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Free to Move: Migration, Tax Competition and Redistribution

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Free to Move: Migration, Tax Competition and Redistribution[1]

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Abstract

We study a model of tax competition between two countries when both skilled and unskilled workers make their migration decisions simultaneously and wages are endogenously determined. If both factors of production are allowed to migrate freely and when the demand for skilled labor is not so elastic, the problem typically predicted in the literature of tax competition that increased mobility of production factors will pose a severe threat to redistribution possibility is less acute than it might first appear. The equilibrium tax rate can be not only positive but also increasing in the degree of mobility of unskilled workers. This is mainly because an initial change in migration flows induced by an increase in the tax rate brings about a higher wage for skilled workers and a lower wage for unskilled workers, which offsets the initial adverse effect. We also show that in contrast to the conventional wisdom in the literature of tax competition decreasing the tax rate invites not only skilled workers but also unskilled workers; unskilled workers always chase skilled workers at the equilibrium.

JEL Categories: D50, F21, H30

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1 Introduction

In almost all countries, personal and household wealth and income are distributed quite unequally, and fiscal authorities are redistributing income and wealth through various fiscal policies. But there is a growing concern that in a globalized world where production factors freely move across the borders, the capacity of redistributive fiscal policies is quite limited, even by a government caring about the welfare of the poor, because redistribution may attract welfare recipients (the poor or unskilled workers) while pushing out sources of the tax base (the rich or skilled workers). (See Epple and Romer (1991), Persson and Tabellini (1992), Rodrik (1997), and Roemer (1997) among others.) Tax competition between fiscal authorities drives the tax rate down because fiscal authorities are playing a Bertrand-type price competition game where each fiscal authority can be better off by undercutting the tax rate of its opponent.

The current paper reexamines the conventional wisdom in the literature of factor mobility and tax competition, using a general equilibrium model with production and free mobility of two production factors – skilled and unskilled labor. We will show that the so-called 'race to the neoliberal bottom' thesis may not hold when the mobility of both unskilled and skilled workers are considered *simultaneously*, and immigranats and natives are *treated equally* in being taxed and receiving benefits. Under the equal treatment principle, the skilled and unskilled workers always move in the same direction. Consequently the exit threat imposed by the rich on the tax-raising country is offset by a countervailing threat imposed by the poor on the tax-undercutting country. Unless the demand for skilled labor is sufficiently elastic, the equilibrium tax rate is not only positive but also increasing in the degree of mobility of unskilled workers.[1]

2 The model

There are two countries located in a unit circle. The distance between the two countries in the circle is $1/2$ in either direction so that the two countries are located

[1] The supply of skilled labor is in general elastic, but the demand for skilled labor may be inelastic.

exactly in the opposite. The Salopian (1979) model specification adopted here is qualitatively identical to the Hotelling's (1929) linear space model when there are only two countries.

There are two types of workers in the world: skilled (S) and unskilled (U) workers.[2] For any type $\theta \in \{U, S\}$, workers are uniformly distributed around the circle and provide e^{θ} efficiency units of labor inelastically. We assume that $e^{U} = 1$ and $e^{S} = e > 1$. The mass of each type of workers is given by m^{θ}, so the total mass of the world population is $m^{U} + m^{S}$. We assume that $m^{U} = 1$ and $m^{S} = m < 1$; unskilled workers outnumber skilled workers. Within each type, there is no ability difference between workers; workers within each type are all identical in the efficiency units of labor they can deliver.

A worker of type-θ who travels distance z^{θ} to work in a country incurs a total travel cost of $d^{\theta} z^{\theta}$, where $d^{\theta} > 0$ is a unit travel cost. The travel cost does not have to be strictly interpreted as a monetary cost. It can be either a worker's dislike for country i or uncertainty that she attaches to the labor market of country i. Due to the presence of the travel cost, workers are imperfectly mobile; even if the post-fisc incomes are smaller in country i, a worker with small $d^{\theta} z^{\theta}$ may still want to migrate into country i.

We interpret the inverse of d^{θ} as the degree of mobility of type-θ workers; a worker is more mobile when she incurs a lower travel cost in moving across the border. We assume $d^{S} \leq d^{U}$, so that skilled workers are more mobile than unskilled workers. When $d^{\theta} = \infty$, workers of type θ are perfectly immobile. As $d^{\theta} \to 0$, on the other hand, the mobility of type-θ workers approaches to a perfect one.

Consider a type-θ worker who is located between countries 1 and 2 in one side of the circle at distance z^{θ} from country 1 and $\frac{1}{2} - z^{\theta}$ from country 2. If her post-fisc income from country 1 is y_1^{θ} and the post-fisc income from country 2 is y_2^{θ}, then she chooses to migrate into country 1 when

$$y_1^{\theta} - d^{\theta} z^{\theta} > y_2^{\theta} - d^{\theta}(\frac{1}{2} - z^{\theta}), \tag{1}$$

[2]Although we develop a model in terms of unskilled and skilled labor, our model is directly applicable to the situation where labor and capital are production factors; we only need to rename the two production factors.

or equivalently

$$z^\theta < \widetilde{z}^\theta = \frac{1}{4} + \frac{y_1^\theta - y_2^\theta}{2d^\theta}. \tag{2}$$

The cutoff point $\widetilde{z}^\theta$ lies in the interior when $\left|y_1^\theta - y_2^\theta\right| \leq d^\theta/2$.

Because there is an identical set of workers (for each type) on the other side of the circle, the total labor supply of type-θ workers for country 1 is

$$\Lambda_1^\theta = m^\theta e^\theta \left(\frac{1}{2} + \frac{y_1^\theta - y_2^\theta}{d^\theta}\right). \tag{3}$$

Since the total supply of type-θ workers in the world is $m^\theta e^\theta$, the labor supply of type-θ workers for country 2 is

$$\Lambda_2^\theta = m^\theta e^\theta \left(\frac{1}{2} + \frac{y_2^\theta - y_1^\theta}{d^\theta}\right). \tag{4}$$

Each country produces a single product, called ‘output’ or ‘income,’ according to a constant returns to scale production function $G_i(L_i^S, L_i^U)$. We normalize the price of the output as 1, and assume that the function G_i enjoys the usual properties of the neoclassical CRS production functions, such as diminishing returns. For a CRS production function, only the ratio of the two production factors matters, so we define $g_i(v_i) \equiv G_i(v_i, 1)$, where $v_i \equiv L_i^S/L_i^U$. We assume the following regarding the functions $g_i(.)$.

Assumption 1 *For all* $i = 1, 2$, $g_i(v_i)$ *is continuous, increasing, concave, and twice-differentiable.*

Labor markets in each country are Walrasian, so that wage rates for each type of workers are determined by their marginal productivity. The profit maximization condition in each country yields

$$w_i^S = g_i^{'}(v_i), \tag{5}$$

and

$$w_i^U = \phi_i(v_i), \tag{6}$$

where $g_i^{'}(v_i)$ is the derivative of $g_i(v_i)$ and $\phi_i(v_i) = g_i(v_i) - v_i g_i^{'}(v_i)$. The concavity of $g_i(.)$ implies that the wage rate of unskilled workers is nondecreasing in v and that of skilled workers is nonincreasing in v. Intuitively, an increase in v increases the marginal productivity of unskilled workers and decreases the marginal productivity of skilled workers. Since $g_i^{'}(v_i)$ is the demand function for skilled labor, we define the elasticity of demand for skilled labor as $\varepsilon_i(v_i) \equiv -g_i^{''}(v_i)v_i/g_i^{'}(v_i) \geq 0$.

The labor market equilibrium condition that labor supply must equal to labor demand for each type of labor yields

$$v_i = \frac{\Lambda_i^S}{\Lambda_i^U} = k\frac{1/2 + \left(y_i^S - y_j^S\right)/d^S}{1/2 + \left(y_i^U - y_j^U)\right)/d^U}, \tag{7}$$

where $k = \frac{m^S e^S}{m^U e^U} = me$.

The government of each country imposes taxes on skilled workers according to a tax function $\tau_i(.)$ and provides transfer payments to unskilled workers according to a transfer function $T_i(.)$ under a balanced budget constraint. Hence the post-fisc income of a skilled worker is

$$y_i^S = w_i^S e - \tau_i(w_i^S e), \tag{8}$$

and that of a unskilled worker is

$$y_i^U = w_i^U + T_i(w_i^U). \tag{9}$$

We assume that the government never levies taxes greater than wages; $w_i^S e - \tau_i(w_i^S e) \geq 0$. This condition is indeed an incentive compatibility condition for skilled workers; otherwise skilled workers will not work.

In the literature of tax competition, many forms of tax and transfer functions have been studied. Some models study tax competition by assuming that $y_i^S = w_i^S e - \overline{\tau}_i$ and $y_i^U = w_i^U + \overline{T}_i$, where $\overline{T}_i$ is lump-sum benefits whereas $\overline{\tau}_i$ is lump-sum taxes. Others have studied with $y_i^S = (1 - t_i)w_i^S e$ and $y_i^U = (1 + s_i)w_i^U$, where $s_i \in [0, 1]$ is the proportional subsidy rate whereas $t_i \in [0, 1]$ is the proportional tax rate. All these models can be succinctly covered by the following forms of tax and transfer functions:

$$T_i(w_i^U) = \beta_0\overline{T_i} + (1 - \beta_0)s_i w_i^U, \tag{10}$$

$$\tau_i(w_i^S e) = \beta_1\overline{\tau_i} + (1 - \beta_1)t_i w_i^S e. \tag{11}$$

If $\beta_0 = \beta_1 = 1$, then $y_i^S = w_i^S e - \overline{\tau}_i$ and $y_i^U = w_i^U + \overline{T}_i$; hence lump-sum transfers are financed through lump-sum taxes. If $\beta_0 = \beta_1 = 0$, then $y_i^S = (1 - t_i)w_i^S e$ and

$y_i^U = (1+s_i)w_i^U$, so that the fiscal policy is implemented by proportional taxes and subsidies. If $\beta_0 = 0$ and $\beta_1 = 1$, then $y_i^S = w_i^S e - \overline{\tau}_i$ and $y_i^U = (1+s_i)w_i^U$; proportional subsides are financed by lump-sum taxes. Finally if $\beta_0 = 1$ and $\beta_1 = 0$, then lump-sum transfers are financed by proportional taxation. Although other combinations are possible, the present paper focuses mainly on the above-mentioned four cases.[3] Hence we assume $T_i \in \{\overline{T_i}, s_i w_i^U\}$ and $\tau_i \in \{\overline{\tau_i}, t_i w_i^S e\}$, where $s_i, t_i \in [0,1]$ and $\overline{\tau}_i, \overline{T}_i$ lie in some nonnegative compact intervals.[4] Note that these four cases are qualitatively all identical when w_i^θ is *exogenously* given. In the model of tax competition, the policy variable is either $\overline{\tau}_i$ or t_i. We denote the policy variable of country i by p_i; hence $p_i \in \{\overline{\tau}_i, t_i\}$.

Because the government budget is balanced, we must have

$$T_i \Lambda_i^U = \tau_i \Lambda_i^S / e. \tag{12}$$

The term Λ_i^U is the total number of unskilled workers in country i, whereas Λ_i^S/e is the total number of skilled workers. Note that this relationship holds whatever the forms of the tax and transfer functions.

Substituting equations (8) and (9) into (7) and using (5), (6), and the relationship $T_i = \tau_i v_i / e$, we obtain the following system of two equations: for $i = 1,2$ and $j \neq i$

$$v_i = k \frac{1/2 + \left((g_i'(v_i)e - \tau_i) - (g_j'(v_j)e - \tau_j)\right)/d^S}{1/2 + \left((\phi_i(v_i) + \tau_i v_i/e) - (\phi_j(v_j) + \tau_j v_j/e)\right)/d^U}, \tag{13}$$

where $\tau_i \in \{\overline{\tau}_i,\ t_i w_i^S e\}$. Note that what matters in the model of tax competition is the form of the tax function; the form of the transfer function is irrelevant. Hence we need to study only two cases, depending on the form of the tax function.

[3]Some models take the form of $y_i^\theta = (1-a_i)w^\theta e^\theta + b_i$, where $a_i \in [0,1]$ and $b_i \geq 0$. This case can be easily studied in our framework if we allow $\overline{\tau}_i$ and s_i to take negative values. By setting $\beta_0 = \beta_1 = \frac{1}{2}$, $\overline{T}_i = -\overline{\tau}_i = 2b_i$, and $-s_i = t_i = 2a_i$, we have $T_i = b_i - a_i w_i^U$ and $\tau_i = -b_i + a_i w_i^S e$. The budget balance equation yields $b_i = \frac{e}{v_i+e}(w_i^U + w_i^S v_i)a_i$ and therefore $\tau_i = -b_i + a_i w_i^S e = a_i(g_i' e - \frac{e}{v+e}(\phi_i + g_i' v_i))$. Computation of the equilbirum with this τ_i is starightforward. Although we do not report here, the equilibrium tax rate depends again on the elasticity of labor demands for skilled workers.

[4]The fact that $\overline{\tau}_i, \overline{T}_i$ lie in compact intervals is a corrolary of our assumption that taxes cannot be greater than wages.

The system of equations (13) determines a vector $\mathbf{v} = (v_1, v_2)$ given parameters $\mathbf{\Theta} = (k, d^S, d^U, e)$ and tax policies $\mathbf{p} = (p_1, p_2)$. By construction, the right hand side expression of equation (13), $\frac{\Lambda_i^S}{\Lambda_i^U}$, is never less than zero. Hence $v_i \geq 0$. Each equation is well-defined if $\Lambda_i^U \neq 0$. The solution to the above system of equations, if exists, will be a function of a policy vector $\mathbf{p} = (p_1, p_2)$ and other parameter values Θ.

In each country the tax policy is determined by maximizing the weighted average of the post-incomes of unskilled and skilled workers; that is, the social welfare function in each country is given by

$$W_i(\mathbf{p}) = \alpha_i(w_i^U(v_i(\mathbf{p})) + \tau_i(\mathbf{p})v_i(\mathbf{p})/e) + (1-\alpha_i)(w_i^S(v_i(\mathbf{p}))e - \tau_i(\mathbf{p})), \quad (14)$$

where $\tau_i(\mathbf{p}) \in \{\overline{\tau_i}, t_i w_i^S(v_i(\mathbf{t}))e\}$.[5] The coefficient $\alpha_i \in [0,1]$ measures the bargaining power of unskilled workers. If $\alpha_i = 0$, then the government in country i reflects only the interest of skilled workers. If $\alpha_i = 1$, then the government in country i reflects only the interest of unskilled workers. We do not model how the bargaining power between the two types of workers is determined in each country.

We completed the description of our model. We now define an equilibrium concept that we will employ in the current paper.

Definition 2 *Suppose $v_i(p_1, p_2)$ is a solution of the equation system given by (13). An* ***equilibrium*** *is a policy vector (p_1^*, p_2^*), a vector of the ratio of the two types of workers (v_1^*, v_2^*), and vectors of equilibrium wage rates $(w_1^{*\theta}, w_2^{*\theta})$ for $\theta \in \{U, S\}$ such that for all $i = 1, 2$ and $j \neq i$*

$$\begin{aligned} p_i^* &\in \arg\max_{p_i} W_i(p_i, p_j^*), \\ v_i^* &= v_i(p_i^*, p_j^*), \\ w_i^{*U} &= \phi(v_i^*), \\ w_i^{*S} &= g'(v_i^*). \end{aligned}$$

Hence each worker chooses a country based on her rationally anticipated policy position that each country will take, and each country sets the policy based on its rational expectation about migration flows.

[5] Alternatively, one can specify the social welfare function as a Nash product. Our result is qualitatively invariant to this alternative specification.

It is in general difficult to obtain a closed-form solution for the system of equations (13) even with a very simple production function. Because of this, we will apply the implicit function theorem in the analysis. We first establish the formula for $\frac{\partial v_i}{\partial p_i}$ and $\frac{\partial v_i}{\partial p_j}$, which will be repeatedly used in the following analysis.

Theorem 3 *Let $\tau_i \in \{\overline{\tau_i},\ t_i g_i' e\}$ be the tax function and $p_i \in \{\overline{\tau_i},\ t_i\}$ the policy instrument of country i. Let*

$$\begin{aligned}
A_i &= \Lambda_i^U - \frac{v_i}{d^U}(v_i g_i''(v_i) - \frac{\partial(\tau_i v_i/e)}{\partial v_i}) - \frac{k}{d^S}(g_i''(v_i)e - \frac{\partial \tau_i}{\partial v_i}),\\
\Lambda_i^U &= \frac{1}{2} + (\phi_i(v_i) + \tau_i v_i - \phi_j(v_j) - \tau_j v_j)/d^U, j \neq i,\\
B_i &= \frac{v_i}{d^U}(v_j g_j''(v_j) - \frac{\partial(\tau_j v_j/e)}{\partial v_j}) + \frac{k}{d^S}(g_j''(v_i)e - \frac{\partial \tau_j}{\partial v_j}), j \neq i,\\
D_{ii} &= (\frac{v_i}{d^U}\frac{v_i}{e} + \frac{k}{d^S})\frac{\partial \tau_i}{\partial p_i},\\
D_{ij} &= -(\frac{v_i}{d^U}\frac{v_j}{e} + \frac{k}{d^S})\frac{\partial \tau_j}{\partial p_j}, j \neq i.
\end{aligned}$$

Then for $i = 1, 2$ and $j \neq i$

$$\begin{aligned}
\frac{\partial v_i}{\partial p_i} &= \frac{-1}{|\mathbf{K}|}(A_j D_{ii} - B_i D_{ji}),\\
\frac{\partial v_i}{\partial p_j} &= \frac{-1}{|\mathbf{K}|}(A_j D_{ij} - B_i D_{jj}),
\end{aligned}$$

where $|\mathbf{K}| = A_1 A_2 - B_1 B_2$.

Proof. The two equations from (13) are

$$\begin{aligned}
v_i &= k\frac{\frac{1}{2} + \left((g_i'(v_i)e - \tau_i) - (g_j'(v_j)e - \tau_j)\right)/d^S}{\frac{1}{2} + \left((\phi_i(v_i) + \tau_i v_i/e) - (\phi_j(v_j) + \tau_j v_j/e)\right)/d^U},\\
v_j &= k\frac{\frac{1}{2} + \left((g_j'(v_j)e - \tau_j) - (g_i'(v_i)e - \tau_i)\right)/d^S}{\frac{1}{2} + \left((\phi_j(v_j) + \tau_j v_j/e) - (\phi_i(v_i) + \tau_i v_i/e)\right)/d^U},
\end{aligned}$$

for $i = 1, 2$ and $j \neq i$. Differentiation of the above two equations with respect to p_i yields

$$\begin{aligned}
A_i\frac{\partial v_i}{\partial p_i} + B_i\frac{\partial v_j}{\partial p_i} + D_{ii} &= 0,\\
B_j\frac{\partial v_i}{\partial p_i} + A_j\frac{\partial v_j}{\partial p_i} + D_{ji} &= 0.
\end{aligned}$$

Solving simultaneously, we have

$$\begin{bmatrix} \frac{\partial v_i}{\partial p_i} \\ \frac{\partial v_j}{\partial p_i} \end{bmatrix} = -\begin{bmatrix} A_i & B_i \\ B_j & A_j \end{bmatrix}^{-1} \begin{bmatrix} D_{ii} \\ D_{ji} \end{bmatrix}$$
$$= -\frac{1}{|\mathbf{K}|}\begin{bmatrix} A_j & -B_i \\ -B_j & A_i \end{bmatrix} \begin{bmatrix} D_{ii} \\ D_{ji} \end{bmatrix},$$

which completes the proof. ($\frac{\partial v_i}{\partial p_j}$ is obtained by reversing i and j from $\frac{\partial v_j}{\partial p_i}$.) ■

Remark 4 *For the sake of later computations, we derive the various variables appeared in Theorem 3.*

(1) If $\tau_i = t_i g_i' e$ and $p_i = t_i$, then $\frac{\partial(\tau_i v_i/e)}{\partial v_i} = t_i(g_i' + g_i'' v_i) = t_i g_i'(1-\varepsilon_i)$, $\frac{\partial \tau_i}{\partial v_i} = t_i g_i'' e$, and $\frac{\partial \tau_i}{\partial p_i} = g_i' e$. Hence $A_i = \Lambda_i^U + \frac{v_i}{d^U}(\varepsilon_i + (1-\varepsilon_i)t_i)g_i' + \frac{e}{d^S}\frac{k}{v_i}(1-t_i)\varepsilon_i g_i'$, $B_i = -\frac{v_i}{d^U}(\varepsilon_j + (1-\varepsilon_j)t_j)g_j' - \frac{e}{d^S}\frac{k}{v_j}(1-t_j)\varepsilon_j g_j'$, $D_{ii} = (\frac{v_i}{d^U}\frac{v_i}{e} + \frac{k}{d^S})g_i' e$ and $D_{ij} = -(\frac{v_i}{d^U}\frac{v_j}{e} + \frac{k}{d^S})g_j' e$ for $j \neq i$.

(2) If $\tau_i = \overline{\tau}_i$ and $p_i = \overline{\tau}_i$, then $\frac{\partial(\tau_i v_i/e)}{\partial v_i} = \overline{\tau}_i/e$, $\frac{\partial \tau_i}{\partial v_i} = 0$, and $\frac{\partial \tau_i}{\partial p_i} = 1$. Hence $A_i = \Lambda_i^U - \frac{v_i}{d^U}(v_i g_i'' - \overline{\tau}_i/e) - \frac{k}{d^S} g_i''(v_i)e$, $B_i = \frac{v_i}{d^U}(v_j g_j'' - \overline{\tau}_j/e) + \frac{k}{d^S} g_j'' e$, $D_{ii} = (\frac{v_i}{d^U}\frac{v_i}{e} + \frac{k}{d^S})$ and $D_{ij} = -(\frac{v_i}{d^U}\frac{v_j}{e} + \frac{k}{d^S})$ for $j \neq i$.

3 Tax competition with identical countries

To obtain analytically tractable results, we will mainly study the symmetric equilibrium under the assumption that the two countries are identical. Elsewhere we showed by simulations that the qualitative results derived from the symmetric equilibrium carry over to asymmetric cases (Lee, 2003). Also to see the intuition behind the model more clearly, we will study with the two extreme forms of the social welfare function, corresponding to $\alpha_i = 1$ and $\alpha_i = 0$. Analysis of the intermediate cases is straightforward. We first compute the symmetric Nash equilibrium when the policy instrument is a proportional tax rate.

Theorem 5 *Suppose $\alpha_i = 1$ and $g_i() = g(.)$ for all i. Suppose the policy instrument of each country is a proportional tax rate t_i. Then the equilibrium tax rate at the symmetric equilibrium is*

$$t_i^* = \begin{cases} 1 & \text{if } \frac{1}{2g'} + \frac{k}{d^U} \geq \frac{e}{d^S} \\ \frac{\frac{1}{2g'} + (\frac{k}{d^U} + \frac{e}{d^S})\varepsilon}{(\frac{k}{d^U} + \frac{e}{d^S})\varepsilon + (\frac{e}{d^S} - \frac{k}{d^U})} & \text{if } \frac{1}{2g'} + \frac{k}{d^U} < \frac{e}{d^S}, \end{cases}$$

where ε is evaluated at k. The equilibrium tax rate is increasing in $\frac{1}{d^U}$ if $\frac{\varepsilon-1}{\varepsilon} < \frac{4g^{'}e}{d^S}$; otherwise it is decreasing in $\frac{1}{d^U}$. The equilibrium tax rate is always decreasing in $\frac{1}{d^S}$.

Proof. At the symmetric equilibrium, we must have $\varepsilon_i = \varepsilon$, $t_i = t$, $\Lambda_i^U = \frac{1}{2}$, $\Lambda_i^S = \frac{k}{2}$, $v_i = k$, $B_i = B = -\frac{k}{d^U}(\varepsilon + (1-\varepsilon)t)g^{'} - \frac{e}{d^S}(1-t)\varepsilon g^{'}$, $A_i = A = \frac{1}{2} - B$, and $D_{ii} = -D_{ij} = kg^{'}(k)(\frac{k}{d^U} + \frac{e}{d^S})$. (See Remark 4-(1).) Therefore from Theorem 3 we compute $|\mathbf{K}| = A^2 - B^2 = \frac{1}{4} - B = \frac{1}{4} + (1-t)(\frac{k}{d^U} + \frac{e}{d^S})\varepsilon g^{'} + t\frac{k}{d^U}g^{'} > 0$ and $\frac{\partial v_i}{\partial t_i} = \frac{-1}{|\mathbf{K}|}(A+B)D_{ii} = \frac{-kg^{'}}{2|\mathbf{K}|}(\frac{k}{d^U} + \frac{e}{d^S})$. Now

$$\begin{aligned}
\frac{\partial W_i|_{\alpha_i=1}}{\partial t_i} &= (\phi_i^{'} + t_i g_i^{''} v_i + t_i g_i^{'})\frac{\partial v_i}{\partial t_i} + g_i^{'} v_i \\
&= \left((\varepsilon + (1-\varepsilon)t)\left(\frac{-kg^{'}}{2|\mathbf{K}|}(\frac{k}{d^U} + \frac{e}{d^S})\right) + k\right) g^{'} \\
&= \frac{kg^{'}}{2|\mathbf{K}|}\left(-(\varepsilon + (1-\varepsilon)t)(\frac{k}{d^U} + \frac{e}{d^S})g^{'} + 2|\mathbf{K}|\right) \\
&= \frac{k(g^{'})^2}{2|\mathbf{K}|}\left(\left(\frac{1}{2g^{'}} + (\frac{k}{d^U} + \frac{e}{d^S})\varepsilon\right) - t\left(\varepsilon(\frac{k}{d^U} + \frac{e}{d^S}) - (\frac{k}{d^U} - \frac{e}{d^S})\right)\right).
\end{aligned}$$

The term $\left(\varepsilon(\frac{k}{d^U} + \frac{e}{d^S}) - (\frac{k}{d^U} - \frac{e}{d^S})\right)$ is positive since $\frac{k}{d^U} = \frac{me}{d^U} < \frac{e}{d^S}$. Hence $\frac{\partial W_i}{\partial t_i}$ is positive up to $\frac{\frac{1}{2g^{'}} + (\frac{k}{d^U} + \frac{e}{d^S})\varepsilon}{(\frac{k}{d^U} + \frac{e}{d^S})\varepsilon + (\frac{e}{d^S} - \frac{k}{d^U})}$ and then negative. If $\frac{\frac{1}{2g^{'}} + (\frac{k}{d^U} + \frac{e}{d^S})\varepsilon}{(\frac{k}{d^U} + \frac{e}{d^S})\varepsilon + (\frac{e}{d^S} - \frac{k}{d^U})}$ is less than 1, then it is the interior equilibrium tax rate; otherwise the equilibrium tax rate is 1 because $\frac{\partial W_i}{\partial t_i} > 0$ for all t_i.

Differentiation of t_i^* with respect to $\frac{1}{d^U}$ yields

$$\frac{\partial t_i^*}{\partial(\frac{1}{d^U})} = \frac{k(2\varepsilon\frac{e}{d^S} - (\varepsilon-1)\frac{1}{2g^{'}})}{\left((\frac{k}{d^U} + \frac{e}{d^S})\varepsilon + (\frac{e}{d^S} - \frac{k}{d^U})\right)^2}.$$

This is positive if $\frac{\varepsilon-1}{\varepsilon} < \frac{4g^{'}e}{d^S}$.

Similarly, differentiation of t_i^* with respect to $\frac{1}{d^S}$ yields

$$\frac{\partial t_i^*}{\partial(\frac{1}{d^S})} = \frac{-e(2\varepsilon\frac{k}{d^U} + \frac{\varepsilon+1}{2g^{'}})}{\left((\frac{k}{d^U} + \frac{e}{d^S})\varepsilon + (\frac{e}{d^S} - \frac{k}{d^U})\right)^2} < 0,$$

which completes the proof. ■

Several remarks can be made regarding Theorem 5.

First, since $\frac{\varepsilon-1}{\varepsilon}$ is increasing in $\varepsilon \geq 0$, our result implies that the equilibrium tax rate is increasing in the mobility of unskilled workers if the labor demand for skilled workers is not so elastic. In particular, if $\varepsilon = 0$, then the equilibrium tax rate is always increasing in the mobility of unskilled workers. If the demand for skilled labor is very elastic, on the other hand, the tax rate decreases as unskilled workers become more mobile. Note that when $\varepsilon \leq 1$, the inequality $\frac{\varepsilon-1}{\varepsilon} < \frac{4g^{'}e}{d^S}$ always holds, so the turning point value of ε is greater than 1.

Second, the equilibrium tax rate when unskilled workers are perfectly immobile ($d^U = \infty$) becomes $t_i^*|_{d^U=\infty} = \min[\frac{\frac{1}{2g^{'}}+\varepsilon(\frac{e}{d^S})}{(\varepsilon+1)\frac{e}{d^S}}, 1]$, and therefore the tax rate is decreasing in $\frac{1}{d^S}$. This result is consistent with the standard result that a higher mobility of skilled workers lowers the equilibrium tax rate. On the other hand, if skilled workers are perfectly immobile ($d^S = \infty$) while only unskilled workers are mobile, then the equilibrium tax rate becomes $t_i^*|_{d^S=\infty} = 1$, because $\frac{1}{2g^{'}} + \frac{k}{d^U} \geq \frac{e}{d^S}$ when $d^S = \infty$. This is due to the fact that the governments are caring only about unskilled workers.

Third, it is straightforward to verify that $t_i^*|_{d^U=\infty} < t_i^* \leq t_i^*|_{d^S=\infty}$.

Our results therefore are very different from the results derived from the models that consider the mobility of only one production factor. In particular, the mobility of unskilled workers offsets the adverse effect of the mobility of skilled workers on the tax rate. What is the intuition behind our results then? To better understand Theorem 5, we derive Lemma 6.

Lemma 6 *Suppose $\alpha_i = \alpha$ and $g_i() = g(.)$ for all i. If the policy instrument of each country is a proportional tax rate, then at the symmetric equilibrium the following results hold.*

(1)$\frac{\partial v_i}{\partial t_i} = \frac{-k}{2|\mathbf{K}|}(\frac{k}{d^U} + \frac{e}{d^S})g^{'} < 0$, *where* $|\mathbf{K}| = \frac{1}{4} + (1-t)(\frac{k}{d^U} + \frac{e}{d^S})\varepsilon g^{'} + t\frac{k}{d^U}g^{'} > 0$;

(2)$\frac{\partial v_i}{\partial t_j} = -\frac{\partial v_i}{\partial t_i} > 0$;

(3) $\frac{\partial \Lambda_i^S}{\partial t_i} = \frac{-ke}{|\mathbf{K}|d^S}(\frac{1}{4} + \frac{ktg^{'}}{d^U})g^{'} < 0$; *and*

(4) $\frac{\partial \Lambda_i^U}{\partial t_i} = \frac{k}{|\mathbf{K}|d^U}(\frac{1}{4} - \frac{tg^{'}e}{d^S})g^{'} \lesseqgtr 0$ *if* $\frac{1}{4} \lesseqgtr \frac{tg^{'}e}{d^S}$.

Proof. We already derived part (1) in Theorem 5. Part (2) is straightforward because $D_{ii} = -D_{ij}$ at the symmetric equilibrium. To prove part (3), note that $\Lambda_i^S = k(\frac{1}{2} + \frac{(g_i^{'}(v_i)e-\tau_i)-(g_j^{'}(v_j)e-\tau_j)}{d^S})$, where $\tau_i = t_i g_i^{'} e$. Hence

$$
\begin{aligned}
\frac{\partial \Lambda_i^S}{\partial t_i} &= \frac{k}{d^S}\left((1-t_i)g_i^{''}e\frac{\partial v_i}{\partial t_i} - (1-t_j)g_j^{''}e\frac{\partial v_j}{\partial t_i} - g_i^{'}e\right) \\
&= \frac{k}{d^S}(2(1-t)g^{''}e\frac{\partial v_i}{\partial t_i} - g^{'}e) \\
&= \frac{k}{d^S}\left(2(1-t)g^{''}e(\frac{-k}{2\left|\mathbf{K}\right|}(\frac{k}{d^U}+\frac{e}{d^S})g^{'}) - g^{'}e\right) \\
&= \frac{-k}{d^S\left|\mathbf{K}\right|}\left(-(1-t)(\frac{k}{d^U}+\frac{e}{d^S})\varepsilon g^{'} + \left|\mathbf{K}\right|\right)g^{'}e \\
&= \frac{-ke}{\left|\mathbf{K}\right|d^S}(\frac{1}{4}+\frac{ktg^{'}}{d^U})g^{'} < 0.
\end{aligned}
$$

Since $v_i = \frac{\Lambda_i^S}{\Lambda_i^U}$, we have $\frac{\partial v_i}{\partial t_i} = \frac{\partial \Lambda_i^S}{\partial t_i}(\frac{1}{\Lambda_i^U}) - \frac{v_i}{\Lambda_i^U}\frac{\partial \Lambda_i^U}{\partial t_i}$. At the symmetric equilibrium $\frac{1}{\Lambda_i^U} = 2$ and $\frac{v_i}{\Lambda_i^U} = 2k$. Hence $\frac{\partial \Lambda_i^U}{\partial t_i} = \frac{1}{k}\frac{\partial \Lambda_i^S}{\partial t_i} - \frac{1}{2k}\frac{\partial v_i}{\partial t_i} = \frac{k}{\left|\mathbf{K}\right|d^U}(\frac{1}{4} - \frac{tg^{'}e}{d^S})g^{'}$, which completes the proof. ■

The first part of Lemma 6 states that as country i increases the tax rate, the ratio of skilled workers to unskilled workers (v) falls. One might *intuitively* think that this is due to the *emigration* of skilled workers from the tax-increasing country *plus* the *immigration* of unskilled workers to that country. But parts (3) and (4) of Lemma 6 shows that this intuition is not always true. Part (3) shows that skilled workers always migrate *out* as the tax rate increases. Part (4) of Lemma 6 shows, however, that the migration flow of unskilled workers depends on the tax rate and the degree of mobility of skilled workers. If the tax rate is sufficiently low or skilled workers are not so mobile (that is, if t/d^S is sufficiently low), then unskilled workers migrate *in* to the tax-increasing country ($\frac{\partial \Lambda_i^U}{\partial t_i} > 0$). If the tax rate is sufficiently high or skilled workers are sufficiently mobile, however, unskilled workers migrate *out* from the tax-increasing country ($\frac{\partial \Lambda_i^U}{\partial t_i} < 0$). What this means is that under certain circumstances skilled and unskilled workers can move *in the same direction.*

The intuition behind this result can be obtained by looking at the post-fisc income of unskilled workers. The first effect of an increase in the tax rate is, by increasing the transfer payment through the tax rate effect, to increase the number of unskilled workers and reduce the number of skilled workers. This first effect reduces v, and so increases w^S. Intuitively, a decrease in the supply of skilled workers increases the wage rate for them. Therefore the effect of an increase in the tax rate on the tax base – which is equal to $w_i^S v_i$– is ambiguous. Furthermore, w^U declines as the number of unskilled workers increases. The combination of the wage rate effect and the tax base effect induced by the first effect provides an opposite incentive for unskilled workers. If skilled workers are not so mobile or the tax rate is not high, then the second combined effects will be smaller than the first effect, so that unskilled workers continue to come in. But if skilled workers are sufficiently mobile or the tax rate is high, then the combined effects will dominate the tax rate effect, so that unskilled workers, looking for higher wages and higher tax bases, will chase skilled workers wherever the latter go.

Indeed this is what happens at the equilibrium. To see this, we compute

$$\frac{1}{4} - \frac{t^* g' e}{d^S} = \frac{4\varepsilon(\frac{k}{d^U} + \frac{e}{d^S})(\frac{1}{4} - \frac{g' e}{d^S}) - (\frac{e}{d^S} + \frac{k}{d^U})}{4((\frac{k}{d^U} + \frac{e}{d^S})\varepsilon + (\frac{e}{d^S} - \frac{k}{d^U}))} < 0, \tag{15}$$

since $(\frac{1}{4} - \frac{g' e}{d^S}) < (\frac{1}{2} - \frac{g' e}{d^S}) < 0$ and $\frac{e}{d^S} - \frac{k}{d^U} > 0$.[6] Hence $\frac{\partial \Lambda_i^U}{\partial t_i} < 0$ at the equilibrium, which implies that increasing the tax rate pushes out not only skilled workers but also unskilled workers. Our result is therefore sharply in contrast to the conventional wisdom in tax competition that skilled workers will migrate *out* whereas unskilled workers migrate *in* to the tax-raising country.

The fact that skilled and unskilled workers can move in the same direction has a dramatic implication for the equilibrium tax rate. When unskilled workers always chase skilled workers, it is not always the best response of a country to undercut the tax rate of the other country, because undercutting the tax rate invites not only skilled workers but also unskilled workers. Hence the undercutting procedure does not continue forever, which implies that each country can propose a positive tax rate at the equilibrium even in the world of free mobility.

But the fact that unskilled and skilled workers move together does not automatically imply that an increase in the mobility of unskilled workers increases the equilibrium tax rate. Although skilled and unskilled workers are moving together in our model, an increase in the mobility of unskilled workers increases the tax rate only when the labor demand for skilled workers is not so elastic.

Whether the demand for skilled labor is elastic or inelastic, of course, depends on a form of the production function. If the production function is that of a CES type, $g(v) = A(\gamma v^\rho + (1-\gamma))^{1/\rho}$, where $A > 0$, $\gamma \in (0,1)$, and $\rho \in (-\infty, 1)$, then

[6] The condition $\frac{1}{2} - \frac{g' e}{d^S} < 0$ is derived from the fact that $t_i^* \leq 1$, which imples that $\frac{1}{2} - \frac{g' e}{d^S} \leq \frac{-kg'}{d^U}$.

the elasticity of labor demand is related with the elasticity of substitution. It is straightforward to show that at the symmetric equilibrium the elasticity of labor demand becomes

$$\varepsilon = \frac{(1-\rho)(1-\gamma)}{\gamma k^{\rho} + 1 - \gamma}. \tag{16}$$

Therefore ε becomes larger as ρ becomes smaller. If $\rho = 1$, the production function is linear and $\varepsilon = 0$. If $\rho = 0$, the production function is Cobb-Douglas and $\varepsilon = 1-\gamma$. Finally if $\rho = -\infty$, then the production function is Leontieff and $\varepsilon = \infty$. Since ρ is the elasticity of substitution between the two production factors, the elasticity of demand for skilled labor in the CES production function becomes smaller when the two production factors are substitutes and larger when the production factors are complements. Therefore another interpretation of our result is that a higher mobility of unskilled workers are conducive to a higher tax rate if unskilled and skilled workers are substitutes. If they are strong complements in production, then a higher mobility of unskilled workers are more likely to decrease the tax rate. Whether the two production factors are substitutes or complements is of course an empirical issue.

When the policy instrument is a lump-sum tax, however, the mobility of unskilled workers is always conducive to a higher tax rate regardless of the elasticity of labor demand.

Theorem 7 *Suppose $\alpha_i = 1$ and $g_i() = g(.)$ for all i. Let the policy instrument of each country be a lump-sum tax. Then the equilibrium tax rate at the symmetric equilibrium is*

$$\overline{\tau}_i^* = \min[\frac{\frac{1}{2} + (\frac{k}{d^U} + \frac{e}{d^S})\varepsilon g'}{(\frac{e}{d^S} - \frac{k}{d^U})/e}, g'(k)e].$$

The equilibrium tax rate is always decreasing in $\frac{1}{d^S}$ and increasing in $\frac{1}{d^U}$.

Proof. At the symmetric equilibrium, we have $\overline{\tau}_i = \overline{\tau}$, $\Lambda_i^U = \frac{1}{2}$, $\Lambda_i^S = \frac{k}{2}$, $v_i = k$, $B_i = B = -(\frac{k}{d^U} + \frac{e}{d^S})\varepsilon g' - \frac{k\overline{\tau}/e}{d^U}$, $A_i = A = \frac{1}{2} - B$, $D_{ii} = -D_{ij} = \frac{k}{e}(\frac{k}{d^U} + \frac{e}{d^S})$. (See Remark 4-(2).) Therefore Theorem 3 implies that $|\mathbf{K}| = \frac{1}{4} - B = \frac{1}{4} + (\frac{k}{d^U} + \frac{e}{d^S})\varepsilon g' +$

$\frac{k\overline{\tau}/e}{d^U} > 0$ and $\frac{\partial v_i}{\partial \overline{\tau}_i} = \frac{-1}{|\mathbf{K}|}(A+B)D_{ii} = \frac{-1}{2|\mathbf{K}|}\frac{k}{e}(\frac{k}{d^U} + \frac{e}{d^S})$. Hence

$$\begin{aligned}\frac{\partial W_i|_{\alpha_i=1}}{\partial \overline{\tau}_i} &= (\phi_i^{'} + \frac{\overline{\tau}_i}{e})\frac{\partial v_i}{\partial \tau_i} + \frac{v_i}{e} \\ &= (\varepsilon g^{'} + \frac{\overline{\tau}}{e})\frac{-1}{2\,|\mathbf{K}|}\frac{k}{e}(\frac{k}{d^U} + \frac{e}{d^S}) + \frac{k}{e} \\ &= \frac{1}{2\,|\mathbf{K}|}\frac{k}{e}\left(-(\varepsilon g^{'} + \frac{\overline{\tau}}{e})(\frac{k}{d^U} + \frac{e}{d^S}) + 2\,|\mathbf{K}|\right) \\ &= \frac{1}{2\,|\mathbf{K}|}\frac{k}{e}\left((\frac{1}{2} + (\frac{k}{d^U} + \frac{e}{d^S})\varepsilon g^{'}) - \frac{1}{e}(\frac{e}{d^S} - \frac{k}{d^U})\overline{\tau}\right).\end{aligned}$$

The term $(\frac{e}{d^S} - \frac{k}{d^U})$ is positive. Hence $\frac{\partial W_i}{\partial \overline{\tau}_i}$ is positive up to $\frac{\frac{1}{2}+(\frac{k}{d^U}+\frac{e}{d^S})\varepsilon g^{'}}{(\frac{e}{d^S}-\frac{k}{d^U})/e}$ and then negative. The incentive compatibility condition implies that $\overline{\tau_i} \leq g^{'}(k)e$. If $\frac{\frac{1}{2}+(\frac{k}{d^U}+\frac{e}{d^S})\varepsilon g^{'}}{(\frac{e}{d^S}-\frac{k}{d^U})/e}$ is less than $g^{'}(k)e$, then it is the interior equilibrium tax rate; otherwise the equilibrium tax rate is $g^{'}(k)e$.

Differentiation of $\overline{\tau}_i^*$ with respect to $\frac{1}{d^U}$ yields

$$\frac{\partial \overline{\tau}_i^*}{\partial(\frac{1}{d^U})} = \frac{\frac{k}{2e} + \frac{2k^2}{d^S}\varepsilon g^{'}}{\left((\frac{e}{d^S} - \frac{k}{d^U})/e\right)^2} > 0.$$

Similarly, differentiation of $\overline{\tau}_i^*$ with respect to $\frac{1}{d^S}$ yields

$$\frac{\partial \overline{\tau}_i^*}{\partial(\frac{1}{d^S})} = \frac{-(\frac{1}{2} + \frac{2k}{d^S}\varepsilon g^{'})}{\left((\frac{e}{d^S} - \frac{k}{d^U})/e\right)^2} < 0,$$

which completes the proof. ■

Theorem 7 tells us that the equilibrium tax rate is always increasing in $\frac{1}{d^U}$. Hence when the transfer payments are financed by lump-sum taxes, an increase in the mobility of unskilled workers always increases the tax rate whatever the elasticity of demand for skilled workers. This is largely due to the fact that the total tax revenue $\frac{\overline{\tau}_i v_i}{e}$ does not directly depend on w^S in the case of lump-sum taxes.

Lemma 8 is a counterpart to Lemma 6 when taxes are lump-sum. The proof of Lemma 8 is omitted because it is basically the same as the proof of Lemma 6.

Lemma 8 *Suppose $\alpha_i = \alpha$ and $g_i() = g(.)$ for all i. If the policy instrument of each country is a lump-sum tax, then at the symmetric equilibrium the following results hold.*

(1)$\frac{\partial v_i}{\partial \overline{\tau}_i} = \frac{-1}{2|\mathbf{K}|}\frac{k}{e}(\frac{k}{d^U}+\frac{e}{d^S}) < 0,$ *where* $|\mathbf{K}| = \frac{1}{4}+(\frac{k}{d^U}+\frac{e}{d^S})\varepsilon g^{'} + \frac{k\overline{\tau}/e}{d^U} > 0;$
(2)$\frac{\partial v_i}{\partial \overline{\tau}_j} = -\frac{\partial v_i}{\partial \overline{\tau}_i} > 0;$
(3) $\frac{\partial \Lambda_i^S}{\partial \overline{\tau}_i} = \frac{-k}{|\mathbf{K}|d^S}(\frac{1}{4}+\frac{k\overline{\tau}/e}{d^U}) < 0;$ *and*
(4) $\frac{\partial \Lambda_i^U}{\partial \overline{\tau}_i} = \frac{1}{|\mathbf{K}|d^U}\frac{k}{e}(\frac{1}{4}-\frac{\overline{\tau}}{d^S}) \lesseqgtr 0$ *if* $\frac{1}{4} \lesseqgtr \frac{\overline{\tau}}{d^S}$.

Again at the symmetric equilibrium, we have

$$\frac{1}{4}-\frac{\overline{\tau}^*}{d^S} = -\frac{(\frac{e}{d^S}+\frac{k}{d^U})\frac{1}{e}+4(\frac{k}{d^U}+\frac{e}{d^S})\frac{\varepsilon g^{'}}{d^S}}{4(\frac{e}{d^S}-\frac{k}{d^U})\frac{1}{e}} < 0, \tag{17}$$

so unskilled workers chase skilled workers.

Until now we have studied the model with $\alpha_i = 1$. Next we turn to another extreme case: $\alpha_i = 0$.

Theorem 9 *Suppose* $\alpha_i = 0$ *and* $g_i() = g(.)$ *for all* i. *Whether the policy instrument is the proportional tax rate or lump-sum taxes, the equilibrium tax rate at the symmetric equilibrium is always zero.*

Proof. (1) Suppose $\tau_i = t_i g_i^{'} e$.

$$\begin{aligned}\frac{\partial W_i|_{\alpha_i=0}}{\partial t_i} &= e(-g_i^{'}+(1-t_i)g_i^{''}\frac{\partial v_i}{\partial t_i}) \\ &= e(-g^{'}+(1-t)g^{''}\left(\frac{-kg^{'}}{2|\mathbf{K}|}(\frac{k}{d^U}+\frac{e}{d^S})\right)) \\ &= \frac{-eg^{'}}{2|\mathbf{K}|}(2|\mathbf{K}|-(1-t)(\frac{k}{d^U}+\frac{e}{d^S})\varepsilon g^{'}) \\ &= \frac{e(g^{'})^2}{2|\mathbf{K}|}\left(-(\frac{1}{2g^{'}}+(\frac{k}{d^U}+\frac{e}{d^S})\varepsilon)-t\left(\frac{2k}{d^U}-(\frac{k}{d^U}+\frac{e}{d^S})\varepsilon\right)\right).\end{aligned}$$

The term $-(\frac{1}{2}+(\frac{k}{d^U}+\frac{e}{d^S})\varepsilon)$ is always negative whereas the sign of the term $\frac{2k}{d^U}-(\frac{k}{d^U}+\frac{e}{d^S})\varepsilon$ is indeterminate. But since $sgn\frac{\partial W_i|\alpha_i=0}{\partial t_i}|_{t_i=1} = sgn(-\frac{1}{2g^{'}}-2\frac{k}{d^U}) < 0$, $\frac{\partial W_i}{\partial t_i}$ is always negative for $t_i \in [0,1]$. Hence the equilibrium tax rate is zero.

(2) Suppose $\tau_i = \overline{\tau}_i$.

$$\begin{aligned}\frac{\partial W_i|_{\alpha_i=0}}{\partial \overline{\tau}_i} &= g_i^{''}e\frac{\partial v_i}{\partial \overline{\tau}_i}-1 \\ &= g^{''}e\frac{-1}{2|\mathbf{K}|}\frac{k}{e}(\frac{k}{d^U}+\frac{e}{d^S})-1 \\ &= \frac{1}{2|\mathbf{K}|}\left((\frac{k}{d^U}+\frac{e}{d^S})\varepsilon g^{'}-2|\mathbf{K}|\right) \\ &= \frac{1}{2|\mathbf{K}|}\left(-(\frac{1}{2}+(\frac{k}{d^U}+\frac{e}{d^S})\varepsilon g^{'})-2\frac{ke}{d^U}\overline{\tau}\right) < 0.\end{aligned}$$

Hence the equilibrium tax rate is zero. ■

The result is not so surprising. Because skilled workers always lose from a higher tax rate, if the governments care only about skilled workers, they will set zero tax rates. Indeed Theorem 9 is a corollary of the fact that the labor supply of skilled workers is decreasing in the tax rate. To see this, consider the case of a proportional tax rate. Since $W_i|_{\alpha_i=0} = (1-t_i)g_i^{'}e$, $\frac{\partial W_i|_{\alpha_i=0}}{\partial t_i} = (1-t_i)g_i^{''}e\frac{\partial v_i}{\partial t_i} - g_i^{'}e$. Since $\Lambda_i^S = k(\frac{1}{2} + \frac{(1-t_i)g_i^{'}(v_i)e-(1-t_j)g_j^{'}(v_j)}{d^S})$, we have $\frac{\partial \Lambda_i^S}{\partial t_i} = \frac{k}{d^S}\left((1-t_i)g_i^{''}e\frac{\partial v_i}{\partial t_i} - g_i^{'}e - (1-t_j)g_j^{''}e\frac{\partial v_j}{\partial t_i}\right)$. Because $g_j^{''} < 0$ and $\frac{\partial v_j}{\partial t_i} > 0$ (part (2) of Lemma 6), $\frac{\partial W_i|_{\alpha_i=0}}{\partial t_i} < \frac{d^S}{k}\frac{\partial \Lambda_i^S}{\partial t_i}$. Hence $\frac{\partial \Lambda_i^S}{\partial t_i} < 0$ (part (3) of Lemma 6) implies $\frac{\partial W_i|_{\alpha_i=0}}{\partial t_i} < 0$, which proves the claim. Similar arguments can be made for the case of the lump-sum taxes.

We have studied the model with two extreme cases. Computation of the equilibrium tax rate in the intermediate cases is straightforward. If α_i is sufficiently large, we have similar results like Theorems 5 and 7. If α_i is sufficiently small, we have the result like Theorem 9.

4 Conclusion

Our results here show that the equilibrium tax rate is not always driven down to zero when the mobility of two factors are considered simultaneously and wages are endogenously determined. The problem of tax competition can be more broadly seen as a cross-border fiscal externality problem. The externality occurs because an increase (a decrease) in the tax rate in one jurisdiction causes an outflow (inflow) of a production factor to other jurisdictions that increases (decreases) their tax revenues. (See Mansoorian and Myers (1993) and Wildasin (1989) for further discussion.) When both factors move simultaneously and wages are endogenously determined, however, the externality problem is less severe because a tax-cutting country imposes not only a *negative* externality to the other country by decreasing the latter's number of tax base but also a *positive* externality by increasing the per-capita income of the tax base.

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