Social Segregation and the Dynamics of Group Inequality

by

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Working Paper 2006-02
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January 27, 2006

Abstract

We explore the dynamics of group inequality when segregation of social networks places the initially less affluent group at a disadvantage in acquiring human capital. Extending Loury (1977), we demonstrate that (i) group differences in economic success can persist across generations in the absence of either discrimination or group differences in ability, provided that social segregation is sufficiently great, (ii) there is threshold level of integration above which group inequality cannot be sustained, (iii) this threshold varies systematically but non-monotonically with the population share of the disadvantaged group, (iv) crossing the threshold induces convergence to a common high level of human capital if the less affluent population share is sufficiently small (and the opposite, otherwise), and (v) a race-neutral policy that reduces the cost of acquiring human capital can expand the range over which reducing segregation can be Pareto-improving.

Keywords: segregation, networks, group inequality, human capital

JEL Classification Codes: D31, Z13, J71

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*This material is based upon work supported by the National Science Foundation under Grant No. SES-0133483, by the Russell Sage Foundation, and by the Behavioral Sciences Program at the Santa Fe Institute. We thank Shelly Lundberg for helpful comments on an earlier version.

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1 Introduction

In *Brown v. Board of Education* the U.S. Supreme Court (1954) struck down laws enforcing racial segregation of public schools on the grounds that ‘separate educational facilities are inherently unequal’. The justices thus endorsed the testimony of the psychologist Kenneth Clark that ‘like other human beings who are subjected to an obviously inferior status,’ African American children in segregated schools ‘have been definitely harmed in the development of their personalities’ (Beggs, 1995, pp. 10-11).

Many hoped that the demise of legally enforced segregation and discrimination against African Americans during the 1950s and 1960s coupled with the apparent reduction in racial prejudice among whites would provide an environment in which significant social and economic racial disparities would not persist. But while substantial racial convergence in incomes did occur from the 50s to the 70s, little progress has since been made in raising the median annual income of full time year round male and female African American workers relative to their white counterparts (President’s Council of Economic Advisors, 1991 and 2003). Similarly, the significant racial convergence in years of schooling attained and cognitive scores at given levels of schooling that occurred prior to 1980 appears not to have continued subsequently (Neal 2005). Significant racial differences in mortality, wealth, subjective well being, and other indicators also persist (Deaton and Lubotsky, 2001, Wolff, 1998, Blanchflower and Oswald, 2004).

Enduring discriminatory practices are no doubt part of the explanation (Bertrand and Mullainathan, 2004). These may be motivated by racial prejudice or hostility, but need not be—statistical discrimination is sufficient, as the contributions of Arrow (1973), Phelps (1972), and others have shown. Here we explore what may be another aspect of the explanation: racial assortation in social networks. Group differences in economic success may persist across generations in the absence of discrimination against the less affluent group because racial segregation of friendship networks, mentoring relationships, neighborhoods, workplaces and schools places the less affluent group at a disadvantage in acquiring the things – contacts, information, cognitive skills, behavioral attributes – that contribute to economic success. In doing this we follow Glenn Loury (1995):

...the full economic opportunity of any individual does not just depend on his own income; it is also determined by the incomes of those with whom he is socially affiliated. The patterns of such affiliation in our society are not arbitrary but derive in part from ethnic and social class identity.
We know from Schelling (1971) and the subsequent literature that equilibrium racial sorting does not require discrimination against the less affluent group and may occur even with strongly pro-integrationist preferences (Young 1998, Sethi and Somanathan 2004). Preferentially associating with members of one’s own kind (known as homophily) is a common human trait (Tajfel, Billig, Bundy, and Flament, 1971) and is well documented for race and ethnic identification, religion, and other characteristics. A survey of recent empirical work reported that:

We find strong homophily on race and ethnicity in a wide range of relationships, ranging from the most intimate bonds of marriage and confiding, to the more limited ties of schoolmate friendship and work relations, to the limited networks of discussion about a particular topic, to the mere fact of appearing in public or ‘knowing about’ someone else. ...Homophily limits peoples’ social worlds in a way that has powerful implications for the information they receive, the attitudes they form, and the interactions they experience (McPherson, Smith-Lovin, and Cook, 2001, pp. 415, 420).

In a nationally representative sample of 130 schools (and 90,118 students) same race friendships were almost twice as likely as cross-race friendships, controlling for school racial composition (Moody, 2001). In this sample, by comparison to the friends of white students, the friends of African American students had significantly lower grades, attachment to school, and parental socioeconomic status. There is also evidence that peer effects such as penalties for ‘acting white’ among African American students provide disincentives for academic achievement (Fryer and Torelli, 2005).

While there are many channels through which the racial assortation of social networks might disadvantage members of the less well of group, statistical identification of these effects often is an insurmountable challenge. The reason is that networks are selected by individuals and as a result plausible identification strategies for the estimation of the causal effect of exogenous variation in the composition of an individual’s networks are difficult to devise. Hoxby (2000) and Hanushek, Kain, and Rivkin (2002) use the year-to-year cohort variation in racial composition within grade and school to identify racial network effects, finding large negative effects of racial assortations on the academic achievement of black students. Studies using randomized assignment of college roommates have also found some important behavioral and academic peer effects (Kremer and Levy, 2003, Sacerdote, 2000, Zimmerman and Williams, 2003). A study of annual work hours using longitudinal data and individual fixed effects found strong neighborhood effects especially
for the least well educated individuals and the poorest neighborhoods (Weinberg et al., 2004). An experimental study documents strong peer effects in a production task, particularly for those with low productivity in the absence of peers (Falk and Ichino, 2004). On the other hand, a number of studies using randomized assignment or other compelling identification strategies have failed to detect important peer effects (Oreopoulos, 2003, Jacob, 2003). No available studies to date allow an estimate of the extent (if any) to which social network effects explain racial differences in economic success.

But the logic expressed by Loury is plausible, and an exploration of the dynamics of group inequality in the presence of racial assortation of social networks may therefore be illuminating. In a classic paper, Loury (1977) identified empirically plausible conditions under which social stratification by income alone could not sustain group inequality in the long-run. Under these conditions, group inequality can persist only if patterns of interaction reflect some degree of segregation, over and above that implied by income sorting alone. Similar conclusions have been drawn by Lundberg and Startz (1998) and Durlauf (1999, 2005). We build on this work by addressing the following series of questions. First, can the racial assortation of social affiliates support permanent group inequality in the absence of discrimination against the less affluent group? Second, is there a threshold level of integration above which group inequality cannot be indefinitely sustained? Third, how does this threshold, if it exists, vary with the population share of the disadvantaged group? Fourth, if integration proceeds beyond such a threshold, resulting in convergence of group incomes, does this equalization arise through reductions in the income of the initially advantaged group, or through increases in the income of the initially disadvantaged group? And fifth, are there Pareto-improving policy interventions that can reduce both segregation and group inequality?

To address this set of interrelated questions we model a setting in which the initial characteristics of the social affiliates of individuals in the less affluent group differ from those of individuals in the more affluent group sufficiently to induce the latter to acquire higher income-earning characteristics than the former. Because the intergenerational human capital transmission process is characterized by regression to the mean (Bowles and Gintis, 2002), it is not obvious that such a situation is dynamically stable. In the next section we present a model of the way individuals acquire income-earning traits. We assume the subjective cost of human capital investment varies inversely with an individual trait (‘ability’) and the mean level of human capital in one’s social network. Group differences in social network human capital are determined by the level of human capital in each group and the degree of segregation. There are no group differences in the cost function for acquiring
human capital or in the ability distribution, and the benefits of human capital are identical for all individuals.

In section 3 we study the case where all individuals are of the same ability and identify the conditions on the degree of segregation and the relative size of the two groups such that a stable asymmetric equilibrium may exist, namely one in which group differences in economic success persist indefinitely. We are also able to identify conditions under which, starting from such a stable asymmetric equilibrium, reductions in segregation will induce a transition to a symmetric equilibrium characterized by either high or low levels of human capital for both groups. We show that there exists a critical population share of the less affluent group above which the low level symmetric equilibrium will occur. Section 4 generalizes the model to the case of heterogeneous ability identically distributed in the two groups, again identifying conditions under which group differences will be permanent. We simulate a group-divided population to explore the conditions under which the groups will converge, and if they do whether it is to a high or low level equilibrium. Our penultimate section explores policies that by addressing group inequality and segregation simultaneously can increase the scope for Pareto-improving desegregation.

2 The Model

We model the process of acquiring human capital as a discrete choice, the private costs of which are influenced by the level of human capital among one’s social affiliates. Consider a single population consisting of two groups, black and white, with population shares \( \beta \) and \( 1 - \beta \) respectively. Each individual has a set of social affiliates that is drawn (in a manner to be described below) from this population. Let \( s_b(t) \) and \( s_w(t) \) denote, for blacks and whites respectively, the proportion of individuals in the group who have acquired human capital in period \( t \). The mean level of human capital in the overall population at time \( t \) is then

\[
\bar{s}(t) = \beta s_b(t) + (1 - \beta) s_w(t).
\]  

(1)

In a perfectly integrated society, the mean level of human capital in one’s social network would simply equal \( \bar{s}(t) \) on average, regardless of one’s own group membership. When networks are characterized by some degree of racial assortment, however, the mean level of human capital in the social network of an individual belonging to group \( j \) will lie somewhere between \( \bar{s}(t) \) and \( s_j(t) \), where \( j \in \{b, w\} \). Suppose that for each individual, a proportion \( \eta \) of social affiliates is drawn from the group to which that individual belongs, while the remaining \( (1 - \eta) \) are randomly drawn from
the overall population of potential social affiliates. We assume that \( \eta \) is the same for both groups. Then a proportion \( \eta + (1 - \eta) \beta \) of a black individual’s social affiliates will also be black, while a proportion \( \eta + (1 - \eta) (1 - \beta) \) of a white individual’s affiliates will be white. These proportions are referred to as isolation indexes in the empirical segregation literature.

The parameter \( \eta \) is sometimes refereed to as the correlation ratio (Denton and Massey, 1988), and in this model is simply the difference between whites and African Americans in the racial composition their social networks. In the Texas schools studied by Hanushek, Kain, and Rivkin (2002), for example, 39 percent of Black third grade students’ classmates were Black, while only 9 percent of White students classmates were Black, so if one’s classmates were the relevant network, then \( \eta = 0.3 \). The relevant social network depends on the question under study: for the acquisition of human capital, parents and (to a lesser extent) siblings and other relatives are among the strongest influences. Because family members are most often of the same group, the social networks relevant to our model may be very highly segregated.

Let \( \sigma_j(t) \) denote the mean level of human capital in the social network of an individual belonging to group \( j \). This depends on the levels of human capital in each of the two groups, as well as the extent of segregation \( \eta \) as follows:

\[
\sigma_j(t) = \eta s_j(t) + (1 - \eta) \bar{s}(t).
\]

(2)

Except in the case of perfect integration \( (\eta = 0) \), \( \sigma_b \) and \( \sigma_w \) will differ as long as \( s_b \) and \( s_w \) differ.

The benefits of human capital accumulation are assumed to be constant and identical across groups, independent of group human capital shares. This means that there is no statistical discrimination in the labor market. Without loss of generality, we can normalize these benefits to equal 0.

The costs of acquiring skills depend on one’s ability, as well as the mean human capital within one’s social network. By ‘ability’ we do not intend simply learning capacity, IQ or similar cognitive measures, but rather any personal characteristic of the individual affecting the costs of investing in human capital, including such things as the tolerance for classroom discipline, the anxiety one may experience in school and the like. Ability \( a \) is distributed independently and identically across all individuals regardless of group membership. In assuming the equality of ability distributions across groups we adopt Loury’s (2002) axiom of anti-essentialism, which is standard in the economic literature on statistical discrimination and group inequality (see also Loury, 1977). The (common) distribution of ability is given by the distribution function \( G(a) \). The costs of acquiring human
capital are represented by \( c(a, \sigma) \), where \( c \) is decreasing in both arguments. For any given level of social network human capital \( \sigma \), therefore, there is some threshold ability level \( \tilde{a}(\sigma) \) such that those with ability above this threshold accumulate human capital and those below do not. Since the benefits of human capital accumulation have been normalized to 0, this threshold is defined implicitly as the \( \tilde{a} \) that satisfies \( c(a, \sigma) = 0 \) for a given value of \( \sigma \). Note that \( \tilde{a}(\sigma) \) is decreasing: individuals belonging to groups exposed to higher levels of human capital will themselves accumulate human capital at lower ability thresholds relative to individuals in groups with initially lower levels of human capital. This difference will be greater when segregation levels are high.

The share of each group \( j \) that acquires human capital in period \( t + 1 \) is just the fraction of the group that has ability greater than \( \tilde{a}(\sigma_j(t)) \). Thus we obtain the following dynamics for each \( j \in \{b, w\} \):

\[
s_j(t + 1) = 1 - G(\tilde{a}(\sigma_j(t))). \tag{3}
\]

A steady state pair of skill shares \( (s^*_b, s^*_w) \) is such that, for each \( j \in \{b, w\} \),

\[
s^*_j = 1 - G(\tilde{a}(\sigma^*_j)) = 1 - G(\tilde{a}(\eta s^*_j + (1 - \eta) (\beta s^*_b + (1 - \beta) s^*_w))).
\]

It is easily seen that no matter what the initial human capital levels \( s_b(0) \) and \( s_w(0) \) happen to be, steady state levels must be equal in the two groups in the case of perfect integration. That is, if \( \eta = 0 \), then \( \sigma_b(t) = \sigma_w(t) = \bar{s}(t) \) in all periods \( t \), which implies

\[
s_b(t + 1) = s_w(t + 1) = 1 - G(\tilde{a}(\bar{s}(t))).
\]

Hence the two groups have equal human capital levels in all periods subsequent to the initial period. Group inequality cannot persist under complete integration in this model.

Consider next the other extreme of complete segregation, corresponding to \( \eta = 1 \). In this case \( \sigma_j(t) = s_j(t) \) for each group \( j \) and so

\[
s_j(t + 1) = 1 - G(\tilde{a}(s_j(t))). \tag{4}
\]

In any steady state, we must have

\[
s^*_j = 1 - G(\tilde{a}(s^*_j)), \tag{5}
\]

so group inequality can persist if and only if this equation admits multiple solutions, a necessary condition for which is the existence of social network effects. Stable steady states of this dynamic are those for which \( ds_j(t + 1)/ds_j(t) < 1 \). In general the existence of multiple solutions will depend on details of the distribution and cost functions which we will explore presently. But to clarify the logic of the model, we begin with a simple case in which all individuals have the same ability.
3 Homogeneous Ability

When all individuals have the same ability, we may suppress the dependence of costs on ability and write the cost function simply as $c(\sigma)$, with $c' < 0$. In this case the only stable steady states involve homogeneous skill levels within groups. (There may exist equilibria in which members of a group are all indifferent between acquiring human capital or not, and make heterogeneous choices in the exact proportions that maintain this indifference. Such equilibria will be dynamically unstable, and we ignore them in what follows.) Suppose that $c(1) < 0 < c(0)$, which ensures that the steady states both $(s_b, s_w) = (0, 0)$ and $(s_b, s_w) = (1, 1)$ are stable steady states at all levels of segregation $\eta$. Condition (6) also implies that under complete segregation ($\eta = 1$), the distribution $(s_b, s_w) = (0, 1)$ is a stable steady state. This is the only stable steady state under complete segregation in which black human capital levels are lower than white.

Since there exists an asymmetric stable steady state under complete segregation but none under complete integration, one may conjecture that there is a threshold level of segregation such that persistent group inequality is feasible if and only if the actual segregation level exceeds this threshold. In order to establish that this is indeed the case, define $\tilde{\beta}$ as the black population share at which $c(1 - \tilde{\beta}) = 0$. This is the value of $\beta$ for which, under complete integration, the costs of acquiring human capital are zero for both groups. (This is because, if $\eta = 0$ and $(s_b, s_w) = (0, 1)$, $\sigma_j = 1 - \beta$ for both groups.) There is a unique $\tilde{\beta} \in (0, 1)$ satisfying this condition since $c(\cdot)$ is a decreasing function satisfying (6). We then have (see the appendix for all proofs):

**Proposition 1.** Given any $\beta \in (0, 1)$, there exists a unique $\tilde{\eta}(\beta)$ such that the stable asymmetric equilibrium $(s_b, s_w) = (0, 1)$ exists if and only if $\eta > \tilde{\eta}(\beta)$. The function $\tilde{\eta}(\beta)$ is positive and decreasing for all $\beta < \tilde{\beta}$, positive and increasing for all $\beta > \tilde{\beta}$, and satisfies $\tilde{\eta}(\tilde{\beta}) = 0$.

Proposition 1 establishes that group inequality can persist if segregation is sufficiently high, where the threshold level of segregation itself depends systematically on the population share $\beta$ of the disadvantaged group. If segregation declines to a point below this threshold, group inequality can no longer be sustained. In this case convergence to a symmetric steady state must occur. However, there are two of these in the model, since both $(s_b, s_w) = (0, 0)$ and $(s_b, s_w) = (1, 1)$ are stable steady states at all levels of segregation $\eta$. Convergence to the former implies that equality is attained through declines in the human capital of the initially advantaged group. Convergence to
the latter, in contrast, arise through increases in the human capital of the initially disadvantaged group. The following result establishes that convergence to the high human capital state occurs if and only if the population share of the initially disadvantaged group is sufficiently low.

**Proposition 2.** Suppose that the economy initially has segregation \( \eta > \hat{\eta}(\beta) \) and is at the stable steady state \((s_b, s_w) = (0, 1)\). If segregation declines to some level \( \eta' < \hat{\eta}(\beta) \), then the economy converges to \((s_b, s_w) = (1, 1)\) if \( \beta < \tilde{\beta} \), and to \((s_b, s_w) = (0, 0)\) if \( \beta > \tilde{\beta} \).

![Figure 1. Effects of segregation and population shares on persistent inequality](image)

Propositions 1-2 are summarized in Figure 1, which identifies three regimes in the space of parameters \( \beta \) and \( \eta \). For any value of \( \beta \) (other than \( \tilde{\beta} \)), there is a segregation level \( \hat{\eta}(\beta) \in (0, 1) \) such that group inequality can persist only if segregation lies above this threshold. If segregation drops below the threshold, the result is a sharp adjustment in human capital and convergence to equality. This convergence can result from a decline in the human capital of the initially advantaged group if the initially disadvantaged group is large enough. Alternatively, it can result from a rise in the human capital of the disadvantaged group if it’s population share is sufficiently small. The threshold segregation level itself varies with \( \beta \) non-monotonically. When \( \beta \) is small, \( \hat{\eta}(\beta) \) is the
locus of pairs of $\eta$ and $\beta$ such that $c(\sigma_b) = 0$ at the state $(s_b, s_w) = (0, 1)$. Increasing $\beta$ lowers $\sigma_b$ and hence raises $c(\sigma_b)$, which implies that $c(\sigma_b) = 0$ holds at a lower level of $\eta$. Hence $\hat{\eta}(\beta)$ is decreasing in this range, implying that higher values $\beta$ require higher levels of integration before the transition to equality is triggered. When $\beta$ is larger than $\tilde{\beta}$, however, $\hat{\eta}(\beta)$ is the locus of pairs of $\eta$ and $\beta$ such that $c(\sigma_w) = 0$ at the state $(s_b, s_w) = (0, 1)$. Increasing $\beta$ lowers $\sigma_w$ and hence raises $c(\sigma_w)$, which implies that $c(\sigma_w) = 0$ holds at a higher level of $\eta$. Hence $\hat{\eta}(\beta)$ is increasing in this range, and higher values of $\beta$ require lower levels of integration in order to induce the shift to equality.

Greater integration within the regime of persistent inequality raises the costs to the advantaged group and lowers costs to the disadvantaged group. In this case one might expect integration to be resisted by the former and supported by the latter. Note, however, that this is no longer the case if a transition to a different regime occurs. When $\beta$ is small, both groups end up better off as a consequence of integration, and hence support for integration may be expected to transcend group boundaries in this case. Similarly, when $\beta$ is large, both groups might be united in opposition to integration policies that would reduce $\eta$ below $\hat{\eta}(\beta)$, since both would end up with higher steady state costs of human capital accumulation. (This feature of the model echoes findings in Chaudhuri and Sethi (2003), which deals with the consequences of integration in the presence of statistical discrimination.)

The simple model with homogeneous ability delivers a number of insights, but also has several shortcomings. There is no behavioral heterogeneity within groups, and all steady states are at the boundaries of the state space. Changes in segregation only affect human capital decisions if they result in a transition from one regime to another; within a given regime changes in social network quality affect costs but do not induce any behavioral response. Furthermore, even when transitions to another regime occur, human capital decisions are affected in only one of the two groups. Finally, convergence to a steady state occurs in a single period. These shortcomings do not arise when the model is generalized to allow for heterogeneous ability within groups, which we consider next.

4 Heterogeneous Ability

When ability is heterogenous within groups (though distributed identically across groups), steady states will typically involve heterogeneous choices within each of the groups. Given the state $(s_b, s_w)$ in period $t$, there will be a mean level of human capital $\bar{s}$ in the population, and hence particular
levels of mean human capital \((\sigma_b, \sigma_w)\) in black and white social networks. These values determine the ability thresholds \(\tilde{a}(\sigma_b)\) and \(\tilde{a}(\sigma_w)\) for blacks and whites respectively, such that all those with abilities above the threshold for the group to which they belong will find it optimal to accumulate human capital. For a given distribution of ability in both groups, these ability thresholds then determine the state \((s_b, s_w)\) for the subsequent period. A steady state is one which results in the same human capital decisions from one period to the next.

As noted above, multiple steady states will exist under complete segregation if and only if there are multiple solutions to equation (5). This in turn depends on details of the ability distribution and cost function. If there exist multiple steady states under complete segregation, then there must also exist an asymmetric steady state under complete segregation, in which the level of human capital is lower among blacks than among whites. As in the homogeneous ability case, asymmetric steady states cannot exist under complete integration. Hence there must be some level of integration above which persistent inequality cannot be sustained.

To see the conditions under which multiple steady states arise under complete segregation, consider the simple case of an ability distribution with density \(g(a)\), having mean 0 and support \((-\infty, \infty)\). Additionally, suppose that the costs of human capital accumulation are linear and separable in both ability and the mean human capital in one’s social network as follows:

\[
c(a, \sigma) = (\alpha - a) - 2\alpha \sigma.
\]

Setting \(c(a, \sigma) = 0\), this specification implies threshold ability levels \(\tilde{a}(\sigma) = \alpha (1 - 2\sigma)\), with thresholds ranging from \(\alpha\) (when \(\sigma = 0\)) to \(-\alpha\) (when \(\sigma = 1\)). Hence individuals with sufficiently high ability (greater than \(\alpha\)) will find it worthwhile to acquire human capital even if none of their social affiliates have done so, and individuals with sufficiently low ability (less than \(-\alpha\)) will not acquire human capital even if all of their affiliates have done so. The parameter \(\alpha\) may be interpreted as the strength of social network effects.

When groups are completely segregated, \(s_j = s_j\) for each group \(j\) in all periods \(t\), and hence

\[
s_j(t + 1) = 1 - G(\tilde{a}(s_j(t)) = 1 - G(\alpha(1 - 2s_j(t))).
\]

Any pair \((s^*_b, s^*_w)\) is a steady state if and only if, for each \(j\),

\[
s^*_j = 1 - G(\alpha(1 - 2s^*_j)).
\]

Since \(G(\alpha) < 1\) and \(G(-\alpha) > 0\), continuity of \(G(\cdot)\) implies that there must exist at least one symmetric steady state \((s^*_b = s^*_w)\). Any symmetric steady state will be stable under the dynamics
if the slope of the function $1 - G(\alpha(1 - 2s_j))$, evaluated at $s_j = s^*_b = s^*_w$ has absolute value strictly below 1. Since this slope is positive, the condition for stability of a symmetric steady state becomes $2\alpha g(\alpha(1 - 2s^*_j)) < 1$.

Clearly $(s^*_b, s^*_w) = (0.5, 0.5)$ is a steady state since the distribution of ability is symmetric with mean 0. This will be stable if $2\alpha g(0) < 1$ and unstable if $2\alpha g(0) > 1$. Since $g(0)$ is independent of $\alpha$, this steady state will be unstable if $\alpha$ is sufficiently high, that is, if social network effects are sufficiently strong. In this case there must exist at least two stable steady states under complete segregation, one on either side of $(s^*_b, s^*_w) = (0.5, 0.5)$. For example, when ability is distributed according to the standard normal, there is a unique stable steady state at $(s^*_b, s^*_w) = (0.5, 0.5)$ when $\alpha = 1$, but two symmetric stable steady states at $(s^*_b, s^*_w) = (0.86, 0.86)$ and $(s^*_b, s^*_w) = (0.14, 0.14)$ when $\alpha = 2$. These two cases are depicted in Figure 2. In the latter case, the asymmetric state $(s^*_b, s^*_w) = (0.14, 0.86)$ must be a stable steady state of the dynamics (4) under complete segregation.

When multiple equilibria arise under complete segregation, racial inequality can persist indefinitely, with each of the two groups having converged to a different stable steady state. Since perfect integration is inconsistent with persistent racial inequality, one might conjecture that there is a threshold level of segregation such that racial inequality can persist indefinitely if and only if segregation exceeds this threshold. Figure 3 illustrates this for the special case of a standard normal ability distribution, with $\alpha = 1.75$ and $\beta = 0.4$. Under complete segregation there is a stable steady
state at \((s^*_b, s^*_w) = (0.06, 0.94)\), and this is used as the initial state in each of the four simulations. The panels in the top row reveal the persistence of racial inequality, although the extent of such inequality is less when integration is greater. When \(\eta = 1\), the population remains at the initial stable steady state (by construction). When \(\eta = 0.9\), convergence occurs to a nearby asymmetric steady state at \((s^*_b, s^*_w) = (0.13, 0.90)\). A further decline in segregation to \(\eta = 0.8\) causes persistent inequality to be unsustainable, an convergence occurs to a symmetric stable steady state at \((s^*_b, s^*_w) = (0.94, 0.94)\). The same happens when \(\eta = 0.7\), although convergence is more rapid. Note that integration can cause a temporary decline in the human capital of whites, through increasing contact with a disadvantaged group, although the decline is reversed as the disadvantage itself starts to erode.

Figure 3. Effects of greater integration when \(\beta = 0.4\).

Figure 4 reveals a strikingly different possibility. This time the black share of the population is set at \(\beta = 0.6\), while all other parameters and initial conditions are left unchanged. Again, persistent inequality is unsustainable once integration proceeds beyond some threshold, but convergence occurs to a steady state with very low levels of human capital. There is a temporary rise in human capital of blacks (through greater contact with whites) but this is reversed as levels of
human capital among whites start to decline.

Figure 4. Effects of greater integration when $\beta = 0.6$.

Figures 3–4 confirm that our findings in the homogeneous ability case extend to the case of heterogeneous ability for the particular numerical specifications. To what extent do these results apply more generally, for arbitrary cost and distribution functions? To explore this, recall that group inequality can persist under complete segregation ($\eta = 1$) if and only if there are multiple solutions to the equation

$$s = 1 - G(\tilde{a}(s)).$$

Suppose that this is indeed the case, and that we have precisely three solutions (as in the right panel of Figure 2). The two extreme solutions, which we now denote $s_l$ and $s_h$, correspond to stable symmetric steady states at all levels of segregation $\eta$. Furthermore, the $(s_l, s_h)$ is an asymmetric stable steady state when $\eta = 1$. Now consider the effects of increasing integration, starting from this state. For any given population composition $\beta$, we shall say that integration is equalizing and welfare-improving if there exists some segregation level $\hat{\eta}(\beta)$ such that for all $\eta < \hat{\eta}(\beta)$ there is no stable asymmetric steady state, and the initial state $(s_l, s_h)$ is in the basin of attraction of the high-investment symmetric steady state $(s_h, s_h)$. Similarly, we shall say that integration is equalizing
and welfare-reducing if there exists some segregation level $\hat{\eta}(\beta)$ such that for all $\eta < \hat{\eta}(\beta)$ there is no stable asymmetric steady state, and $(s_l, s_h)$ is in the basin of attraction of the low-investment symmetric steady state $(s_l, s_l)$. We then have the following result.

**Proposition 3.** There exist $\beta_l > 0$ and $\beta_h < 1$ such that (i) integration is equalizing and welfare-improving if $\beta < \beta_l$ and (ii) integration is equalizing and welfare-reducing if $\beta > \beta_h$.

This result provides a partial generalization of our findings in the homogenous ability case. When local complementarities in the accumulation of human capital are strong enough to allow for multiple stable steady states under complete segregation, integration can have dramatic effects on steady state levels of human capital. Once a threshold level of human capital is crossed, asymmetric steady states may fail to exist resulting in a transition to equality. As in the case of homogeneous ability, this can happen in one of two ways: through a sharp decline in the human capital of the previously advantaged group, or through a sharp increase in the human capital of the previously disadvantaged group. If the population share of the initially less affluent group is small enough, integration can result in group equity (meaning that equally able individuals acquire similar levels of human capital) and higher average incomes for both groups. Under these conditions, one should expect broad popular support for integrationist policies. On the other hand, if the initially disadvantaged group constitutes a large proportion of the total population, equity may be still attained through integration but at the cost of income, as costs are higher and human capital levels in both groups decline.

### 5 Policy Implications

Thus integration may benefit the disadvantaged group without harming the advantaged group, as is suggested by the empirical analysis by Cutler and Glaeser (1997) of the relationship between segregation and high school graduation rates. But integration may also harm both groups. As a result the the policy maker and the public are faced with a difficult challenge.

Since the heterogeneous ability case reproduces the logic of the simpler homogeneous ability case, we will illustrate the policy implications of the model with the latter. Suppose that the reduction in both segregation and between-group inequality are objectives, but the disadvantaged population share is sufficiently great that the outcome of reduced segregation is likely to be a welfare loss by both groups occasioned by a transition to low equilibrium levels of human capital acquisition.
by both groups. Does there exist a policy that will raise the value of $\beta$ for which $c(1 - \beta) = 0$, increasing the range of population shares for which the Pareto-improving transition to equal and high levels of human capital acquisition can result from greater integration? Educational or other policies resulting in a reduction in the cost function of human capital acquisition will accomplish this, shifting $\hat{\eta}(\beta)$ upwards to the left of $\tilde{\beta}$ (because $c(\sigma_b) = 0$ at higher levels of segregation following the educational improvement). To the right of $\tilde{\beta}$ the $\hat{\eta}(\beta)$ function shifts downward for analogous reasons. Hence a race-neutral policy which successfully lowers the costs of human capital accumulation for each individual in the population can make a Pareto-improving transition possible for a broader range of population shares and segregation levels. This effect is illustrated in Figure 5.

Figure 5. Effects of a race-neutral policy lowering costs of human capital accumulation.

Contrary to what we have assumed, costs may differ between groups, the disadvantaged group facing higher costs of acquiring human capital due, for example to less adequate schools or a less supportive in-school environment. Policies to rectify this discrepancy will (as in the race neutral case) shift upwards the left hand segment of the $\hat{\eta}(\beta)$ function; but will leave the right segment unaffected. As a result such policies would be effective in promoting Pareto improving transitions.
to equality for sufficiently low $\beta$. Where the population share of the disadvantaged group is large however, policies to reduce the costs of the advantaged group could be advocated as a means of allowing substantial levels of integration without inducing a mutually disadvantageous transition to equality. (Highly resourced magnet schools combined with bussing to promote integration may provide an example of such policies.) Thus while race-neutral interventions to lower costs of human capital investment can support Pareto-improving outcomes following desegregation, race-based interventions may accomplishing the same ends at lower cost as long as the disadvantaged population share is sufficiently high or low.

How might desegregation be accomplished? Recall that $\eta$ does not reflect the imposition of discriminatory practices by the advantaged on the disadvantaged group, as we have assumed these to be absent. Rather the degree of segregation is the result of equilibrium sorting choices in residence, friendships, couples and the like, arising in the absence of overt discrimination. Are there non-paternalistic ways that a policy maker could legitimately alter these patterns? We think that there are. First, under quite general conditions equilibrium sorting produces levels of segregation that are Pareto-inefficient in the sense that an arbitrary reduction in segregation could enhance the well being of members of both groups (Schelling, 1978). In this case policies to reduce, say, neighborhood segregation do not override individual preferences over aggregate outcomes, but rather allow for their greater satisfaction. Second, segregated networks may be the unintended result of current policies whose reversal would lower $\eta$. For example the degree of racial segregation of friendship networks in schools appears to be affected by the extent of tracking, the degree of cross grade mixing, and the extent of racial mixing in extracurricular activities, all of which are subject to alteration by school policies. (Moody, 2001). However, because the most important social affiliates for the formation of human capital are parents and siblings and these kin networks are highly racially segregated, there may be quite stringent limits to the degree to which segregation of the relevant networks can be reduced in the absence of a reduction in racial assortment in childbearing.

6 Conclusions

In response to a lower court judge’s question about the effects of school segregation on African American children a year prior to the *Brown v. Board of Education* decision by the Supreme Court, Kenneth Clark had concluded “I think it is the kind of injury which would be as enduring or lasting as the situation endured..” (Beggs, 1995, p. 11). Extending earlier work by Loury,
Durlauf, Lundberg and Startz and others we have identified the conditions under which, in a plausible dynamic model, the economic injury of segregation does indeed endure.

We have shown that group differences in economic success can persist across generations in the absence of either discrimination or group ability differences provided that the social segregation of networks is sufficiently great, and that there is a threshold level of segregation below which group inequality cannot be sustained. A challenge to policy makers arises because crossing this threshold may induce a transition to an equilibrium with either equally high or low levels of human capital in both groups. Which of these will occur depends on the population share of the disadvantaged group.

Thus the challenges facing policy makers in an urban area such as Baltimore are quite different from those in Bangor or Burlington. Similarly the challenges of assuring group-equal opportunity in other countries are quite different in New Zealand, where 15 percent of the population are Maori and South Africa where the disadvantaged African population constitutes 78 percent of the total. We have shown that race-neutral educational policies that reduce the costs of acquiring human capital unambiguously increase the range of population shares over which, starting from a segregated and unequal initial state, the process of integration will induce a transition to an equilibrium without group inequality and with high levels of human capital.
Appendix

Proof of Proposition 1. At the state \((s_b, s_w) = (0,1)\), the mean skill share \(\bar{s} = 1 - \beta\) from (1).

Hence, using (2), we get

\[
\begin{align*}
\sigma_b &= (1 - \eta)(1 - \beta), \\
\sigma_w &= \eta + (1 - \eta)(1 - \beta).
\end{align*}
\]

Since \(c(\cdot)\) is a decreasing function, \(c(\sigma_b)\) is increasing in \(\eta\) and \(c(\sigma_w)\) is decreasing in \(\eta\). Under perfect integration (\(\eta = 0\)) we have \(\sigma_b = \sigma_w = 1 - \beta\), and the costs of human capital accumulation are therefore \(c(1 - \beta)\) for both groups. Under complete segregation, \(\eta = 1\) and hence \(\sigma_b = 0\) and \(\sigma_w = 1\). Hence under complete segregation, the costs of human capital accumulation are \(c(0)\) and \(c(1)\) for blacks and whites respectively, where \(c(1) < 0 < c(0)\) from (6).

First consider the case \(\beta < \tilde{\beta}\), which implies \(c(1 - \beta) < 0\). Since \(c(\sigma_w)\) is decreasing in \(\eta\) and is negative at \(\eta = 0\), it is negative for all \(\eta\). Since \(c(\sigma_b)\) is increasing in \(\eta\) and is negative at \(\eta = 0\) and positive at \(\eta = 1\), there exists a unique \(\hat{\eta}(\beta)\) such that \(c(\sigma_b) = 0\). For all \(\eta > \hat{\eta}(\beta)\), we have \(c(\sigma_w) < 0 < c(\sigma_b)\), which implies that \((s_b, s_w) = (0,1)\) is a stable steady state. For all \(\eta < \hat{\eta}(\beta)\), we have \(c(\sigma_w) < c(\sigma_b) < 0\), which implies that \((s_b, s_w) = (0,1)\) cannot be a steady state. Note that any increase in \(\beta\) within the range \(\beta < \tilde{\beta}\) raises \(c(\sigma_b)\). Since \(c(\sigma_b)\) is increasing in \(\eta\), this lowers the value of \(\hat{\eta}(\beta)\), defined as the segregation level at which \(c(\sigma_b) = 0\).

Next consider the case \(\beta > \tilde{\beta}\), which implies \(c(1 - \beta) > 0\). Since \(c(\sigma_b)\) is increasing in \(\eta\) and is positive at \(\eta = 0\), it is positive for all \(\eta\). Since \(c(\sigma_w)\) is decreasing in \(\eta\) and is positive at \(\eta = 0\) and negative at \(\eta = 1\), there exists a unique \(\hat{\eta}(\beta)\) such that \(c(\sigma_w) = 0\). For all \(\eta > \hat{\eta}(\beta)\), we have \(c(\sigma_w) < 0 < c(\sigma_b)\), which implies that \((s_b, s_w) = (0,1)\) is a stable steady state. For all \(\eta < \hat{\eta}(\beta)\), we have \(0 < c(\sigma_w) < c(\sigma_b)\), which implies that \((s_b, s_w) = (0,1)\) cannot be a steady state. Note that any increase in \(\beta\) within the range \(\beta > \tilde{\beta}\) raises \(c(\sigma_w)\). Since \(c(\sigma_w)\) is decreasing in \(\eta\), this raises the value of \(\hat{\eta}(\beta)\), defined as the segregation level at which \(c(\sigma_b) = 0\).

Proof of Proposition 2. First consider the case \(\beta < \tilde{\beta}\). Recall from the proof of Proposition 1 that if the economy is initially at \((s_b, s_w) = (0,1)\), then for all \(\eta < \hat{\eta}(\beta)\), we have \(c(\sigma_w) < c(\sigma_b) < 0\). Hence all individuals in each of the two groups will find it optimal to become skilled, resulting in a transition to \((s_b, s_w) = (1,1)\). This lowers both \(c(\sigma_w)\) and \(c(\sigma_b)\), and hence maintains the condition \(c(\sigma_w) < c(\sigma_b) < 0\). Hence the economy remains at \((s^b, s^w) = (1,1)\) thereafter.

Next consider the case \(\beta > \tilde{\beta}\). Recall from the proof of Proposition 1 that if the economy is
initially at \((s_b, s_w) = (0, 1)\), then for all \(\eta < \bar{\eta}(\beta)\), we have \(0 < c(\sigma_w) < c(\sigma_b)\). Hence all individuals in each of the two groups will find it optimal to remain unskilled, resulting in a transition to \((s_b, s_w) = (0, 0)\). This raises both \(c(\sigma_w)\) and \(c(\sigma_b)\), and hence maintains the condition \(0 < c(\sigma_w) < c(\sigma_b)\). Hence the economy remains at \((s_b, s_w) = (0, 0)\) thereafter.

**Proof of Proposition 3.** Using (1–3), we may write the dynamics of investment levels \(s_b\) and \(s_w\) as follows

\[
\begin{align*}
    s_b(t + 1) &= 1 - G(\bar{a}(\eta s_b(t) + (1-\eta)(\beta s_b(t) + (1-\beta)s_w(t))) \\
    s_w(t + 1) &= 1 - G(\bar{a}(\eta s_w(t) + (1-\eta)(\beta s_b(t) + (1-\beta)s_w(t)))
\end{align*}
\]

For each \(s_w\), define \(h_b(s_w)\) as the set of all \(s_b\) satisfying

\[
s_b = 1 - G(\bar{a}(\eta s_b + (1-\eta)(\beta s_b + (1-\beta)s_w))).
\]

This corresponds to the set of isoclines for the black population, namely the set of points at which \(\Delta s_b \equiv s_b(t + 1) - s_b(t) = 0\) for any given \(s_w\). Similarly, for each \(s_b\), define \(h_w(s_b)\) as the set of all \(s_w\) satisfying

\[
s_w = 1 - G(\bar{a}(\eta s_w + (1-\eta)(\beta s_b + (1-\beta)s_w))).
\]

This is the set of points at which \(\Delta s_w = 0\) for any given \(s_b\). Any state \((s_b, s_w)\) at which \(s_b \in h_b(s_w)\) and \(s_w \in h_w(s_b)\) is a steady state. Now consider the extreme case \(\eta = 0\), and examine the limiting isoclines as \(\beta \to 0\). In this case \(h_b(s_w)\) is the set of all \(s_b\) satisfying

\[
s_b = 1 - G(\bar{a}(s_w))
\]

and \(h_w(s_b)\) is the set of all \(s_w\) satisfying

\[
s_w = 1 - G(\bar{a}(s_w)).
\]

There are exactly three solutions, \(s_l\), \(s_m\), and \(s_h\) to the latter equation. Hence there are three horizontal isoclines at which \(\Delta s_w = 0\), as shown in the left panel of Figure 6. The former equation generates a single isocline \(s_b = h_b(s_w)\) which is strictly increasing, and satisfies \(h_b(0) \in (0, s_l)\), \(h_b(1) \in (s_h, 1)\), and \(h_b(s) = s\) for each \(s \in \{s_l, s_m, s_h\}\), also depicted in the left panel of Figure 6.

As is clear from the figure, only three steady states exist, all of which are symmetric. Only two of these, \((s_l, s_l)\) and \((s_h, s_h)\) are stable. The initial state \((s_l, s_h)\) is in the basin of attraction of the high investment steady state \((s_h, s_h)\). Since the isoclines are all continuous in \(\eta\) and \(\beta\) at \(\eta = \beta = 0\), it follows that for \(\beta\) sufficiently small, integration is equalizing and welfare-improving.
Next consider the limiting isoclines as $\beta \to 1$ (maintaining the assumption that $\eta = 0$). In this case $h_b(s_w)$ is the set of all $s_b$ satisfying
\[
s_b = 1 - G(\tilde{a}(s_b))
\]
and $h_w(s_b)$ is the set of all $s_w$ satisfying
\[
s_w = 1 - G(\tilde{a}(s_b)).
\]

There are exactly three solutions, $s_l$, $s_m$, and $s_h$ to the former equation. Hence there are three vertical isoclines at which $\Delta s_b = 0$, as shown in the right panel of Figure 6. The latter equation generates a single isocline $s_w = h_w(s_b)$ which is strictly increasing, and satisfies $h_w(0) \in (0, s_l)$, $h_w(1) \in (s_h, 1)$, and $h_w(s) = s$ for each $s \in \{s_l, s_m, s_h\}$, also depicted in the right panel of Figure 6. As in the case of $\beta = 0$, only three steady states exist, all of which are symmetric and two of which, $(s_l, s_l)$ and $(s_h, s_h)$, are stable. The initial state $(s_l, s_h)$ is in the basin of attraction of the low investment steady state $(s_l, s_l)$. Since the isoclines are all continuous in $\eta$ and $\beta$ at $\eta = 1 - \beta = 0$, it follows that for $\beta$ sufficiently large, integration is equalizing and welfare-reducing.

Figure 6. Limiting Isoclines for $\eta = 0$, with $\beta \to 0$ (left) and $\beta \to 1$ (right).
References


