \lambda f_\pi \nu \frac{1}{s\sigma\pi}] - k\sigma\lambda f_e \frac{g+\delta}{s\sigma\pi} > 0$
3. $\det(J) = \nu\lambda k \frac{g+\delta}{s\sigma\pi^2} f_e \frac{g+\delta}{s\sigma\pi}\sigma < 0$
4. $-\mathrm{tr}(J)[\det(J_1) + \det(J_2) + \det(J_3)] + \det(J) > 0$

The third condition is always satisfied, and straightforward calculations show that the other three conditions must be satisfied if the employment effect f_e is sufficiently strong.[23]

Comparing the Robinsonian and Kaldorian formulations in sections 4-5, the steady-growth equality between desired and actual utilization - equation (21) - is based on pricing/output behavior in Robinson and on accumulation in Kaldor; to get a steady-growth relation between growth and profitability, conversely, the Robinsonian model uses capital accumulation instead of output growth, as in the Kaldorian equation (27). From a steady-growth perspective these changes in the assignment of pricing and accumulation make no difference.[24] The relative adjustment speeds for output and prices are reversed in the two models, and this reversal affects the short-run dynamics. Both versions, however, have utilization at the desired rate in steady growth, both versions endogenize the profit share and use this endogenization as a stabilizing factor, and both versions yield multiple steady-growth solutions for a dual economy.[25][26]

In behavioral terms I find the Kaldorian version more persuasive and its short-run dynamics fit some important stylized facts. A more detailed discussion of the relative merits of the two versions, however, is beyond the scope of this paper.

[23] The expression in condition 4 is quadratic in f_e.

[24] Steindl (1952) also set up models with multiple steady-growth paths, focusing on the stable high-growth solution. Steindl's verbal argument is close to Robinson's and includes sluggish adjustments in the markup. As shown by Flaschel and Skott (2006), however, his focus on a high-growth solution in a formal model with a fixed markup seems misplaced.

[25] Neither prices not output are completely flexible, and Chiarella et al. (2005) pursue specifications with sluggishness in both prices and output.

[26] Behavioral relations between growth and profitability have been discussed by many other writers, including Penrose (1959), Wood (1975) and Eichner (1976).

6 Conclusion

The Kaleckian growth model has become a standard work horse for the analysis of growth and distribution. The model is simple and tractable and it lends itself to extensions in many directions. The simplicity and tractability, however, comes at a cost. The model includes a questionable stability condition and key predictions of the model, including the accommodating long-run variations in utilization, find little support in empirical evidence. At a methodological level, moreover, the standard Kaleckian approach may have unfortunate consequences since it plays down the need to 'think dynamically'.

Dynamic issues were at the heart of the Keynesian revolution. The fundamental proposition of the General Theory is that even with flexible prices and wages, the market mechanism can not be expected to ensure full employment. A market-clearing neoclassical general equilibrium may exist but is unlikely to be stable, even under hypothetical conditions of highly flexible prices and wages.[27] Harrod extended the dynamic analysis to movements over time of a Keynesian economy, his basic approach consisting "in a marriage of the 'acceleration principle' and the 'multiplier' theory" (Harrod 1939, p. 14). A number of early contributors (including Samuelson 1939, Kaldor 1940, Hicks 1950 and Goodwin 1951), formalized these interactions and although in some ways primitive, the fundamental insights remain valid: steady growth paths of a mature capitalist economy are likely to be locally unstable.

These dynamic issues are glossed over by the standard Kaleckian macro model with its emphasis on stable steady-growth paths, its neglect of lags and its use of utilization rates as an accommodating variable, in the long run as well as the short run. The predominant focus in Kaleckian theory on dual economy regimes, moreover, may threaten the relevance of the analysis with respect to most OECD economies.

This paper has discussed alternatives to the Kaleckian model. Sections 4-5 used endogenous variations in income distribution and/or employment to stabilize an otherwise unstable economy. I consider these mechanisms theoretically and empirically plausible but other solutions to the Harrodian 'instability problem' have been suggested. Shaikh (2007), for instance, denies the inherently unstable tendency in Harrod's argument while Dumenil and Levy (1999) accept the instability tendency but suggest that the stabilizing

[27] 'Old Keynesians' like Tobin have emphasized this point (Tobin 1975).

force comes from monetary policy.[28][29]

In general, the Harrodian alternatives are more complex than the Kaleckian model. They remain tractable, however, and the basic models in this paper can be (and have been) extended in a number of ways; Skott and Ryoo (2008), for instance, analyze the implications of financialization, using models that include explicit financial stocks. Most importantly, in my view, the Harrodian inspired models tell a behavioral story that is more convincing and that fits the empirical evidence better than the Kaleckian model.[30] The current dominance of the Kaleckian model therefore is unfortunate.

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[28] I do not find Shaikh's argument convincing. Leaving out some minor twists, Shaikh (2007) specifies the following investment function

$$\hat{K} = \hat{Y}^e + k(u - u^d) \tag{63}$$

Assuming that short-run expectations are being met (that is, $\hat{Y} = \hat{Y}^e$) and that the technical output-capital coefficient and the desired utilization rate are constant, this equation implies a stable differential equation for u,

$$\hat{u} = -k(u - u^d) \tag{64}$$

and utilization will converge to the desired rate.

This argument is correct but it is based on the assumption of fulfilled expectation at all times, and the Harrodian instability argument is precisely that when all firms reduce investment in order to raise their utilization rate, the outcome will be an unanticipated decline in aggregate demand and a fall in the utilization rate. Shaikh circumvents the instability by *assuming* that the economy is always on a warranted path with $\hat{Y} = \hat{Y}^e$, and his argument shows not the stability of warranted growth but the convergence of a warranted growth path to steady growth with $u = u^d$.

[29] The Dumenil and Levy argument has been discussed in relation to standard Taylor rules by Lavoie and Kriesler (2007). An emphasis on policy is in line with Harrod's analysis but he also suggested that the instability would be bounded even "without the application of monetary and fiscal restoratives" (Harrod 1973, p. 36).

[30] To avoid misunderstanding, let me emphasize that contributors to the post-Keynesian and structuralist literature cannot be neatly categorized into groups of 'Kaleckians', 'Harrodians', 'Robinsonians', etc. Some of the main contributors to the Kaleckian literature have also produced important studies that incorporate Harrodian instability. Conversely, writers, myself included, that may be thought of as critical of the Kaleckian model have used the Kaleckian model in some of their own work.

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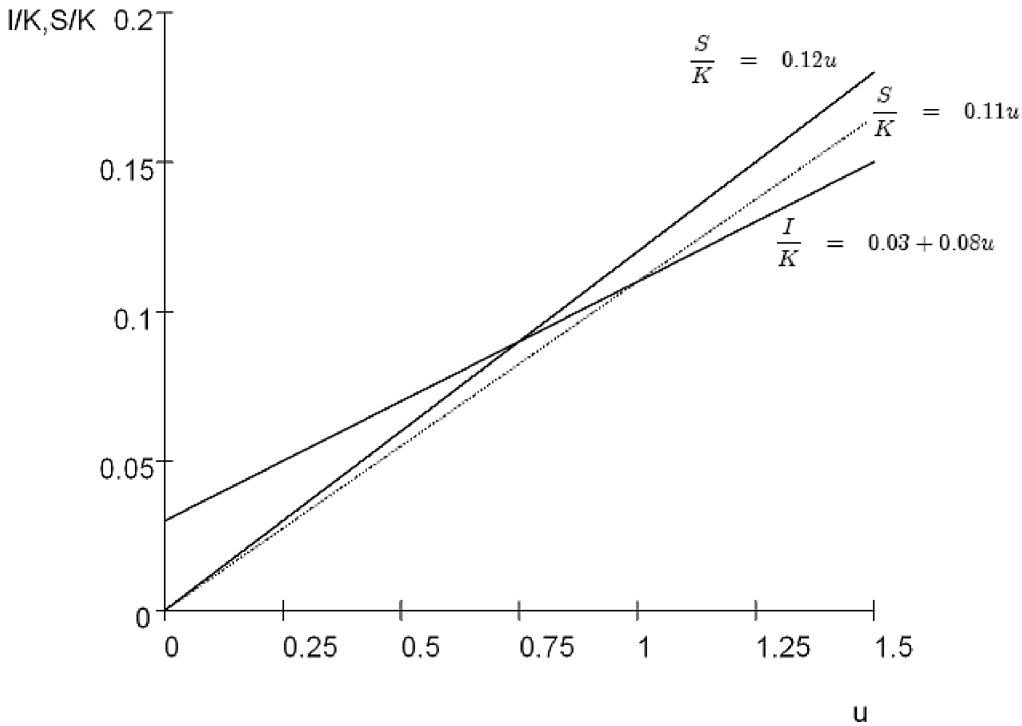


I/K,S/K
0.2
0.15
0.1
0.05
0
0
0.25
0.5
0.75
1
1.25
1.5
u
$\frac{S}{K} = 0.12u$
$\frac{S}{K} = 0.11u$
$\frac{I}{K} = 0.03 + 0.08u$

Figure 1

Figure 2a

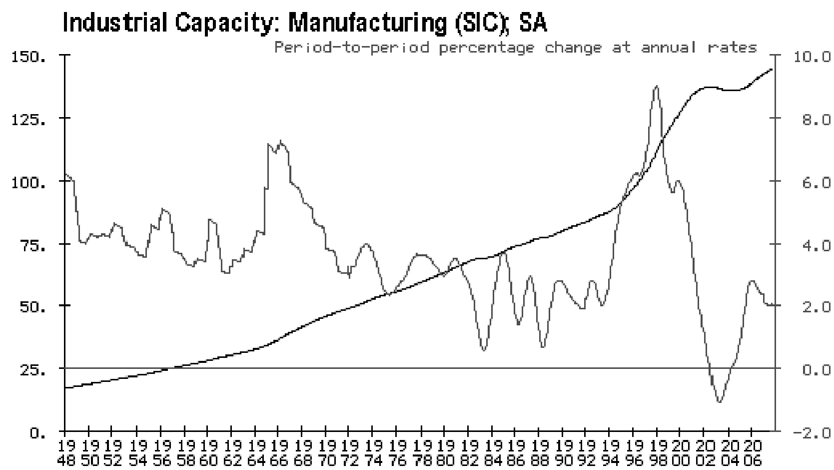
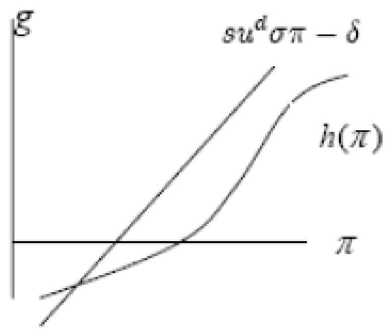


Figure 2b



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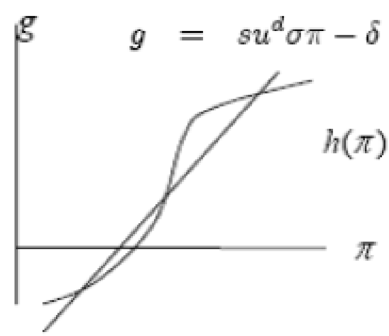

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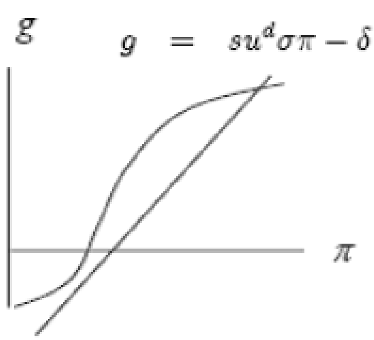

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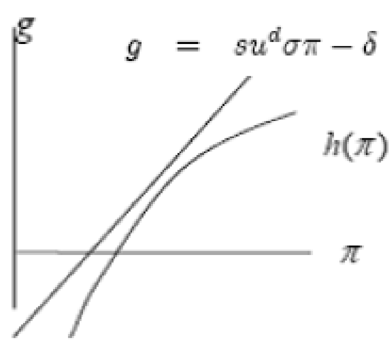

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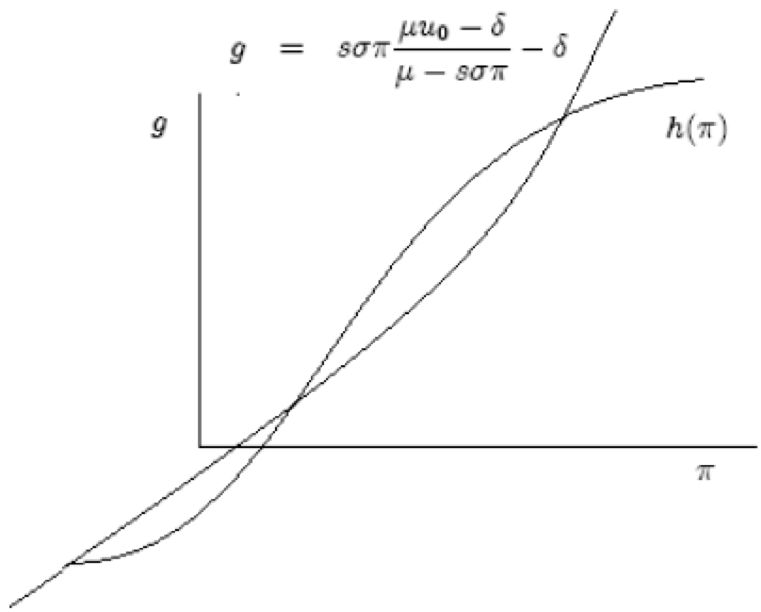

Figure 4

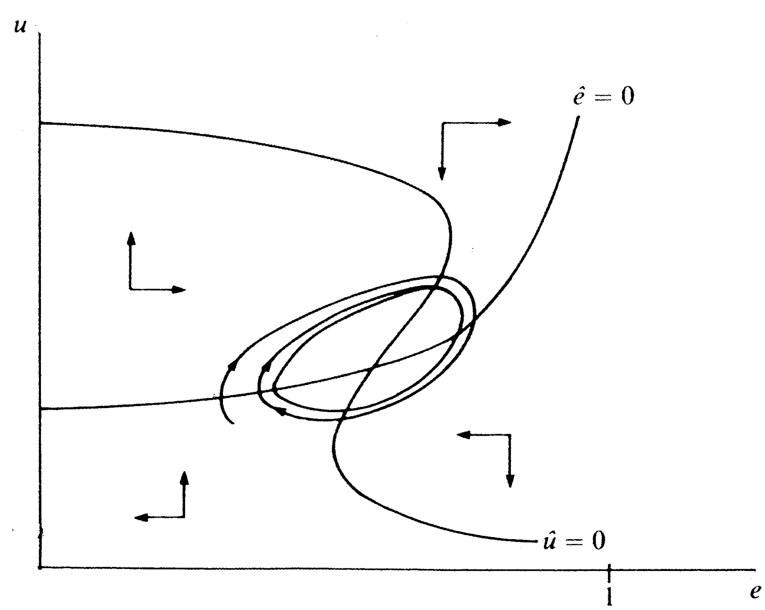

Figure 5