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Keynesian and Neoclassical Closures in an Agent-Based Context

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KEYNESIAN AND NEOCLASSICAL CLOSURES IN AN AGENT-BASED CONTEXT

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Abstract. Since the "closure debate" of the 1980s it is well known that comparative static derivatives in analytical macro models are highly sensitive to the closure rule selected. This led Keynesians to conclude that Keynesian closures were superior to those favored by the orthodoxy and *vice-versa.* It is argued that with the advent of agent-based or multi-agent systems, the closure debate is superseded. While elements of both Keynesian and neoclassical models survive the transition to the more synthetic environment, an agent-based approach eliminates the need for drastic simplification that was at the root of the debate from the beginning.

1. Introduction

The notion of closure, first framed by Sen in 1960, was widely discussed in the literature on applied general equilibrium modeling in the 1980s[1]. A central issue was the comparative statics of aggregate macroeconomic models, which reversed when the closure was changed. A Keynesian model, with an independent investment function, usually calibrated to depend on capacity utilization, the rate of profit or both, responded differently to, say, a change in the wage rate than did a neoclassical model in which savings determined the level of investment. This paper argues that the debate between Keynesians and neoclassicals is effectively over and attempts to revive "old style" Keynesian analysis are fruitless[2]. This is not to announce a victory on the part of the Walrasian system, but rather to argue that the debate has been superseded by the rise of agent-based models, computerized simulations that do not require the simplifying assumptions of the past[3]. The approach has its roots in the late nineteenth century statistical mechanics of Gibbs, Boltzmann and Maxwell (Durlauf, 1999). When coded using widely available software by nonprofessional programers, these models capture fairly complex dynamic social

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[1]See, for example, Sen (1963), Rattso (1982), Dewatripont and Michel (1987) and Robinson (2006) for a recap.

[2]Many heterodox economists still use the Keynesian model without regard to its lack of microfoundations. See for example Dutt (2007) and references cited therein.

[3]See, for example, Arthur et al. (1997), Axtell et al. (2001), Epstein and Axtell (1996) and Wooldridge (2002).

situations without regard to representative agents or rules of thumb (Railsback et al., 2007). Agent-based models are characterized by *emergent properties* that are not generally possible to anticipate using strictly analytical tools[4]. Thus the new generation of models represent not only a break with earlier theories, but a break in how we learn about economies and economics generally.

The paper is organized as follows: section 2 discusses the background to the debate between Keynesians and the proponents of neoclassical models. Section 3 introduces the multi-agent system framework in the context of complexity models. Section 4 addresses the question of closure in simplified Keynesian and neoclassical models. A concluding section argues that while agent-based models incorporate important elements from both theoretical frameworks, the divisions are substantially blurred.

2. Keynes and the Neoclassical Critique

It is a radical thesis to say we should retire Keynes. But it may well be time. The old debates over the nature of macroeconomic aggregates, whether they were savings driven or investment driven, have been made largely irrelevant by multi-agent, dynamical systems that incorporate learning and expectations in a natural and realistic way. What is sacrificed in these models is the idea that representative agents can compute solutions to long-horizon combinatoric optimization models that supposedly guide their actions over the course of their lives. Many problems, much simpler than those we commonly assume that our economic agents can solve, have been shown by research in theoretical computer science to essentially involve an infinite number of steps, which in the words of one researcher means "abandon all hope of finding an efficient algorithm for the exact solution of this problem" (Spiliopoulos, 2007). In agent-based models, agents do indeed optimize but they do so in computationally constrained ways, involving heuristics, approximate solutions and the like. They do well when they can determine an upper bound on the difference between the approximate and actual solutions.

Agent-based models eliminate the over-simplification of the Keynesian model without falling into the "representative agent" trap of the Walrasian system. Neither do they necessarily assume price-taking atomistic agents. Nor do these models serve any particular political ideology, since the objective is to model the economy realistically, as it actually performs, rather than produce welfare theorems applicable only to perfectly competitive systems.

In the 1970s, the profession began to abandon the Keynesian system as essentially anecdotal in its view of agency (Lucas and Sargent, 1978, p 277), (Plosser, 1989). Although dynamic versions of the Keynesian model certainly exist, the fundamental framework was seen as static. But above all, the Keynesian model was considered unrealistic in response to a change in policy. Keynesian agents were regarded as *reactive* only, failing to learn about the economic landscape as it underwent change in response to economic policy. Lucas objected early on, noting that only self-interest would be invariant to policy change; everything else would adjust (Lucas, 1976).

By anecdotal, the critics meant that the underlying agents in the Keynesian system did not conform to the principles of intertemporal rationality. Since then, of

[4] For a general introduction, see Holland (1998) or Waldrop (1994) or for a more technical approach see notes to section 3 below.

course, experimental economics has provided substantial evidence that the rational model is an imperfect foundation on which to build coherent theory (Henrich et al., 2004), (Basu, 1994). This criticism does not, however, eliminate the need to measure social welfare in terms of the well-being of the individual agents.

Ex-ante aggregation is at the core of the problem with the Keynesian system. Since homogeneous agents do not require aggregation, the representative agent approach solves the problem by assuming it away. An economy in which many people are better off by some small measure, yet a few are much worse off, could be judged superior unambiguously. This kind of reasoning could justify very progressive social policy or its opposite and with equal ease. Without specifics as to who precisely is better off and who is not, analysts are left in the dark.

The Walrasian system does not require *ex-ante* aggregation, but is nonetheless unrealistic in its reliance on a perfectly competitive economy as well as unrealistic assumptions about the computational capacities of its agents. If the physical analogy of the Keynesian system is the perfect gas law, the Walrasian system is closer to the approach of statistical mechanics, but with *perfectly elastic* collisions (Durlauf, 1999), that is, with no strategic interactions. Its policy implications all derive from a generalized libertarian philosophical outlook that denies the existence of "society" as separate from its constitutive components. Apparent inconsistencies with Keynesian macroeconomic theories of effective demand were resolved at a very high level, by Sonnenshein, Mantel and Debreu (SMD)[5]. The resolution of the conflict over the shape of the aggregate excess demand curve was simply to abandon the macro in favor of the more trustworthy microeconomic alternative.

The Walrasian system produced welfare theorems of stunning effect, if of limited generality, but it lacked the clarity of the Keynesian policy prescriptions. Just as the perfect gas law is more useful in solving practical engineering problems than is the more sophisticated statistical thermodynamic model, the Keynesian system is still broadly embraced by policymakers worldwide who refer to aggregate demand and job creation.

When the Keynesian model was dominant, the neoclassical closure was considered unrealistic because it follows Say's law, that supply creates its own demand. Too much emphasis is placed on the labor market to determine the magnitude of the main macroeconomic variables. Effective demand only served to change the *composition* of output between savings and consumption. Output was determined by factors of production on the supply side.

For both camps, the critique of the opposition was essentially that the other model was "too simple." Each had elevated one feature to prominence while downplaying the importance of the factor the other held dear. The closure debate was about which was the most essential feature, effective demand or rational choice, that is *structure versus agency*.

Now with the aid of computers, the economy can be re-conceived as an evolving complex adaptive system without the attendant oversimplifications of either the Keynesian or Walrasian systems. These models include the heterogeneity of agents and multi-dimensionality of the Walrasian system while at the same time incorporating social and economic structures present, but largely unexplained, in Keynesian models. Seventy years after the *General Theory*, it may well be time

[5]See Debreu (1974). For an interpretation of SMD theory, see Rizvi (1994).

for Keynes to retire, but it will be seen that his influence is still felt in the more realistic models of the agent-based framework.

3. Multi-Agent Systems

What is a multi-agent system and how does it resolve the closure debate? Formally, the goal of a multi-agent system is to characterize the joint probability distribution for the entire stochastic path that is compatible with the conditional probability distributions for each agent. This entails a number of attractive features that are not entirely obvious from the abstract definition. Agent-based models involve the interaction of a relatively large number of data structures (agents). These data structures interact iteratively with an environment in which they are located. Over time the result can be chaos or order depending upon how the agents adapt to their environment. The resulting models are complex adaptive systems and are now applied in a range of diverse fields, from physics, molecular biology and aerospace to linguistics, sociology, political science, and of course, economics.

Complexity itself may seem to be a vague notion but in fact can be defined fairly precisely, at least computationally (Machta and Machta, 2005). Basic computational theory holds that some problems can be solved in polynomial time, that is in a number of steps that can be represented by a polynomial in some metric of the data[6]. Many interesting and common problems, the classical example is the traveling salesman problem, are known as $NP - complete$, that is, essentially that have been proven to have *no* polynomial that describes the number of steps in their solution. One could search forever.

Complex models are simulation models with the added feature that the laws that describe the behavior of a complex system are qualitatively different from those that govern its units. In Gell-Mann's phrase, "surface complexity arising out of deep simplicity" is what typically characterizes the macro behavior.

Emergence is defined as an unexpected drop in complexity where complexity has to do with the length of the algorithm required to represent the problem, often described in terms of a stylized computing device known as a Turing machine. Relative algorithmic complexity (RAC) is defined as

> the shortest description that a given observer can give of the system, relative to the description tools available to that observer. Emergence occurs when RAC abruptly drops down by a significant amount.(Dessalles et al., 2007)

Phase transition is a well know example of an emergent property. All such transitions have an order parameter, which is zero on one side of the transition and non-zero on the other. There are few restrictions on how the order parameter is defined, but it must "flip" in some observable way. Some examples include when liquid water changes to ice at a constant temperature or in percolation when the fractional size of a spanning cluster reaches a critical value. Transitions may involve continuous change of the order parameter, or not when some amount of energy is required for the transition to occur (such as a latent heat)[7].

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[6] For example, a sorting problem of n numbers can be preformed according to the (first order) polynomial n. More compuationally complex problems correspond to higher order polynomials.

[7] Phase transitions occur in materials as their internal energy progresses through the five states of nature, solid, liquid, gas, plasma and the Bose-Einstein condensate. Crystallization of liquids at their freezing point generates an unexpected drop in complexity inasmuch as the algorithm

One way to characterize a transition is by way of the distribution of the order parameter. Transitional clusters form and the system's properties begin to change according to a power-law distribution. Most of clusters are small, but it is not unusual to encounter an enormous cluster interspersed among the far more numerous tiny agglomerations. Barabási and Albert note that power-law distributions come about when the underlying process, in their case networks, shows preferential attachment, and produce a "rich get richer," effect[8]

> Similar mechanisms could explain the origin of the social and economic disparities governing competitive systems, because the scale-free inhomogeneities are the inevitable consequence of self-organization due to the local decisions made by the individual vertices, based on information that is biased toward the more visible (richer) vertices, irrespective of the nature and origin of this visibility.(Barabási and Albert, 1999, p 512)

Whether specific agent-based models have power-law distributions of any order parameter is an open question. Many models show income or wealth distributions that follow a power-law (Durlauf, 1996), (Gibson, 2007).

Thus, agent-based models are most suited to address how order emerges from disorder rather than simply characterizing the equilibrium. Barabási also gives the example of the image of a Ferrari that could be rendered as the result of some mathematical simulation. A deeper question is what processes were required to build the Ferrari from the beginning? If these activities can be represented in a computation framework that converges to the image, then much more has been learned (Barabási, 2003).

The agent-based framework confers a number of theoretical advantages. The interaction of heterogeneous agents with respect to a wide range of personality parameters is central. Some agents learn quickly, others not; some have high consumption goals, others more modest. Attitudes toward risk, education and reproduction can all vary as well. Inter-agent communication can be error-free or noisy. Rationality is inherently bounded by computational complexity and agents may differ with respect to how long they are willing to search for solutions to combinatorial optimization problems. Some agents are more myopic than others, but all operate with imperfect and limited information. The approach does not bracket externalities, but integrates them in a fundamental way.

Naturally, the artificial intelligence literature offers the most extensive and sophisticated analysis of learning available. Sutton and Barto, for example, provide an analysis of reinforcement learning (RL) made up of four component parts. *Policies* are actions that agents implement, roughly equivalent to methods in object oriented programming languages. A policy is a "mapping from perceived states of the environment to actions to be taken when in those states" and is typically stochastic. In game theory, policies are essentially strategies. The *reward* function is a map of the environment and its associated benefits or costs that may be conferred

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that describes the lattice structure is more compact than that which is required to describe the fluid motion of asymmetric liquid molecules or the random orientation of electron spin as a ferromagnetic materials cool. The same emergence of order and symmetry applies to superconducting ceramics as they reach a thermally induced state of near zero resistance to electron flow. At a temperature near the critical point of phase change, systems vacillate between the states of matter with greater frequency as the critical point is approached.

[8]The result is also known as a Pareto distribution or, more colloquially the 80/20 rule.

upon local agents. The reward function roughly corresponds to the pay-off matrix in game theory and cannot be altered by agents directly. The *value* function is an aggregator of the reward function as rewards accrue to specific agents[9]. *Model* equations describe the dynamic environment and are used by the agents to enhance their learning. It is necessary to specify which model is used by which agents. These range from simple trial-and-error models to sophisticated state-space dynamic programming, Markov decision processes (MDPs) or optimal control models (Sutton and Barto, 1998).

RL is distinguished from more common supervised learning in that agents are not told how to behave but must figure it out on their own. Agents can be either "greedy" or experimental. Agents who adopt experimental strategies are more likely to reach global optima that those who remain in a statisficing, locally optimal state. Models with RL can have rich and realistic trajectories.

4. Closure

Sen describes a particularly simple accounting framework in which the number of equations is one short of the number of unknowns. Formally speaking the model cannot be solved, or "closed", until an additional equation is found and justified as part of the macroeconomic system (Sen, 1963) [10]. Closure then refers to selection of parameters and variables, specifically around the relationship between savings and investment. In a Keynesian closure, an independent investment function is present and savings adjusts to it through changes in output. Consider a system with two accounting equations for income and savings, a consumption function and a production function

$$\begin{aligned} Y &= C + I \\ S &= Y - C \\ C &= \bar{C} + cY \\ L &= lY \end{aligned} \tag{4.1}$$

with the Keynesian variable list $v(Y, S, C, L)$ or income, savings, consumption and employment. The parameter list $p(I, \bar{C}, c, l)$ includes investment, autonomous consumption, the marginal propensity to consume and the labor coefficient from the production function. With four variables and four parameters, there are sixteen comparative static derivatives that characterize the behavior of the system.

To convert the model into a neoclassical closure, parameters and variables in equations 4.1 simply change places. The variable list for the neoclassical closure is $v(Y, S, C, I)$ while the parameter list is now $p(L, \bar{C}, c, l)$. The only change is that I has been upgraded to variable status while L is taken as a parameter representing the constraint on production imposed by the supply of labor. This is most

[9]Evolutionary models and genetic algorithms strictly speaking do not have value functions. If the reward function causes the obliteration of the agent operating a particular policy, then the population learns, even though the individual does not. Agents need not even be able to sense the environment in evolutionary learning.

[10]Closure is related to but not the same thing as a "gap" model, in which there are specific targets for output and employment and either a savings, foreign or fiscal constraint binds (Bacha, 1990), (Taylor, 1994). The gap is determined by the amount by which the constraint would have to be shifted so that internal and external policy objectives could be met.

the simplified version of the notion of closure. In the Keynesian model, investment is the binding constraint; in the neoclassical model it is the supply of labor. This is a fundamental difference that has served to historically distinguish the two approaches.

In multi-agent systems or agent-based models it is *ultimately* agents and their decisions that are responsible for all structure. Much rides, of course, on what is meant by "ultimately" but here is where the approach makes its most important contribution. It is not necessary to *assume* a structure in which agents make choices. The structure embodies previous choices, that is accumulated decisions of the past. All four components of human activity classically identified by Aristotle, form, substance, intention and accomplishment are present, but the last distinction is key. Agents may have specific intentions, but whether they are able to accomplish their goals in the context in which they are undertaken is altogether another matter. The substance given to previous forms through accomplishment changes the underlying process and the cycle begins anew. The model is inherently dynamic.

Is the multi-agent system framework then just an *uber*-choice theoretic model? The principal reason it is not is that agents make decisions in a social context as just noted. It might be argued that this is true in the Walrasian model as well, but here there is an important distinction. Walrasian agents are "atomistic," and make optimal decisions, taking their environment as given. The issue of whether agents are able to solve their optimization problem is never posed. In contrast, in agent-based models, agents are best thought of as *computational entities,* who make decisions based in an informationally constrained environment and with limited computational means in real time. So structure is present, but it is located within the limitations of the human thought process. Thus, agent-based modelers take as a central problem the question of how precisely to describe "approximately rational behaviors in operational, computational terms" (Boutilier et al., 1997, p 2). Since computation itself requires real time, agents must cease their computational effort within an action frame of the model. Frequently, sub-game perfect strategies, common in analytical models, are beyond the reach of agents (Basu, 1994). This amounts to a theoretical break with the hard optimization approach of many formal economic models.

The most stripped down example of an agent-based model that produces emergent properties is the original Schelling neighborhood model (Schelling, 1971). There white liberals decide if they are going to either stay in the current neighborhood or move. A fully rational decision tree would take into account both the state of the current neighborhood as well as the expected characteristics of the destination. In a computationally constrained world, however, one might not be able to determine the latter as easily as the former. Agents are rational, but boundedly so, although in more complex models, their computational abilities can evolve within the model.

The decision rule in the Schelling model is deceptively simple: move if a threshold of racial homogeneity of the neighborhood is reached. That is, white liberals may prefer a mixed neighborhood, but if it becomes too black, then the whites decamp for another. This is the only decision agents make in the model: *stay or move.* At the end of some 40-50 iterations, the model converges to strictly *segregated* neighborhoods: this simple agent-based model has generated an emergent property, segregation, that is not possible to deduce from the characteristics of the agents of the model.

Gibson describes simple model, based on Schelling, in which agents decide whether to take a job or not (Gibson, 2007). "Stay" is to accept a given wage offer and "move" is to reject it. The wage offer might vary from low, say at Starbucks or Wal-Mart, to high, say an assistant to the chief operations officer in a multinational corporation. Agent *job-satisfaction* is the key decision variable. Either the job "works" for the agent, in that it covers expenses and adds to accumulated wealth, or it does not.

A job can be thought of as a bundle of production processes involving capital, intermediate goods and *one unit of labor*, the agent (Axtell, 1999). Hence, the decision the agent must make is whether to operate the production process in front of her. In the Gibson model, both a unit of labor (an agent) and an amount of *finance* are required in order to activate the technology of a given cell. Finance is available from wealth accumulated by agents in the past and is distributed back to cells according to profitability with a random error term. Profit is the difference between wages and output and is returned to agents in proportion to their wealth. The wealth-capital constraint does not imply that the system is constrained "from above" since the wealth is product of the decisions made by individual agents, now and in the past.

The dynamics of the model depend on the wage bargain between agents and the cells on which the agents reside. Cells can compute the marginal product of labor, but agents lack sufficient information. Agents can compute their own reservation wage, based on life-cycle variables, as they age, reproduce and die.

As noted, the decision variable is whether the agent is satisfied with her current job. Job satisfaction depends mostly upon whether wealth is increasing or decreasing, but there are also variables that derive from the RL framework[11]. Agents must learn what the grid as a whole has to offer in terms of consumption possibilities. Unsuccessful agents become "stuck" in relatively low wage jobs either because they do not have the accumulated wealth to finance a move or they lack the education and skills required to take advantage of nearby opportunities.

If agents move, they must then Nash bargain over the wage payment with the new cell. In the Nash bargain, the surplus is defined as the difference between the marginal product of labor and the agent's reservation wage. The outcome of the bargaining process depends on the relative impatience of the agent to the cell. Cells know that unless they are profitable, they will be unable to attract capital and will fall into disuse. Agents realize that if they reject the offered wage they must move again, with all the associated costs and uncertainty. If the agent's reservation wage exceeds the marginal product, cells raise their prices to compensate, provoking inflation. As a result, they are less able to compete for finance for their operations and may experience cell death.

In this simple model the economy grows with less than full employment on a track that underutilizes the available technology. There is very little that is optimal about the model in the traditional sense, but neither is it excessively prone to mass unemployment nor spiraling inflation. As noted, a skewed distribution of income is an emergent property of this simple system. Even if the economy begins with an egalitarian wealth distribution, it will deteriorate over time and eventually follow a

[11] A full description is beyond the scope of this paper. See Gibson (2007).

power-law distribution. Educated agents who secure good jobs early and keep them for a long time end up wealthy. Those who move tend to run down their wealth but they may also succeed in finding a better opportunity.

Is this model Keynesian, neoclassical? At first blush, it seems that the model is more Keynesian in that at any given moment there would be unemployment as the job search proceeds. Markets are certainly not the central feature, as in the neoclassical scheme, in that markets in a formal institutional sense do not even exist. There is no Walrasian auctioneer to announce prices to which the market as a whole can respond. Unemployment in the agent-based view is not different from underemployment in that agents are modeled as always doing something, operating some process whether part-time, casual, informal, illegal or what have you.

The model shares a basic Keynesian feature that demand matters. There are many processes that populate the economic space that could be operated, but if there is no demand for them, they are not viable. Production processes for horseshoes are not viable, for example, but for Ipods, they certainly are. It follows that a demand expansion reduces un(der)employment and causes GDP to rise[12]. The problem is that there is no lever to pull to make demand expand exogenously, no parameter in the model that controls aggregate demand. Government would have to be built in, as perhaps a coalition of agents as in Abdulla and Lesser, who show how agents can learn *through* run-time communication to form effective dynamic coalitions by self-organization (Abdallah and Lesser, 2007). Clusters of demand could then result from the formation of the coalition, but this is not present in the reference agent-based model of this paper.

Since the important decisions here are made in what would appear to the neoclassical mind as a labor market, does this mean that agent-based models are essentially neoclassical? To begin to answer that question, consider the comparative statics of the Keynesian and neoclassical systems. In order to compare the two along a common metric, we can only consider a change in the parameters that are shared by both. The comparative statics of any one of $v(Y, S, C)$ can be evaluated with respect to a change in any one of $p(\bar{C}, c, l)$. That is, we may examine the change in output and its components, consumption and savings, with respect to a change in the demand parameters for either the goods or labor market.

Notice that a rise in $\bar{C}$, the level of autonomous consumption, or c, will increase all variables in the Keynesian view, but will only increase consumption and decrease savings in the neoclassical. As has been seen, this is a direct result of the fact that output is determined in the labor market in the neoclassical model. The models are therefore predicting different reactions to changes in preferences. Also a rise in the labor demand, l, will not affect output in the Keynesian closure but cause output to *fall* in the neoclassical.

In the simple agent-based model, an increase in consumption demand will reduce total savings in the system. It should be clear that there will be no impact in the current period if some agents decide to raise their consumption levels and decrease their savings. But iterative agent-based models are intrinsically dynamic and thus savings in this period must have some impact on the ability to finance production

[12]Underemployment is the relevant concept here since agents can operate processes with very little capital and that offer a wage that is below the agent's reservation wage.

in the next period[13]. Job dissatisfaction is likely to rise in the next period. Similarly a rise in labor productivity, (a decrease in l) in a simple agent-based system would have no impact on current output, as in the Keynesian closure, but would certain have an impact on the following period. In neoclassical models, all this savings is invested and there is an increase in the capital stock. If investment exceeds depreciation, output rises. Keynesian model dynamics are less straight forward since if investment does not adjust to match the rise in savings, output can fall. The subsequent unemployment will deplete aggregate savings, restoring the savings-investment balance. Whether investment increases usually depends on profitability, expectations and the rate of capacity utilization.

How do the dynamics of the agent-based formulation stack up against these two canonical models? This is a bit more complicated to visualize. In every period most agents operate processes and receive a wage. The profit earned in the process is then pooled and used to finance the capital stock for the next round of production.

Is it possible to have an *excess* of the supply of finance over demand? It could come about if the sum of agent wealth is greater than the sum of the demand for financial capital by each of its cells. In a purely Walrasian model, this would not happen since the interest rate would fall and the capital intensity of all processes would instantaneously rise. In the agent-based system, an excess supply is also a disequilibrium, but agents cannot *instantaneously* react. In the next period, producing cells compete for a higher level of available finance. Following the Keynesian framework, they compete on the basis of profitability.

Agents can refuse new finance, but they would do so only when they have something better to do, such as retire or return to school, both of which happen endogenously in the model. When all agents are operating processes and there is still a surplus of savings, some savings may go underutilized. The system runs at suboptimal level, but the inability of agents to move instantaneously to more capital intensive processes is what is responsible, not some given level of investment demand that the analyst regards as too low [14].

On the other hand, a shortage of wealth relative to the capital requirements of the processes in operation can certainly throw some agents out of work. This is similar to the traditional "insufficient aggregate demand" of the Keynesian system. Agents can decide whether to return to school or search for work in the next period. While looking for a better job, they may run down their wealth, reducing the number of processes that can be financed in the next period; again this looks very much like a traditional Keynesian model.

How about a shortage of effective demand? Can agents always sell everything they produce operating the production processes? In the Walrasian model, they can; sellers simply lower their prices until all markets clear simultaneously. Here, again, there is no market *per se*; agents bargain with each other on the basis of what they have individually produced. Trading out of equilibrium is inevitable

[13] If in the Schelling model white liberals were asked to buy a car at the same time they are considering a move to a new neighborhood, this would certainly reduce the probability of moving.

[14] Here the effort to approximate "rational behaviors in operational, computational terms" comes directly into play. If agents are endowed with higher levels of computational capacity, then they can learn more quickly and the system as a whole can perform more rationally. It is not the institutional context, market failure or what have you, but rather the characteristics of the agents themselves that create the suboptimality.

since there is no *ex-ante* price provided by an auctioneer. The trades are zero-sum, however, since any net benefit that accrues to one agent is immediately offset by a loss to the other.

Can there be a *general* shortage of aggregate demand? Yes, but it shows up as a shortage of finance to activate production processes that would satisfy the individual agents, or a shortage of viable technologies. To see this, imagine that an injection of "exogenous expenditure" takes the form of a new weapons system for "government." In that case, a new blueprint would enter the system and the number of potentially producing cells would increase. Let the blueprint reside in cell i and consider the jth agent. If prior to the appearance of the government contract, agent j was satisfied with her job, the process might not be activated because of local labor shortage. But it could also easily be that agent j can now see the new process and will move to cell i in order to operate the process. To insure that the process can be financed, government expenditure might have to "jump the queue", thereby crowding out more profitable private processes. This possibility would have to be built into the coding of a more complete model. Demand would then matter, but there would be no independent aggregate demand *function* as in the Keynesian model.

Multipliers can also be built into agent-based systems, but again this might be complicated to achieve. The standard explanation of the multiplier process is through inventory adjustment. As inventories are depleted, firms increase their demand for goods to restore the desired inventory-sales ratio. The very process by which inventories recover gives rise to an increase in income, which in turn, causes inventories to fall back by some fractional amount. For every step forward, there is a half-step backward as aggregate demand rises. Eventually the process converges to the new equilibrium.

In an agent-based model, the process would unfold somewhat differently and lead to a variable multiplier. As inventories fall, agents might well operate processes to replace them as opposed to some other process that paid a lower wage. The inventory replacement process may, however, block the operation of even more remunerative processes, which agents may subsequently discover. Since agents are always in the process of learning about their economic environment, producing more inventories may mean the agents find the work satisfactory and then break off the search for other activities that may indeed be more productive.

It becomes evident that agent-based frameworks build in technological change in every step of the process. Learning is central. Experimentation is required for agents to discover optimal properties of the economic landscape and the Keynesian adjustment process does not allow for that to occur in each action frame of model. The result in the multi-agent system is a variable multiplier based on technological interactions built into the grid.

Evidently motifs from both closures, Keynesian and neoclassical, easily find their way into agent multi-agent systems. On one hand, savings and wealth drives investment with a lag, as in the neoclassical model, and Say's law holds in approximate form. On the other hand, demand matters and drives technological change through the process of learning. More is to be done, of course, in building realistic macroeconomic models that incorporate connections between technologies as just discussed.

The traditional Keynesian and neoclassical models suppress complexity through aggregation and the use of representative agents. This does not mean that complexity is absent; it is simply repressed. Agent-based models focus on heterogeneity and interaction in complex environments. The neoclassical system models savings and lets investment follow in its path without much comment while the Keynesian system does the reverse. In the agent-based system, both aspects of the problem are incorporated.

5. Conclusion

Multi-agents systems provide an interdisciplinary approach that can integrate results from other disciplines such as sociology, anthropology and political science, as well as the natural sciences. These models can be made consistent with experimental and game theoretic results. Since they do not rely on analytical results for their main findings, there is no need to invoke arbitrary assumptions to obtain existence or stability of equilibria. Running the model reveals whether interesting properties emerge and what happens out of "equilibrium" cannot be safely ignored. Indeed, agent histories cumulate in a path dependent way to give rise to a statistical distribution of outcomes. How that distribution is characterized becomes a fundamental property of the system.

"Closure" is not something that any agent can perceive. It makes no sense to model the decision of heterogeneous agents as responding to whether it is the supply of labor or the level of investment that is given to the system as a whole. In old-school macromodels, closure determined the basic character of the model, its comparative statics and associated dynamics. In the agent-based framework, the character of the model is not imposed from outside, but rather arises from within the equations of motion of the individual agents (Gatti et al., 2008).

Is it time to retire Keynes? In some fundamental sense the answer is *yes.* A new generation of models represents not only a break with earlier theories but a break in how we learn about economies and economics generally. Old-style Keynesian or neoclassical economics that ignore advances in computational theory and practice is astronomy without telescopes. The closure debate drew its energy from the fact that *both* models were fundamentally inadequate. Agent-based models represent a step forward, and at a minimum, allow us to close the debate on closure.

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