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Steindlian models of growth and stagnation

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Abstract

Following an analysis of the relation between a standard Steindlian model of stagnation and Steindl's own analysis, we modify the standard model by introducing endogenous changes in the markup and a reformulation of the investment function. These extensions, which address significant weaknesses of the standard model, find support in Steindl's writing and leave intact some of Steindl's key results. In a further extension, we add a labour market and analyse the stabilizing influence of a Marxian reserve-army mechanism. The implications of the extended model for the effects of increased oligopolization are largely in line with Steindl's predictions.

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1 Introduction

Steindl explained the depression in the interwar period by the inability of the economy "to adjust to low growth rates because its saving propensity is adapted to a high one" (Steindl 1979, p.1) The argument was laid out in Steindl (1952). In the process of capitalist development, he argued, previously competitive industries become oligopolized. This change in competitive conditions puts upward pressure on the profit margin and makes the profit margin less responsive to changes in demand conditions. An increase in the profit margin may provide the trigger for reduced demand and a reduction in growth rates; the insensitivity of the margin to lower demand and the emergence of unwanted excess capacity potentially turn the downturn into secular depression or stagnation. The economy, in his terminology, becomes 'mature', where maturity is defined "as the state in which the economy and its profit function are adjusted to the high growth rates of earlier stages of capitalist development, while those high growth rates no longer obtain" (1979, p. 7).

The postwar economy was revitalized and experienced a golden age with near full employment and high growth rates from the 1950s to around 1970. This golden age, in Steindl's view, was explained by a combination of expansionary policy (large increases in the government sector in all OECD countries), an acceleration of R&D stimulated by the cold war, increased cooperation between western countries, and the potential for technological catch-up in both Europe and Japan. The stimulus from these factors, he argued, was temporary, and other influences also contributed to a re-assertion of stagnationist tendencies in the 1970s. Steindl singles out, in particular, an increasing trend of personal saving and "a changed attitude of governments towards full employment and growth" (1979, p. 12). This latter influence, which is seen as "the most striking feature of the new economic climate" (1979, p. 12), is explained in terms of a Kaleckian political cycle "as a reaction against the long period of full employment and growth which has strengthened the economic position of workers and the power of the trade unions, and has led to demands for workers' participation" (p. 12-13). Writing in 1979, Steindl therefore expected "low growth for some time to come".

It is beyond the scope of this paper to attempt an empirically based evaluation of Steindl's theory.[1] Our aim is more modest and almost entirely theoretical. Steindl's contributions to an understanding of capitalist growth and stagnation have been highly influential but re-reading his original studies, we have been struck by the fact that important aspects of his argument appear to have been left out of subsequent models. In this paper we try to clarify the connection between a 'standard Steindlian model' and Steindl's own analysis. Secondly, and more importantly, we extend the standard model to include some of the aspects of his analysis that have been left out.

[1] We have reservations with respect to some of his claims. The existence of an increasing trend in monopolization, for instance, is debatable (e.g. Semmler (1984), Auerbach (1988) and Auerbach and Skott (1988)).

Most Steindlian models focus on the product market and treat the markup as exogenous. We outline a standard model of this kind in section 2. Unlike Steindl's (1952) own model, which is set up as mixed difference-differential equations and which has multiple steady growth solutions, the standard model is cast entirely in continuous time and has a unique steady growth solution. In some respects the standard model does a good job of capturing Steindl's argument, and the switch to a continuous-time setting simplifies the analysis enormously. However, the standard model also has weaknesses, both on its own terms and from an exegetical perspective. One weakness is the use of an exogenous markup. This assumption clashes with Steindl's verbal analysis of how "elastic profit margins" tend to eliminate undesired excess capacity in competitive industries and how the "growth of the monopolistic type of industry may lead to a fundamental change in the working of the economy: bringing about greater inelasticity of profit margins" (1952, p. ix). A second weakness concerns the specification of the long-run investment function. The standard model differs from Steindl's own specification in this respect and there are, we shall argue, problems with the standard model as well as with Steindl's own analysis.

Section 3 presents a reformulation of the standard model which addresses the two weaknesses. The reformulation, first, introduces Steindlian movements in the markup. Thus, we assume that the markup will be rising when actual capacity utilization exceeds desired utilization. Other models exist, of course, in which the mark-up changes endogenously, but in these models the determination of the changes is rather different. Dutt (1984), for instance, relates changes in the markup to the rate of growth of the economy while Sawyer (1995) allows the *level* of the markup to depend on the rate of utilization (as indeed did Kalecki (1954)). Although still different, the specifications in Taylor (1985) and Lavoie (1995) which relate changes of the markup to the profit rate come closer to the Steindlian position.

Our second extension of the standard model concerns the investment function. We respecify this function to allow for a distinction between the short-run and the long-run sensitivity of the accumulation rate to changes in utilization. This distinction - central to models in a Harrodian tradition and discussed at some length in Skott (1989a) - is included in Steindl's formal 1952 model as well as in Dutt's (1995) more recent formalization of Steindl's theory. Our specification of the function in this paper is much simpler than Steindl's and more general than Dutt's. The main contribution of the extension, however, lies in the combination of the new investment function with a Steindlian markup dynamics.

Both of the extensions in section 3 find support in Steindl's writing, and they significantly influence the properties of the system. If the new investment function is used in the standard model without markup dynamics, the steady growth path is likely to become unstable. The markup dynamics has a stabilizing influence and the combined model may, but need not, produce a stable steady-growth path. In the stable case, increasing oligopolization leads to a decline in both the rate of growth and the utilization of capital but, paradoxically, to a fall in the share of profits. Thus, the stable case leaves intact some but not all of key results of the standard model.

In section 4 we go beyond the analysis of the product market. Both financial and labour markets play important roles in Steindl's verbal argument; financial markets because of Steindl's emphasis on internal finance and changes in household saving, and labour markets because Steindl regarded prolonged full employment in the 1950's and 1960s as a key factor behind the subsequent stagnation. Financial extensions of the standard model have been explored by Dutt (1995) and in this paper we make no attempt to pursue this aspect of Steindl's analysis. Our emphasis, instead, is on the labour market.

A labour market has been introduced into Steindlian models by Dutt (1992), among others. Our specification, however, differs substantially from his. Following Steindl's (1979) argument we let the rate of employment affect firms' investment decisions and show that the implications of this extended model for the effects of increased oligopolization are largely in line with Steindl's predictions, at least for a range of parameter values. Dutt, by contrast, considers the influence of the rate of employment on wage inflation. He assumes that firms' pricing decisions fail to neutralize these nominal changes in the wage. Thus, the labour market enters his model because of its effects on (the rate of change of) the markup. This mechanism is akin to the one in Goodwin (1967) and other models in which a real-wage Phillips curve generates a rising real wage and a falling markup when employment is high. It should be noted that if one assumes that the employment and utilization rates move together and can be represented by the same variable, a real-wage Phillips curve implies an inverse relation between utilization and the change in the markup - the opposite of the Steindlian assumption.[2]

The paper closes, in section 5, with some conclusions and remarks on future work.

2 Steindl and the standard model

2.1 A standard model in continuous time

We consider a closed economy without public sector. Output is produced using two inputs, labour and capital, and the production function has fixed coefficients. It would be straightforward to include Harrod-neutral technical change but we leave out this element to simplify the exposition. It is assumed that firms retain a proportion s_f of profits and distribute the rest to households in the form of interest payments and dividends, and that

[2]Since it simplifies the analysis, the assumption of a (near-)perfect correlation between the rates of employment and capital utilization is common in the literature. The assumption may be legitimate in the short run but the ratio of the capital stock to the labour force is neither constant nor exogenously given, and the assumption can be highly misleading in the long run.

there is a uniform saving rate s out of distributed incomes, including wages.[3] Investment is positively related to the rate of utilization of the capital stock and may also depend positively on retained earnings. Algebraically, the (net) investment and saving functions are given by

$$\frac{I}{K} = a + m(u-1) + bs_f\pi\frac{u}{k} \tag{1}$$

$$\frac{S}{K} = s_f\pi\frac{u}{k} + s(1-\pi+(1-s_f)\pi)\frac{u}{k} = s(\pi)\frac{u}{k} \tag{2}$$

where I, S and K denote investment, saving and the capital stock, k is the capital-output ratio at the desired utilization rate (normalized to one), u the actual utilization rate and π the share of profits in income (u/k and $\pi u/k$ thus define the actual output-capital ratio and the profit rate). The average saving rate out of income is $s(\pi) = s_f(1-s)\pi + s$. All variables are contemporaneous, and the parameters m, b, s_f and s are positive.

The equilibrium condition for the product market can be written

$$a + m(u-1) + b\frac{s_f}{k}\pi u = \frac{u}{k}\left[s_f(1-s)\pi + s\right] = \frac{s(\pi)}{k}u \tag{3}$$

This equation determines the rate of capacity utilization u as a function of the profit share π. The profit share itself is determined by an exogenously given markup on unit labour cost ($\pi = (\beta-1)/\beta$ where β is the markup).

Solving equation (3) for u we get

$$u = \frac{k(a-m)}{s + s_f(1-s-b)\pi - mk} \tag{4}$$

Using standard assumptions for the adjustment process, the stability of this short-run equilibrium requires that investment be less sensitive than investment to variations in output; that is, $mk + bs_f\pi < s(\pi)$. When this 'Keynesian stability condition' is imposed, the constant a in the accumulation function function must satisfy the restriction $a > m$ in order for the model to produce a positive rate of utilization.

Equations (1)-(4) give rise to Marglin-Bhaduri (1990) possibilities of exhilarationist or stagnationist outcomes. Assuming that the Keynesian stability condition holds, an increase in the profit share will lead to a decline in utilization if the 'Robinsonian stability condition' $0 < [s_f(1-s-b)]$ is satisfied; a reversal of this Robinsonian condition implies that u will

[3] A uniform saving rate out of distributed incomes is in line with Steindl's specification (1952, p. 214, equation (40 vii)). In the presence of retained earnings, the aggregate saving rate depends positively on the profit share. Thus, the introduction of differential saving rates s_p and s_w for household saving out of wage income and distributed profits would leave the structure of the model substantively unchanged.

rise with π and thus will fall with increases in the real wage.[4] The effects on growth are ambiguous. Differentiating $g = \frac{s(\pi)}{k}u(\pi)$ with respect to π, we get

$$\begin{aligned}\frac{\partial g}{\partial \pi} &= \frac{u}{k}\left[s(\pi)\frac{-s_f(1-s-b)}{s+s_f(1-s-b)\pi - mk} + s_f(1-s)\right] \\ &= \frac{us_f}{k}\frac{sb-(1-s)mk}{s+s_f(1-s-b)\pi - mk}\end{aligned}$$

Hence, if both the Keynesian short-run stability condition and the Robinsonian stability condition are met, an increase in the profit share will have a negative impact on growth if

$$sb < (1-s)mk.$$

This ambiguous conclusion mirrors the results in Steindl (1952). Thus, Steindl (1952, p. 224) finds that a rise in the markup depresses growth if the direct effect of the profit share on investment is small relative to the effect of utilization on investment, a condition which is similar to the condition above.

Overall, a linear model with a constant term in the investment function and parameter restrictions that ensure Keynesian and Robinsonian stability might appear to capture the spirit of Steindl's argument. It is not surprising therefore that following early contributions by Rowthorn (1981) and Dutt (1984) and subsequent work by, among others, Taylor (1985), Sawyer (1985) and Marglin and Bhaduri (1990), a model along these lines has become the standard formalization of the Kalecki-Steindl theory.

The rest of this section examines the relation between the standard model and Steindl's (1952) formalization in greater detail. Readers with no interest in this relation may skip directly to section 3.

2.2 The 1952 argument

Steindl's formal model of an economy with variable utilization (1952, pp. 211-228) is cast in terms of mixed difference-differential equations. The key investment equation (equation (39), p. 213) can be written

$$I_{t+\theta} = \gamma\dot{C}_t + q\left(C_t - g_0K_t\right) + m(kY_t - u_0K_t) \tag{5}$$

where θ is a discrete investment lag, k is the ratio of the stock of capital to productive capacity and u_0 the desired utilization rate; the impact of financing conditions are captured by the retained earnings $\dot{C}$ and the stock of "entrepreneurs' capital" C; g_0 is the inverse

[4] Although logically possible and used in some models, the condition for Robinsonian instability seems implausible: empirical evidence suggests that the *impact* effect of changes in real wages falls mainly on consumption, rather than investment.

of the desired gearing ratio; a dot over a variable is used to denote a rate of change (i.e. $\dot{x} = \frac{dx}{dt}$) and the parameters γ, q and m are all positive.[5] Using assumptions similar those of the standard model concerning the determination of retained earnings and personal saving, Steindl derives a dynamic equation for the evolution of the capital stock,

$$\ddot{K}_{t+\theta} - L\ddot{K}_t + M\dot{K}_t + NK_t = 0 \tag{6}$$

where the composite parameters L, M and N can be expressed in terms of the underlying parameters from the functions describing investment and saving.

To solve equation (6), Steindl assumes that the equation represents "a long-run model of moving averages" (p. 227) and that long-run movements may plausibly be described by exponential trends determined by the real roots of (6). Thus, implicitly it is assumed that the initial conditions (i.e. the trajectory of the system over a time interval corresponding to the discrete lag θ) can be written

$$K(t) = \sum c_i \exp \rho_i t \tag{7}$$

where the c_i 's are constants and the ρ_i 's represent the real roots of the characteristic equation

$$\rho^2 \exp(\theta\rho) - L\rho^2 + M\rho + N = 0 \tag{8}$$

Given these initial conditions, the full solution to equation (6) also takes the form (7).

The next step is to find the real roots of (8). It turns out that in order to get any positive roots, additional restrictions on the parameter values have to be introduced. These restrictions *reverse* the Keynesian stability condition in the standard model. Thus using the notation of the standard model, Steindl's necessary condition for positive roots (p. 219) is that

$$\frac{mk + b\pi s_f}{s(\pi)} > 1$$

which is the condition for Keynesian *instability* in the standard model.

Assuming that positive roots exist, the equation will have three real roots and the movements of the capital stock can be described by

$$K(t) = c_1 e^{\rho_1 t} + c_2 e^{\rho_2 t} + c_3 e^{\rho_3 t}$$

where $\rho_1 < 0 < \rho_2 < \rho_3$. Asymptotically, the largest of the three roots dominates the movements in K and, Steindl concludes, the capital stock must therefore grow asymptotically at the high rate ρ_3.[6]

[5] Steindl (1952, p. 213) uses Z rather than the standard notation K to denote the capital stock.

[6] Steindl (incorrectly) suggests that it is the root which is largest in absolute value that will dominate. No harm is done, however, since the large root ρ_3 happens to be the largest in absolute value. The analysis

The comparative statics of the steady growth path associated with ρ_3 can now be examined. From a Steindlian perspective, the effects of increasing oligopolization are particularly interesting. Increasing ologopolization is associated with an upward shift of the profit function (that is, the markup).[7] This shift, Steindl finds, produces a decline in the rate of growth, as long as the expansionary financial effects on investment (represented by the parameters γ and q in equation (5)) are weak relative to the effect of utilization (represented by mk in equation (5)). This condition seems plausible, and the results are strengthened if the rise in the degree of monopoly also leads to increased fears of excess capacity in the industry and a corresponding increase in the desired utilization rate u_0.[8] Thus, the model appears to support Steindl's central conclusion:

> On the basis of the present model it is thus possible to demonstrate that the development of monopoly may bring about a decline in the rate of growth of capital. I believe that this is, in fact, the main explanation of the decline in the rate of growth which has been going on in the United States from the end of the last century. (p. 225)

Unfortunately, the empirical application of the model raises difficulties, and Steindl is refreshingly forthright and clear about these difficulties He points out that "if plausible values are given to the structural coefficients ... then it appears that the limiting rate of growth thus obtained is very big" (p. 226). This problem is serious since it implies that it "is difficult to explain, on the basis of my model, moderate rates of growth, such as has been observed in the history of capitalism" and "either the model requires modifications in important respects in order to be realistic, or else, it follows that an exponential trend in the strict mathematical sense is not a proper description of long-run growth" (p. 226).

(p. 220) is slightly flawed also by a failure to realize that the capital stock will be declining from some point onwards (and reach zero in finite time) if the coefficient c_3 associated with the dominant root is negative. Meaningful non-negative solutions for the long-run capital stock require that the initial conditions are such that $c_3 > 0$ (or, alternatively, such that either $c_2 > c_3 = 0$ or $c_1 > c_2 = c_3 = 0$); implicitly, Steindl's analysis presumes that $c_3 > 0$.

Note finally that although the stability analysis is conditional on very restrictive assumptions concerning the initial conditions, it is not quite correct, as suggested by Dutt (1995, p. 17), that Steindl "does not discuss the dynamic properties of his model" but "only the limiting (or equilibrium) state of the economy".

[7] Steindl included overhead cost and, assuming that these costs are proportional to the capital stock, the profit function links the profit share to the markup and the rate of utilisation.

[8] This second mechanism - introduced partly, perhaps, to get around the ambiguity of the direct effect of changes in the profit share - seems doubtful. If anything, one might expect a *decline* in desired utilization following an increase in oligolization: excess capacity may serve as a deterrent to new entry and the higher the mark-up, the more excess capacity may be required to deter entry. This type of argument is used in a formal model of growth and cycles by Skott (1989a).

2.3 A simplified 1952 model

The complex nature of mixed systems of differential equations with discrete lags makes it difficult to ascertain the reasons for this empirical anomaly in the model. The reasons become clearer if one considers a simplified version of the model in a discrete-time setting. Thus, let

$$I_{t+1} = m(kY_t - K_t) \tag{9}$$

where (to simplify notation) the desired utilization rate has been normalized to unity and k is the capital-output ratio at the desired rate of capital utilization. Aside from the switch to a pure discrete-time system, equation (9) differs from (5) by leaving out the effects of retained earnings and the gearing ratio on accumulation. These effects, it may be recalled, were assumed small relative to the effects of the utilization rate, and it simplifies matters to leave them out altogether.

Combining equation (9) with (a discrete–time version of) the saving function (2), the equilibrium condition $I = S$ implies that

$$\frac{I_t}{K_t} = m\frac{K_{t-1}}{K_t}(u_{t-1} - 1) = m\frac{1}{1 + \frac{s(\pi}{k})u_{t-1}}(u_{t-1} - 1) = s(\pi)\frac{Y_t}{K_t} = \frac{s(\pi)}{k}u_t = \frac{S_t}{K_t}$$

or

$$u_t = \frac{mk}{s(\pi)}\left(\frac{u_{t-1} - 1}{1 + \frac{s(\pi)}{k}u_{t-1}}\right) \tag{10}$$

where $u_t = kY_t/K_t$ is the actual rate of utilization. It is readily seen (see Appendix A) that generically this difference equation has either no stationary point or two stationary points. Furthermore, the existence of stationary points requires (as a necessary condition), that

$$\frac{mk}{s(\pi)} > 1$$

In the case with two stationary points, the high equilibrium is locally stable; the low is unstable. Qualitatively, these conclusions mirror Steindl's results: positive steady growth rates require that the ratio of mk to the average saving rate is sufficiently high.

The outcome is illustrated in figure 1 which uses the parameter values $m = 0.2, k = 2, s(\pi) = 0.1$. Using (10) and figure 1, it is readily seen that a rise in the saving rate $s(\pi)$ (associated with an increase in profit share) generates a shift in the expression on the right hand side of (10) and a decline of the stable solution for u. The growth rate su/k also suffers. To see this, note that the growth rate can be written

$$g = \frac{s(\pi)u}{k} = m\left(\frac{\frac{s(\pi)u}{s(\pi)} - 1}{1 + \frac{s(\pi)}{k}u}\right) = m\left(\frac{\frac{gk}{s(\pi)} - 1}{1 + g}\right)$$

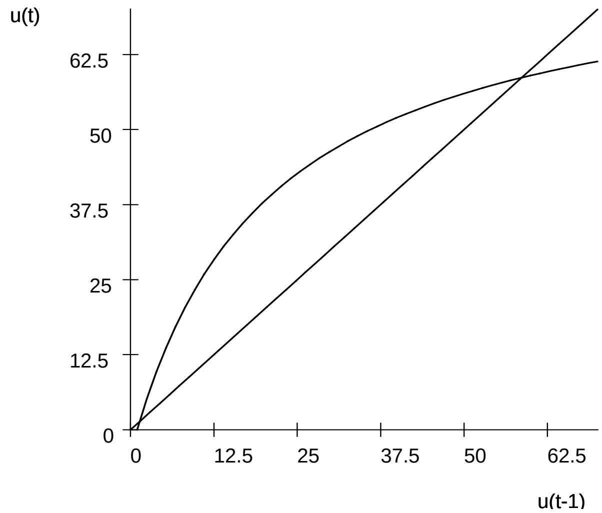

Figure 1: *The two stationary solutions*

The existence of two solutions for the utilization rate implies that this equation in g will also have two solutions; graphically the picture is similar to figure 1. The expression on the extreme right hand side of the equation is decreasing in $s(\pi)$, and it follows that a rise in $s(\pi)$ leads to a decline in the high solution for g.

The stability of the high solutions for u and g may suggest that these, rather than the low and unstable solutions, are the relevant ones. This indeed is the reasoning that guided Steindl's analysis. But consider the special case where the sensitivity m of investment to changes in utilization goes to infinity. The stable solution goes to infinity as $m \to \infty$ and we get a unique, unstable u solution: $u* = 1$. For finite values of m, a high and locally stable solution may exist, but Steindl's problem re-emerges in this simplified setup: for plausible parameter values, the high solution becomes unreasonably high and, as a corollary, the growth rate also becomes too high.[9]

The reason for this problem is transparent in the simplified version. The stable equilibrium owes its existence to the non-linearity on the right hand side of (10). This non-linearity is quite weak, especially for realistic, small values of $s(\pi)$. Hence, the high equilibrium value necessarily becomes large. In figure 1, for instance, the high equilibrium yields a utilization rate of over 58, with desired utilization normalized at unity. Since it is hard

[9] Dutt (1995) also obtains two steady-state equilibria for some parameter values in his formalization for Steindl's theory. Again, the low equilibrium is unstable while the high is stable. Dutt does not comment explicitly on the plausibility of the high equilibrium but notes (p. 28, n.7) that "the model will cease to apply" if the economy hits the full capacity constraint.

to envisage an economy that experiences steady growth with utilization significantly above the desired rate, these observations indicate the empirical and theoretical irrelevance of the high solution.

In support of this conclusion, it should be noted that the economic logic behind the specific non-linearity in equation (10) is difficult to justify. It arises because the investment function (9) imposes a lag: it is investment at time $t+1$ rather than at time t that is determined in period t. The existence of this lag may be reasonable, but it would seem plausible to suppose that firms take into account expected changes in output as well as the changes in the capital stock that are already in the pipeline when they form their investment plans. Thus, we may want to respecify the investment function as

$$I_{t+1} = g^e_{t+1} K^e_{t+1} + m(kY^e_{t+1} - K^e_{t+1})$$

or

$$\frac{I_{t+1}}{K^e_{t+1}} = g^e_{t+1} + m(\frac{kY^e_{t+1}}{K^e_{t+1}} - 1)$$

where g^e_{t+1} is the expected growth rate of demand between periods $t+1$ and $t+2$ (when period-$(t+1)$ investment enters service as part of the productive capital stock) and where K^e_{t+1} and Y^e_{t+1} denote the expected values of the capital stock and the level of output in period $t+1$.

The specification in (9) is obtained as a special case when firms expect both output and the capital stock to remain unchanged so that $K^e_{t+1} = K_t, Y^e_{t+1} = Y_t, g^e = 0$. Changes in the capital stock, however, have already been planned by past - and known - investment decisions. Thus, the capital stock at time $t+1$ should also be known and $K^e_{t+1} = K_{t+1} = K_t(1 + \frac{I_t}{K_t}) = K_t(1 + \frac{s(\pi)}{k} u_t)$. The assumption of static output expectations seems questionable, too, in a long-run model with positive growth rates. It would seem more reasonable to suppose that the expected output growth between period t and period $t+1$ is positively related to actual growth in output between periods $t-1$ and t.[10] Since output at period t is proportional to investment in period t ($I_t = s(\pi)Y_t$), the growth rate between periods $t-1$ and t, in turn, will be positively dependent on the accumulation rate I_t/K_t. Combining these observations, the rate of accumulation may be determined by

$$\begin{aligned} \frac{I_{t+1}}{K_{t+1}} &= g^e_{t+1} + m(u_t \frac{1 + g^e_t}{1 + \frac{s(\pi)}{k} u_t} - 1) \\ g^e_t &= f(\frac{s(\pi)}{k} u_t, z) \end{aligned}$$

where z captures other influences on expected output growth. If, as a simple benchmark, $f(\frac{s(\pi)}{k} u, z) = \frac{s(\pi)}{k} u$ and the growth rates further into the future - between $t+1$ and $t+2$ -

[10] Other factors, including output movements before period $t-1$ may influence expectations, too. These complications are irrelevant for present purposes.

are treated as a constant, we have $g^e_{t+1} = a$ and

$$\frac{I_{t+1}}{K_{t+1}} = a + m(u_t - 1) \tag{11}$$

Using the specification (11), the equilibrium condition $I = S$ yields

$$u_t = \frac{k}{s(\pi)}[a + m(u_{t-1} - 1)] \tag{12}$$

The non-linearity now is gone and there is a unique stationary solution. If we impose Steindl's parameter restriction, $mk > s(\pi)$, this solution is unstable, and an increase in the saving rate raises the equilibrium solutions for both utilization and the growth rate.

The assumptions underlying (11) are, we would argue, at least as plausible as the ones underlying (10). Of course, one may reject both sets of simplifying assumptions. In a more general specification, however, u_t may become either convex or concave in u_{t-1}. Convexity - which may arise when $(1+g^e_t)/(1+I_t/K_t)$ is increasing in u_t - seems as likely as concavity and convexity rules out a high and locally stable solution.

These conclusions (the irrelevance of the high equilibrium and the instability of the low solution) may seem at odds with the general tenor of Steindl's argument. His vision of long-term stagnation would appear to require a stable equilibrium with slow growth and/or high unemployment. In order to achieve this outcome, the sensitivity of investment to variations in utilization needs to be reduced. Indeed, this is what happens through the back door at the high equilibrium in the non-linear case depicted in figure 1: a first-order Taylor approximation of the reduced-form relation between the rate of accumulation and the lagged value of the utilization rate around the high equilibrium has a positive constant and a small coefficient on utilization. As a result, the Keynesian stability condition is satisfied at the high solution.

Having rejected Steindl's high solution, a unique and stable steady-growth solution can be obtained by using (11) instead of (10). All that is required is a reversal of the parameter restriction $mk/s(\pi) > 1$. Moreover, using $a \approx s(\pi)/k$, it is possible to ensure that the steady-growth value of the utilization rate will be in the neighbourhood of one, thus avoiding the anomaly of excessive utilization and growth rates. The lag in the investment function, finally, is of no real importance in the stable case; the long-run results would be the same with a contemporaneous formulation.

Putting together these conclusions, our analysis of the weaknesses of Steindl's own formalization lead us, it might seem, to the simple and transparent formulation of the standard model.

3 A core model of the product market

3.1 Two shortcomings

Although the standard model replicates key Steindlian conclusions, its assumptions violate Steindl's verbal argument in some respects. Two areas of conflict seem particularly prominent.

The first area concerns the specification of the accumulation function. The standard model assumes that the long-run sensitivity of investment to changes in utilization is small and that the constant term is positive. This imposition of the Keynesian stability condition on the long-run investment and saving functions contradicts Steindl's (1952, p. 219) own parameter restrictions. Moreover, both his graphical illustration on p. 128 and his formal specification on p. 214 clearly show that he expected negative accumulation rates for low values of the utilization rate, contrary to the investment function in the standard model.

Steindl's verbal argument is also in line with this conclusion. On p. 123, for instance, he argues that "[i]f the entrepreneur finds himself with more excess capacity than he wants to hold ... he will be strongly discouraged from undertaking any expansion. This discouragement will be even stronger if he knows that such an unusual degree of excess capacity is fairly general in his industry". The destabilizing implications of this argument are clearly stated (p. 123): "The individual entrepreneur may think that by reducing investment he will cure his excess capacity, but in fact for industry as a whole this strategy has only the effect of making excess capacity even greater". Similar statements about the cumulative process arising from the interaction between investment and utilization can be found throughout chapters 9-10 (e.g. p. 115 and p. 135-137), and in chapter 12 (p. 174-5) Steindl goes out of his way to argue that "a cumulative process, with the trend rate of accumulation decreasing more and more" was avoided in the late 19th and early 20th centuries only because "the fall in the profit rate was largely compensated by the cheapening of the terms on which share finance could be obtained".

As pointed out in sections 2.2-2.3, Steindl's own long-run analysis focused on the properties of a questionable steady-growth solution with implausibly high values for the rates of growth and capacity utilization. But this weakness in his formal analysis hardly justifies attributing to him a view of accumulation that he clearly did not hold.

Disregarding questions of exegetical accuracy, the specification of the accumulation function in the standard model seems questionable. There are two separate but closely related issues. The relative insensitivity of investment, first, is plausible in the short run. But a weak impact effect of a change in utilization on investment (which is required for the stability of the short-run Keynesian equilibrium) does not guarantee that the long-term effects of a sustained increase in utilization will be weak, too. Thus, the standard model can be criticized because of its implausible extension to the long run of a restriction - the insensitivity of investment to fluctuations in the utilization rate - that is perfectly reasonable for the short run.[11] Second, like Steindl, we find it hard to conceive of a steady growth path where firms are content to accumulate at a constant rate if, along this path, they have significantly more (or less) excess capacity than they desire. Thus, if the desired rate of utilization were constant, the long-run accumulation function should be infinitely elastic at this desired rate. Managerial constraints or other bottlenecks may make it difficult or costly to expand at high rates and the desired utilization rate, consequently, may not be constant. Instead, it may depend, inter alia, on the share of profits and the rate of accumulation. This complication may modify the analysis and affect some conclusions. Within the relevant range of steady-growth solutions for the rates of accumulation and utilization, however, we find it implausible to assume that the long-run accumulation function will be anything but highly elastic with respect to the rate of utilization.[12]

The second area of conflict between the standard model and Steindl's analysis concerns the determination of the markup. In his verbal discussion Steindl devoted a lot of attention to the influence of demand on the markup. He did not succeed, however, in developing a formal model which incorporated the possibility of adjustments in both the profit margin and the rate of utilization. Instead, he set up two distinct models: one with constant utilization and a flexible profit function (that is, a flexible markup) and the other with variable utilization rates but a fixed markup. He considered neither of these models fully satisfactory: the first was deficient since "the underlying hypothesis of a prompt re-establishment of a given degree of utilisation is not realistic" but the second was not realistic either since "in reality there may be some adjustment of the profit function" (p. 211). Thus, "the actual behaviour of the system will probably be somewhere in between the two extreme cases" (p.212). These concerns are reiterated in Steindl's comments on the results of the second model (p. 228):

> The profit function which I assumed constant in my long run model should not really be so. In reality there will be a certain elasticity of the profit margins, that

[11] Steindl's original specification is preferable in this respect. Although he viewed the long-run accumulation rate as being highly sensitive to variations in utilization, this high sensitivity did not apply to the short run. In his formal model, investment at time t is determined by utilization at time $t - \theta$. Thus, investment is predetermined at any moment, and the impact effect of changes in utilization is zero. As a result, his theory could allow investment to be highly sensitive to changes in utilization in the long run without jeopardizing *short-run* Keynesian stability.

[12] A large literature has developed on the long-run relation between actual and desired utilization rates. Kurz (1986) and Auerbach and Skott (1988) are among those who have insisted that actual utilization must equal desired utilization on a steady growth path; Lavoie (1995) surveys the debate.

Following Amadeo (1986), Lavoie also suggests that the equalization of actual and desired utilization can be reconciled with the long-run variability of the utilization rate: simply treat the desired rate of utilization itself as an endogenous variable whose rate of change is proportional to the difference between actual and desired utilization. As a result, the desired utilization becomes an accommodating variable that can take any value in the long run. From a logical perspective this argument is clearly correct but we find the approach unconvincing, Dutt's (1997) attempt to provide a rationale for the adjustment process notwithstanding.

> is the profit function will depend on the degree of utilisation (a high utilisation shifting it upwards, and a low utilisation downwards). My mathematical model does not include this complication, and it is in this respect poorer than the verbal exposition of the theory in the earlier chapters.

The movements in the markup (which in our simple version is identical to the profit share) will be strong in competitive regimes. The transition to oligopolistic regimes weakens the adjustment mechanism, but according to Steindl a tendency remains for the markup to rise (resp. fall) when utilization is above (resp. below) the desired level.[13] Thus, we see no justification for assuming a fixed markup in Steindlian models of growth and stagnation.[14]

Of course, even if a fixed markup cannot be attributed to Steindl, one might still view this assumption as a reasonable representation of real-world pricing. It is beyond the scope of the present paper to consider this question in any detail. It should be noted, however, that at the micro level there is evidence of significant variability in prices and markups. A study by Levy et el. (1997), for instance, found that a sample of US supermarkets changed an average of 16% of their prices every week and that most of these changes were unrelated to cost changes. Thus, Steindl's model might err by attributing too little rather than too much flexibility to the markup. An alternative, Marshallian approach reverses the adjustment speeds of output and the profit margin. Using this approach, Skott (1989a, 1989b) treats the profit margin as a fast variable and output as a gradually adjusting state variable.

In the remainder of this section we address the two areas of conflict between Steindl's own analysis and the standard model. We extend the standard model by adding dynamic equations to describe induced shifts in both the markup and the accumulation function. The implications of these shifts and their interactions will be examined in subsection 3.4. First, however, the two types of dynamic adjustment are considered separately in subsections 3.2-3.3.

3.2 Markup dynamics

The endogenous movements in the profit share can be captured by an expectations-augmented price Phillips curve which relates price inflation to deviations of actual from

[13] The adjustment of price margins to maintain a 'normal' or desired rate of utilization can also be found in Joan Robinson's writings, e.g. Robinson (1962, p. 46): "let us suppose that competition (in the short-period sense) is sufficiently keen to keep prices at the level at which normal capacity output can be sold."

[14] It should be noted that Steindl regarded the flexibility of the profit function is a long-run property. While unsatisfactory for long-run analysis, "the rigidity of the profit function is probably realistic for the short run model" (1952, p. 228) where the short run model is defined as one designed to explain "the ordinary business cycle".

desired utilization (rather than to conditions in the labour market).[15] Thus, let

$$\hat{p} = f(u,\pi) + \hat{w}^e; f_u > 0, f_\pi \leq 0, f(u,0) > 0, f(u,1) < 0$$

where $\hat{p} = \dot{p}/p$ is the rate of inflation and $\hat{w}^e$ the expected rate of wage inflation. Utilization reflects current demand in the product market relative to firms' capacity and has a positive effect on price inflation. It is assumed that firms always aim for a profit share between zero and one (that is, $f(u,0) > 0, f(u,1) < 0$ for any value of the utilization rate), and a non-positive feedback from the current profit share is included to ensure this property. We assume that the rate of wage inflation is correctly anticipated by firms ($\hat{w}^e = \hat{w}$). The rate of wage inflation therefore has no impact on the share of profits. As in other simple Keynesian models, moreover, aggregate demand is invariant with respect to changes in both the level and the rate of change of the money wage. Thus, for present purposes there is no need to specify a wage Phillips curve.

Using $\hat{w}^e = \hat{w}$, the price Phillips curve implies that

$$\dot{\pi} = -\frac{d}{dt}(\frac{w}{p}\frac{L}{Y}) = -\frac{w}{p}\frac{L}{Y}(\hat{w} - \hat{p}) = (1-\pi)f(u,\pi) \tag{13}$$

Combining equation (13) with the equilibrium condition for the product market, equation (4), the result is a first-order differential equation in π,[16]

$$\dot{\pi} = (1-\pi)f(u(\pi),\pi) = \phi(\pi); \quad \phi(0) > 0; \phi(\pi) < 0 \text{ for } \bar{\pi} < \pi < 1 \tag{14}$$

This equation has at least one locally stable stationary solution between zero and one. Uniqueness is ensured if the Robinsonian stability condition is met since in this case $\phi'(\pi) < 0$. If the Robinsonian stability condition fails to be satisfied, the derivative of ϕ cannot be unambiguously signed, and there may be multiple solutions. Even with multiple solutions, we still get convergence of the profit share to a stationary point but initial conditions will determine which one. Using (4), it follows that $u \to u(\pi^*)$ if $\pi \to \pi^*$.

The comparative statics are straightforward. At a locally stable stationary point

- a marginal upward shift in the investment function (a rise in a) leads to an increase in both u^* and π^*. The growth rate also increases.
- a marginal increase in the saving propensity s leads to a decline in both u^* and π^*, and the growth rate also suffers.

[15] See Flaschel and Krolzig (2002) for a general analysis of the specification and interaction of wage and price Phillips curves.

[16] If $f(u,\pi)$ is continuous and $f(u,1) < 0$, the inequality $\phi(\pi) = (1-\pi)f(u,\pi) < 0$ must hold for values of π above some threshold $\bar{\pi}$ (that is, for $\bar{\pi} < \pi < 1$).

From a mathematical perspective this adjustment equation is similar to Lavoie's (1995, p.811) specification of changes in the 'target rate of return'.

- a marginal weakening of competition (an upward shift in the f-function) leads to an increase in π^*. Utilization falls if the Robinsonian stability condition is met, and the growth effects of weaker competition and higher profit margins are ambiguous, as in the standard model.

3.3 Accumulation dynamics

The combination of a low short-run but high long-run sensitivity of investment to changes in utilization can be captured by introducing dynamic adjustments in the constant term a in the investment function. These changes in a are related to the discrepancy between actual and desired utilization but, in accordance with our discussion in section 2, we allow for the possibility that the desired rate of utilization may depend on both profitability and the growth rate. Thus, let

$$\dot{a} = h(u, \pi, g); \quad h_1 > 0, h_2 \geq 0, h_3 \leq 0 \tag{15}$$

where g is the current rate of accumulation and desired utilization is defined implicitly by the stationarity condition, $h(u, \pi, g) = 0$.

The formulation in (1) and (15) generalizes the approach used by, among others, Dutt (1995). Dutt takes actual accumulation g as predetermined at each moment while the desired accumulation rate is determined by utilization and profitability (as well as the gearing ratio, a variable that we have left out). The change in g is assumed proportional to the difference between desired and actual accumulation. Thus,

$$\dot{g} = \theta(g^d(u, \pi) - g)$$

Setting $m = b = 0$ in the accumulation function (1), Dutt's specification emerges as a special case of equations (1) and (15).

Combining (1), (4) and (15) - and treating π as constant - we get a one-dimensional differential equation for the movements in 'animal spirits',

$$\dot{a} = h(u(a, \pi), \pi, g(a, \pi)) = \psi(a)$$

The sign of ψ' is ambiguous: both utilization and accumulation depend positively on the value of a, and the net feedback from a to the rate of change in a therefore depends on the partials h_u and h_g that describe the relative strength of the effects of u and g. In principle there could be multiple stationary points (or no stationary points), and even in the case of a unique stationary point the stability properties are undetermined. But since, as argued above, the effects of utilization are likely to be strong and the negative feedback effects from changes in the growth rate weak within the relevant range, the most likely outcome is one with a unique, unstable stationary point.

As a simple example, consider the Harrodian case in which desired utilization is exogenously given and constant, that is $h_2 \equiv h_3 \equiv 0$, Normalizing desired utilization at unity and assuming that the change in a is proportional to the difference between actual and desired utilization, the shift in a is given by

$$\dot{a} = \lambda(u-1); \qquad \lambda > 0$$

Substituting for u, we get

$$\dot{a} = \lambda(\frac{k(a-m)}{s + s_f(1-s-b)\pi - mk} - 1)$$

This equation has a unique, unstable stationary solution

$$a = \frac{s + s_f(1-s-b)\pi - mk}{k} - m$$

The warranted growth rate at the associated (unstable) growth path is given by the standard Harrodian expression $g = \frac{s(\pi)}{k}$. This growth rate is a continuous-time analogue to the empirically relevant, unstable solution in Steindl's model. Comparative statics can be readily derived (and in fact are well-known from the Harrodian literature). These comparative statics are interesting insofar as one has some indication that forces outside the model keep the economy near the steady growth path. Policy intervention, for instance, or feedback effects from the labour market may play this role. The next subsection, however, considers an alternative Steindlian mechanism: the stabilizing influence of induced changes of the markup.

3.4 Combining markup and accumulation dynamics

Using equations (1), (4) and (14)-(15), we get a two-dimensional system of differential equations in a and π:

$$\begin{aligned} \dot{a} &= h(u(a,\pi), \pi, g(a,\pi)) = H(a,\pi); H_a > 0 && (16) \\ \dot{\pi} &= (1-\pi)f(u(a,\pi),\pi) = G(a,\pi); G_a > 0, G_\pi < 0 && (17) \end{aligned}$$

The properties of this system depend on the functions H and G. We assume that both the Keynesian and the Robinsonian stability conditions are met so that $u_\pi < 0$. Hence,

$$\begin{aligned} G_a &= (1-\pi)f_u u_a > 0 && (18) \\ G_\pi &= (1-\pi)(f_u u_\pi + f_\pi) - f \\ &= (1-\pi)(f_u u_\pi + f_\pi) < 0 \text{ at a stationary point with } \dot{\pi} = f = 0 && (19) \end{aligned}$$

Turning to the partials of H, we have

$$\begin{aligned} H_a &= h_u u_a + h_g g_a > 0 & (20) \\ H_\pi &= h_u u_\pi + h_\pi + h_g g_\pi & (21) \end{aligned}$$

The first term in the expression for H_a is positive and the second negative. In line with the discussion in subsection 3.3, however, we assume that the first term will dominate and that the pure accumulation dynamics is destabilizing; that is, we consider the case in which $H_a > 0$. The partial H_π, on the other hand, is difficult to sign on either theoretical or empirical grounds. As a result, qualitatively diverse dynamic scenarios are possible.

In terms of the reduced forms H and G, the Jacobian of the system is given by

$$J(a, \pi) = \begin{pmatrix} H_a & H_\pi \\ G_a & G_\pi \end{pmatrix}$$

and the stationary point is locally stable if (evaluated at the stationary point) we have

$$\begin{aligned} Det(J) &= H_a G_\pi - G_a H_\pi > 0 \\ Tr(J) &= H_a + G_\pi < 0 \end{aligned}$$

Saddlepoint instability is obtained if $Det(J) < 0$, and the system generates an unstable node or focus if $Det(J) > 0$ but $Tr(J) < 0$.

In Appendix B we illustrate these possibilities. One case allows for a stable steady growth path and generates roughly Steindlian results. This case demonstrates that a system which has an unstable equilibrium when the markup is exogenous may be stabilized by endogenous changes in the markup, provided the adjustments of the markup are sufficiently fast. A second case (also based on assumptions that seem plausible a priori) implies saddlepoint instability. This case shows that fast adjustment in the markup may not suffice to stabilize the system. Both cases are characterized using assumptions concerning the underlying functions f, g and h.[17]

The phase diagram in figure 2 depicts the dynamics when the system (16)-(17) generates a node or a focus and, assuming stability, the figure can be used to examine the effects of changes in competition. According the Steindl, a decline in competition puts upward pressure on the markup and, secondly, leads to a decline in the adjustment speed of the markup. The first effect corresponds to an upward shift in the Phillips curve (the f-function) while the second can be parameterized by introducing a multiplicative constant μ in the equation for the change in the profit share. Thus, the effects of changes in the degree of competition can be captured by re-writing equation (17) as

$$\dot{\pi} = (1 - \pi)\mu \left[f(u(a, \pi), \pi) + \nu \right] = G(a, \pi; \mu, \nu)$$

[17] The two cases do not exhaust the set of possibilities with respect to these underlying functions.

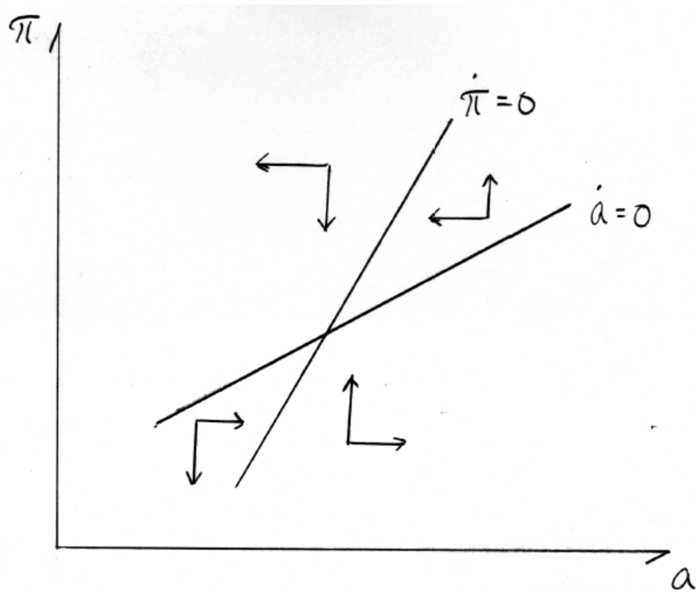

Figure 2: *Stabilizing markup dynamics*

The benchmark degree of competition in (17) is associated with $\mu = 1$ and $\nu = 0$; increased oligopolization leads to a reduction in μ (slower adjustment speeds) and a rise in ν (upward pressure on the markup).

Consider now the effects of increased oligopolization. The rise in ν implies an upward shift in the $\dot{\pi} = 0$ locus while changes in μ have no effects on the slope or position of either of the two loci. The upward shift in the $\dot{\pi} = 0$ locus entails a decline in the stationary solutions of both a and π (see figure 2), and the rates of utilization and accumulation must fall too. To see this, note that by assumption $f_\pi \leq 0$, and the rise in ν must therefore be associated with a decline in u if $f(u, \pi) + \nu$ is to remain equal to zero. The fall in the rate of accumulation now follows from the decline in both u and π (since $g = \frac{s(\pi)}{k}u$).

Although changes in the parameter μ have no effects on the shapes and positions of the $\dot{\pi} = 0$ and $\dot{a} = 0$ loci, these changes may still be of critical importance. By assumption the pure accumulation dynamics is unstable ($H_a > 0$), and there is therefore a critical value $\bar{\mu}$ such that $Tr \gtreqless 0$ for $\mu \lesseqgtr \bar{\mu}$. It follows that a decline in competition may destabilize a previously stable equilibrium.

Overall then, assuming stability of the stationary solution, the Steindlian system (16)-(17) with the restrictions (18)-(20) implies that increased oligopolization will (i) produce a decline in the equilibrium values of utilization and the rate of accumulation, and (ii) endanger the local stability of the equilibrium. These implications are consistent with Steindl's conclusions. But somewhat surprisingly - and contrary to Steindl's analysis - increased oligopolization (an upward shift in the price Phillips curve) ultimately generates a fall in the profit share.[18] Of course, one could re-define increased oligopolization as a rise in the markup, rather than an upward shift in the price Phillips curve. This alternative definition evades the paradox but produces an un-Steindlian positive relation between oligopolization and growth. In any case, the paradoxical rise in the profit share following an upward shift in the price Phillips curve is reversed when we add reserve-army effects.

4 Adding a labour market

4.1 The reserve army of labour

The dynamic systems developed so far have focused on the product market and the interaction between investment, saving, finance and pricing decisions. The neglect of the labour market is striking but not entirely un-Steindlian. Steindl (1952, p. 168) for instance points out that, since it is strongly influenced by immigration, the growth of the working population is as much an effect as a cause of the trend in accumulation. The same conclusion is reached in his discussion of Marx on pp. 233-34. According to this position, the growth of the labour force is endogenous and does not constrain accumulation.

It is hard to square this dismissal of any role of the labour supply with Steindl's (1979, p. 12) argument that "the most striking feature of the new economic climate" is the way prolonged near-full employment "has strengthened the economic position of workers and the power of trade unions, and has led to demands for workers' participation". As a result of these demands, he argues, the attitudes of governments and big business alike have changed:

> Formerly there was a general conviction in most countries that the government would intervene to prevent a prolonged depression; this reduced uncertainty and therefore made for higher and more stable private investment. This confidence has been shattered. Here is another reason why the function ϕ [the investment function] has shifted downwards (1979, p. 13)

Steindl's seemingly contradictory suggestions with respect to labour market conditions may be reconciled by noting that although there have been periods of low official unemployment

[18] This paradoxical result is in line with Dumenil and Levy's (1996) data on US trends in profitability after the civil war. Steindl, who did not have profit data for this period, argued that "towards the end of the last century ... the American economy had undergone a transition which gave considerable weight to the oligopolistic pattern in the total economy" (p. 191). He suggested that this transition would have raised profit margins. According to Dumenil and Levy's data, however, the profit share declined at an average annual growth rate of 0.4 percent between 1869 and 1910.

both before the big depression and in the 1950s and 1960s, the supply of labour to the modern, capitalist part of the economy was quite elastic. Up until the 1960s most OECD countries had hidden reserves of unemployment in agriculture, in parts of the service sector and among women, and, as pointed out by Steindl, immigration also helped alleviate any shortages of labour. The hidden reserve army gradually became depleted, however, and immigration was hampered by growing political resistance. As a result, the economy became mature in Kaldor's (1966) sense of the word: its growth rate became constrained by the growth in the labour force.

We formalize this argument (which may or may not be a reasonable representation of Steindl's thinking) by including an effect of labour market conditions on the shifts in the investment function. Thus, let

$$\begin{aligned} \dot{a} &= h(u(a,\pi),\pi,g(a,\pi),e); \quad h_1 > 0, h_2 \geq 0, h_3 \leq 0, h_4 < 0 \\ &= H(a,\pi,e) \end{aligned} \tag{22}$$

where e is the measure of labour market conditions. We shall refer to this variable simply as the employment rate. In any empirical application, however, the role of hidden unemployment as well as the possibility of obtaining workers through immigration must be taken into account. The equation describes how 'animal spirits' suffer under full employment, leading to a gradual, downward shift in the investment function. These employment effects are likely to be non-linear: negligible at high levels of unemployment but very substantial when the economy approaches full employment. Equation (22) captures, we believe, Steindl's main point - a point which is closely related to Kalecki's (1943) insights that persistent high employment undermines "the social position of the boss" and "the self assurance and class consciousness of the working class" grows (quoted from Kalecki (1971, p. 140-1).

The movements in the employment rate depend on changes in the labour force, output and technology. Assuming Harrod-neutral technical progress, we have

$$\begin{aligned} \dot{e} &= e(\hat{u} + g - n) \\ &= e\left[\frac{u_a(a,\pi)}{u(a,\pi)}\dot{a} + \frac{u_\pi(a,\pi)}{u(a,\pi)}\dot{\pi} + g(a,\pi) - n\right] \end{aligned} \tag{23}$$

where n is the growth rate of the labour force in efficiency units. For simplicity we shall ignore the possibility of induced changes in the growth of the labour supply and take n to be constant.

4.2 Employment and accumulation dynamics

Equations (22)-(23) and (17) constitute a three-dimensional system in a, π, e. First, however, we shall consider the special case with a constant markup, that is, the case in which

$\dot{\pi} \equiv 0$. We then have the following two-dimensional system

$$\begin{aligned} \dot{a} &= H(a,e); H_a > 0, H_e < 0 \\ \dot{e} &= e\left[\frac{u_a(a)}{u(a)}\dot{a} + g(a) - n\right] = F(a,e) \end{aligned}$$

where it is assumed, as in section 3, that the pure accumulation dynamics is unstable ($H_a > 0$).

At a stationary point we get

$$\begin{aligned} F_a &= e\left[\frac{u_a(a)}{u(a)}H_a + g'(a)\right] > 0 \text{ since } g'(a) > 0, \frac{u_a(a)}{u(a)} > 0, H_a > 0 \\ F_e &= e\frac{u_a(a)}{u(a)}H_e < 0 \end{aligned}$$

Hence, evaluated at a stationary point, the Jacobian takes the following form

$$J(a,e) = \begin{pmatrix} H_a & H_e \\ e\left[\frac{u_a(a)}{u(a)}H_a + g'(a)\right] & e\frac{u_a(a)}{u(a)}H_e \end{pmatrix}$$

and

$$\begin{aligned} Det(J) &= -eg'(a)H_e > 0 \\ Tr(J) &= H_a + e\frac{u_a(a)}{u(a)}H_e \end{aligned}$$

It follows that the stationary solution represents a node or a focus. Unlike the system with markup and accumulation dynamics, saddlepoint instability can be excluded. But analogously to the case of a node/focus in section 3.4, stability is ensured if animal spirits adjust slowly relative to the adjustment in the stabilizing variable, in this case the employment rate. A Marxian reserve army effect, in other words, may help to stabilize the economy.

4.3 Employment, accumulation and markup dynamics

Now consider the full three dimensional system consisting of (22)-(23) and (17). If $g_\pi(a,\pi) \geq 0$, there is (at most) one stationary solution.[19] To see this, note that stationarity requires

$$\begin{aligned} g(a,\pi) &= n; & g_a > 0, g_\pi \geq 0 \\ G(a,\pi) &= 0; & G_a > 0, G_\pi < 0 \end{aligned}$$

[19] Without the restriction $g_\pi \geq 0$ there may (but need not) be multiple solutions. The restriction is satisfied in the special case analyzed by Dutt (1995) - who assumes $g = a$ and $g_\pi = 0$ - as well as by all exhilirationist cases.

These two equations cannot have more than one solution for a^* and π^*. Having found a^*, π^*, the equilibrium solution for the employment rate can be derived from

$$\dot{a} = H(a^*, \pi^*, e) = 0; H_e < 0$$

Evaluated at a stationary point the Jacobian of the three-dimensional system takes the following form

$$J(a, \pi, e) = \begin{pmatrix} H_a & H_\pi & H_e \\ G_a & G_\pi & 0 \\ e(\frac{u_a}{u} H_a + \frac{u_\pi}{u} G_a + g_a) & e(\frac{u_a}{u} H_\pi + \frac{u_\pi}{u} G_\pi + g_\pi) & e\frac{u_a}{u} H_e \end{pmatrix}$$

The necessary and sufficient Routh-Hurwitz conditions for local stability are that

1. $Tr(J) = H_a + G_\pi + e\frac{u_a}{u} H_e < 0$
2. $Det(J_1) + Det(J_2) + Det(J_3) = eH_e[\frac{u_a}{u} G_\pi - \frac{u_\pi}{u} G_a - g_a] + H_a G_\pi - G_a H_\pi > 0$
3. $Det(J) = eH_e[g_\pi G_a - g_a G_\pi] < 0$
4. $-Tr(J)[Det(J_1) + Det(J_2) + Det(J_3)] + Det(J) > 0$

The second condition will be satisfied if the stabilizing effect of the reserve army is sufficiently strong (that is, for sufficiently large absolute values of H_e). To see this, note that using (18)-(19) the term in square brackets can be written

$$\frac{u_a}{u} G_\pi - \frac{u_\pi}{u} G_a - g_a = \frac{u_a}{u}(1 - \pi) f_\pi - g_a$$

Since $u_a > 0, f_\pi \leq 0$ and $g_a > 0$ the right hand side of this equation is unambiguously negative.

The third condition is satisfied if the expression in the square brackets is positive and, when combined with the signs of the other partials, our earlier condition for uniqueness ($g_\pi \geq 0$) is sufficient to ensure that this is the case. If the second and third conditions are met, finally, the first and fourth will also hold if the absolute value of H_e is large. To see that the fourth condition will be met, note that $-Tr(J)[Det(J_1) + Det(J_2) + Det(J_3)]$ is quadratic in H_e while $Det(J)$ is linear.

It may be interesting to look briefly at the comparative statics of increasing oligopolization for the three-dimensional systems. Increasing oligopolization corresponds to an upward shift in the $G(a, \pi)$-equation that describes the mark-up dynamics. We have $G_a > 0$ and $G_\pi < 0$, and this upward shift therefore has to be offset by an increase in π and / or a decline in a. Since the long-run rate of growth $g(a, \pi)$ must remain equal to n and since by assumption $g_a > 0$ and $g_\pi \geq 0$, it follows that a and π cannot move in the same direction. Thus, π must increase while the change in a must be non-positive. Utilization, which is increasing in a but decreasing in π, therefore must fall. The effect on employment, finally, can be found from the stationarity condition for a: $h(u, \pi, g, e) = 0$. Since $g = n$ is unchanged we have

$$0 = h_u(u_a da + u_\pi d\pi) + h_\pi d\pi + h_e de$$

or

$$de = \frac{[h_u(u_a da + u_\pi d\pi)] + h_\pi d\pi}{-h_e}$$

The denominator of the expression on the right hand side is positive and it follows that employment falls iff the numerator is negative. The term in square brackets is unambiguously negative and, as argued above, desired utilization is likely to be very insensitive to changes in profitability; that is, $h_\pi d\pi$ will be small. Thus, although a positive employment effect cannot be ruled out, the most likely outcome is one where increasing oligopolization leads to a rise in unemployment. Intuitively, a larger reserve army is needed to boost animal spirits in order to make up for the depressing effects of lower utilization.[20] These effects, are consistent with Steindl's predictions: increased oligopolization raises the profit share but generates stagnation in the form of lower employment and capital utilization.

The long-term effects of a transition, finally, from a stage of large hidden unemployment (in which $h_g = 0$) to one of Kaldorian maturity may be analyzed by comparing a stationary point of the two-dimensional accumulation-mark-up dynamics (with $g > n$) to a stationary point of the three-dimensional system. But for the comparison to be meaningful, it must be assumed that the stationary points are stable, and the case of saddlepoint instability in the two-dimensional system is therefore excluded. Assuming that initially we are in the stable two-dimensional case, the negative effect of the employment rate on the rate of change of animal spirits as the economy reaches a mature stage can be depicted as a rightward shift of the $\dot{a} = 0$ locus in figure 2. The result is a decline in both a and π. The rate of utilization then must fall (or, if $f_\pi = 0$, remain unchanged).[21] Thus, the transition to a new stationary point associated with a constant employment rate implies a fall in the rate of accumulation to bring it into line with the growth of the labour force ($g = n$), and a decline in both the rate of utilization and the profit share.

[20] These results mirror the effects obtained by Skott (1989a, pp. 151-153). In Skott's Marshallian setting, changes in output are related to the difference between realized and target profit margins and, by raising the target, increased monopolization therefore depresses output for any given realized profit margin.

[21] To see this, observe that $f(u, \pi) = 0$ at the stationary solution. Total differentiation yields $f_u du + f_\pi d\pi = 0$, and the result now follows from $f_u > 0, f_\pi \leq 0$.

5 Conclusions and extensions

We opened this paper by comparing a standard Steindl-Kalecki model with Steindl's (1952) analysis. This comparison revealed several differences, and both the standard model and Steindl's own formalization had significant weaknesses, we argued. Steindl himself noted a puzzling and unsatisfactory aspect of his model: it generated unreasonably high values of the (locally) stable steady-state solutions for utilization and the rate of growth. One contribution of this paper is to demonstrate that weak and questionable non-linearities lie behind this problematic feature of his model. The main contribution, however, lies in the presentation and analysis of extended Steindlian models that address the weaknesses of the standard model.

Using a continuous-time framework, we first incorporated the interaction between markup dynamics and accumulation dynamics. Steindl, more than any other contributor to the post Keynesian tradition, has emphasized the influence of competitive conditions on the sensitivity of the markup to changes in utilization, and he has consistently combined this emphasis with a keen awareness of the possibilities of Harrodian instability arising from strong, lagged effects of utilization on the rate of accumulation. In our view the dynamic interaction between accumulation and the markup therefore constitutes the core of a Steindlian model.

We formalized the interaction between markup dynamics and accumulation dynamics in the form of a two-dimensional system of differential equations, one for shifts in the markup and one for shifts in 'animal spirits'. Consistent with Steindl's vision, we find that fast adjustment of the markup may (but need not) contribute to a stabilization of the steady growth path. The model also supports Steindl's position on the stagnationist effects of increased oligopolization: an upward shift in the dynamic equation for the markup generates a decline in both utilization and growth. Paradoxically, however, in the stable case it also leads to a decline in the stationary solution for the markup.

The core model - developed in section 3 - can be extended in various ways. Our extension in section 4 focuses on the Marxian influence of the reserve army. There is a tension in Steindl's views on this issue. Steindl (1952) largely dismisses the idea that accumulation could be constrained by a declining reserve army. In Steindl (1979), however, his position on this issue appears to have changed since in this paper the effect of prolonged near-full employment on accumulation plays a key role. In any case, the inclusion of a reserve army effect tends to stabilize the economy (as in Skott (1989a, 1989b)), and the effects of increasing oligopolization are quite Steindlian: increasing oligopolization must leave the growth rate unchanged - since it is tied to the growth of the labour force in this model - but oligopolization has stagnationist effects in the form of a fall in both the employment rate and the rate of capital utilization. It should be noted also that in this three dimensional system, which incorporates employment dynamics as well as markup and accumulation dynamics, an upward shift of the dynamic equation for the markup raises the stationary value of the markup. Thus, the paradox that characterizes the two-dimensional system without a labour market disappears in the extended system.

Our extensions of the standard model capture, we believe, important Steindlian insights and overcome some shortcomings of earlier formalizations. But our extended models clearly have weaknesses and limitations. Financial factors, for instance, play a very limited role. We have allowed for retained earnings to stimulate investment. The 'principle of increasing risk' - the cost and riskiness of high degrees of external finance - may provide a rationale for the role of retained earnings. But with a constant retention rate, retained earnings might also appear in the investment function simply because high current profitability signals the profitability of additions to the capital stock. Furthermore, the principle of increasing risk suggests that in terms of financial constraints, the gearing ratio rather the flow of retained profits may be the more important variable. Thus, following Steindl (1952), one may extend the model by including the gearing ratio in the investment function.[22] The gearing ratio, indeed, is a key variable in Dutt's (1995) examination of the interaction between the product market and financial aspects. His analysis, which leaves out markup dynamics and labor market effects, can be seen as complementary to the one presented in this paper.

Other prominent aspects of Steindl's verbal analysis also suggest further extension of the model. We have taken all saving rates as well as firms' financial environment as constant. These assumptions could be relaxed to allow for the presence of stock markets and capital gains as well as endogenous changes in saving behaviour (emphasized by Steindl in several contributions, e.g. Steindl (1982)), the effects of institutional influences on saving (e.g. Pitelis (1997)) or evolving standards of financial behaviour along the lines suggested by Minsky (e.g. Steindl (1990 [1989], p. 173)). From an applied perspective, however, the most severe shortcomings probably arise from the neglect of policy, both fiscal and monetary, and the closed-economy assumption. Thus, the possibility of export and profit led growth clearly increases in an open economy setting (e.g. Blecker (1989) and Bhaduri and Marglin (1990)). We leave extensions in these and other directions for future research.

6 Appendices

6.1 Appendix A

Let

$$f(u) = \frac{mk}{s}(\frac{u-1}{1+su}); u \geq 0$$

[22] These financial aspects are left out in Steindl (1979). This more recent analysis instead emphasizes labour market effects and markup dynamics, although neither of these factors are included in the formal equations.

The function $f(u)$ is increasing and strictly concave: $f'(u) > 0, f'' < 0$. Furthermore,

$$\begin{aligned} f(u) &\gtreqless 0 \text{ for } u \gtreqless 1 \\ f(u) &\to \frac{mk}{s}\frac{1}{s} \text{ for } u \to \infty \end{aligned}$$

Hence, $u > f(u)$ both when u is small and when u is sufficiently large. The inequality, however, may be reversed for intermediate values of u. Since $f(u) < \frac{mk}{s}u$ for all u, however, the parameter restriction $\frac{mk}{s} > 1$ is a necessary condition for this to happen.

6.2 Appendix B

A stable case: Assume that

- current accumulation depends non-negatively on profitability, that the change in a depends negatively on the current growth rate, and that there are no direct effects of profitability on the change in a; that is $h_g < 0, h_\pi \equiv 0$ and $g_\pi \geq 0$.
- Price inflation is completely insensitive to small variations in the profit share in the neighborhood of the stationary point; that is $f_\pi \equiv 0$ in this neighbourhood.
- market conditions are competitive in the Steindlian sense that adjustments in the markup are sensitive to deviations of actual utilization from desired utilization. Moreover, the speed of markup adjustment is fast relative to shifts in the accumulation function ($f_u >> h_u$).

Using the first two assumptions, we get

$$\frac{d\pi}{da}_{|\dot{\pi}=0} = -\frac{G_a}{G_\pi} = \frac{u_a}{-u_\pi} > \frac{u_a + \frac{h_g}{h_u}g_a}{-u_\pi - \frac{h_g}{h_u}g_\pi} = -\frac{H_a}{H_\pi} = \frac{d\pi}{da}_{\dot{a}=0} > 0$$

It follows that the $\dot{\pi} = 0$ locus is steeper than the $\dot{a} = 0$ locus and, using (17)-(19), that $Det(J) > 0$. Thus, assuming the existence of a stationary solution, the stationary point must be a node or a focus. Furthermore, the third assumption on the relative adjustment speeds of a and π ensures that the second stability condition will also be met. To see this, note that

$$Tr(J) = H_a + G_\pi = h_u u_a + [h_g g_a + (1-\pi) f_u u_\pi]$$

The term in square brackets in the expression for the trace is negative while the first term is positive. Stability - quite intuitively - can be undermined if the destabilizing adjustments of investment function are fast relative to the speed of stabilizing markup adjustment.

A saddlepoint: Our second case shows that fast price adjustments will not always suffice to stabilize the system. Assume that

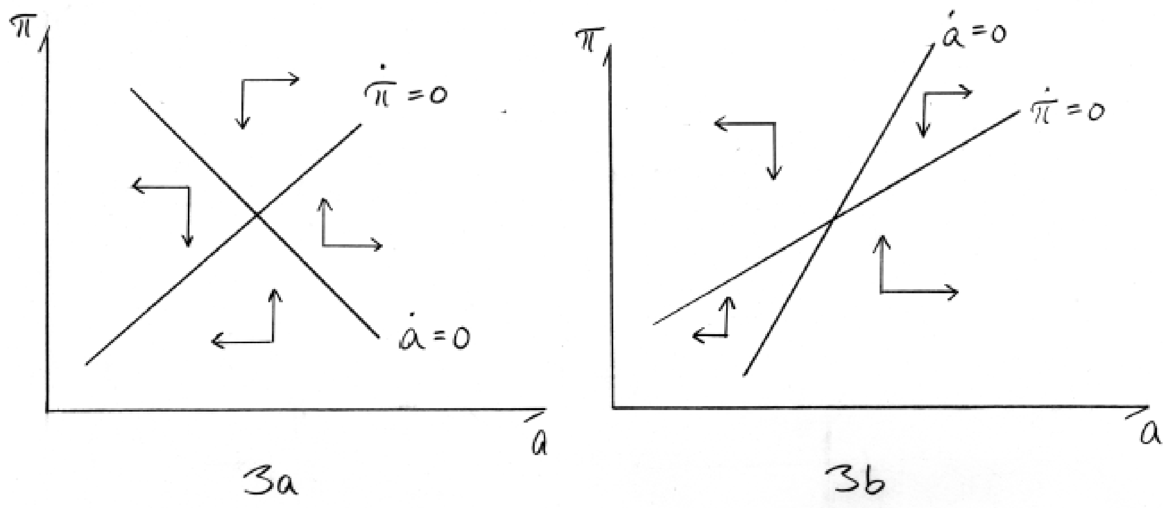

Figure 3: *Saddlepoint dynamics; 3a:* $H_\pi > 0$; *3b:* $H_\pi < 0$

- There is no negative feedback from the current accumulation rate to the change in a (that is, $h_g = 0$)
- The profit share exerts a positive effect on the change in a and/or a direct negative effect on price inflation ($h_\pi > 0$ and/or $f_\pi < 0$)

It is readily seen that with these assumptions the determinant of the Jacobian becomes negative and the stationary point is a saddlepoint. Figures 3a and 3b illustrate the outcome. In figure 3a, $H_\pi > 0$ and the $\dot{a} = 0$ locus is negatively sloped. In figure 3b, $H_\pi < 0$, and both loci are positively sloped; the slope of the $\dot{a} = 0$ locus, however, is steeper than that of the $\dot{\pi} = 0$ locus. In both figures the stationary point exhibits saddlepoint instability, and global analysis is needed to decide what will happen over the longer run.

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