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The Current Non-Status of General Equilibrium Theory

Donald W. Katzner

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Abstract

This paper argues that the abandonment of general equilibrium theory by microeconomists was a mistake. It provides counter arguments to two of the reasons for that abandonment – lack of both generality and consistency with methodological individualism in uniqueness and stability analysis of equilibria – and urges microeconomists to refocus some of their attention on it.

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*Department of Economics, University of Massachusetts / Amherst, Amherst, MA 01002

The Current Non-Status
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The theory of microeconomic, general (as opposed to partial) equilibrium does not have a long history. Although the use of simultaneous equations in economics dates at least to Isnard in 1781, the first reasonably complete presentation of it was given by Walras [18] in 1874. For the next 75 years development was rather slow. The contributions of Cassel, Hicks, Samuelson, Wald, and others illuminated various issues relating to it but, by and large, general equilibrium theory remained on the back benches of economic discourse. It was not until the 1950's that interest in general equilibrium theory really took off. Perhaps the event that most closely marks this surge of interest is the publication in *Econometrica* of the paper "Existence of Equilibrium for a Competitive Economy" by Arrow and Debreu [1] in 1954. Suddenly, it seemed, everyone was doing general equilibrium or something related to it. And the search for conditions under which competitive equilibrium exists and is unique and globally stable, conditions which must be imposed if there is to be any hope of linking the general equilibrium system to the real economy, became of paramount importance.

Then, in the late 1980's, almost as quickly as economists had raised its banner, interest in microeconomic, general equilibrium theory abruptly waned. Arguably its death knell was best sounded in the *Economic Journal* in 1989 by Kirman [11] in a paper entitled "The Intrinsic Limits of Modern Economic Theory: The Emperor Has no Clothes." Essentially, Kirman made two claims: First, as of 1989, general equilibrium theory had not yet been able to achieve what was expected of it. That is, economists had not yet been able to employ its theoretical structure to provide a satisfactory understanding of one of the most fundamental and important questions that constitute the primary reason for embracing it. In particular, no acceptable analysis of the uniqueness and global stability of competitive equilibrium had yet been found and, hence, even with respect to tâtonnement dynamics, a full and theoretically satisfying explanation of price determination in a competitive economy, *i.e.*, an explanation of the means by which market-equilibrium prices are determined from all appropriate out-of-equilibrium positions, could not be given. Kirman's second claim was that there was little hope that an adequate theory of price determination could ever emerge from general equilibrium theory. In the years since the publication of Kirman's paper, general equilibrium theory has once again been relegated to the back benches, and one has to look hard to find papers relating to it in the current economics literature.

Now, clearly, the first of Kirman's claims that economists had no satisfactory explanation of price determination as of 1989 is true. That is so because the criterion of satisfactoriness invoked by economists required explanations that

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[1] The author would like to thank Peter Skott for his help in preparing this paper. A number of the ideas contained in it may be found in expanded form in Ch. 10 of my *Introduction to the Economic Theory of Market Behavior: Microeconomics from a Walrasian Perspective* (Cheltenham: Elgar, forthcoming).

were consistent with methodological individualism, and in which utility and production functions were "general" in that they should not be assumed to exhibit specific forms. And as of 1989, all known explanations of price determination violated both of these precepts. But it is possible to entertain doubts about whether the generality requirement is not too strong. And it is also possible to take issue with Kirman's second claim, that there is little hope of finding a satisfactory explanation of price determination, at least with respect to the requirement of consistency with methodological individualism. Moreover, on these and other grounds, I believe it was a serious mistake for microeconomists to abandon general equilibrium theory. Each of these matters will be addressed below.

Consider the latter, which is more easily and quickly addressed, first. In what follows, I will try to show that it is too early to give up on efforts to construct an adequate analysis of uniqueness, global stability, and price determination in the context of a general equilibrium, competitive system. Just because something has not yet been demonstrated does not mean that it can never be confirmed. Indeed, an attempt will be made to convince you, the reader, that things are not nearly as bad as Kirman's article suggests. But there are more important reasons why the abandonment of general equilibrium theory was a mistake.

First, the general equilibrium construction as originally set out by Walras is a significant component of the theoretical basis of, in its most idealized form, capitalism – an arrangement that, for better or worse, orders much of our modern world. Public discussions in the media, in political arenas, and elsewhere that relate to general equilibrium in part or in toto are commonplace. For economists to turn their backs on such matters is to disassociate themselves from an important aspect of current affairs to which they have a professional obligation to contribute. Second, most economists are trained in at least the fundamentals of Walrasian general equilibrium theory, and there is no other widely understood vocabulary for conducting research and communicating among economists (and others) about the microeconomy at large. Abandoning the general equilibrium approach in their work, then, forces economists to focus their attention on fragments or limited combinations of fragments of the microeconomy rather than on the whole. None of this is to say that there are not, there should not be, and there should not be developed alternatives to the Walrasian vision, in part or in its entirety, that provide what might be regarded as a more realistic picture of what goes on in a modern microeconomy.[2] But it does suggest that until there is widespread acceptance of a better "substitute" among economists, the Walrasian system remains the only means for communicative discourse on the microeconomy as a totality.

To describe what went wrong with the general equilibrium research program it is useful to begin by summarizing the general equilibrium system itself.

[2] Some inquiries into overlapping generations models, the notion of temporary equilibrium, and the dynamics of fixed prices and old Keynesian macroeconomics can be seen as movements in this direction. An approach to greater realism based on the work of Shackle has been proposed as a fully developed alternative to the Walrasian system by Katzner [9].

For both practical and pedagogical reasons, only exchange systems with no production will be considered, and attention will focus only on tâtonnement dynamics in which trading is not permitted until equilibrium is reached.[3] Before proceeding, however, it is appropriate to digress for a moment to consider the manner in which explanation is achieved in economic analysis.

1. Explanation in Economics

When trying to explain an economic phenomenon, the standard procedure is to build a model. A *model* of something — call the thing T — is a construct having enough in common with the observable facets of T that insight into T can be obtained by studying the construct. Einstein and Infeld [4, p. 33] gave the following example: Imagine an individual is asked to explain how a particular clock with rotating hands works but is not permitted to remove its cover. After some thought, the person might obtain appropriate springs, gears, and whatever else might be required, and build a physical "model" of the clock whose "observable" behavior duplicates the observed behavior of the original. He could then give an explanation of how his model behaves and say that the original clock works analogously to, or *as if* it were, his model. Clearly, there are many different as-if models, and hence explanations, that could be built. But all explanations operate by identifying something in the model (in the example, the movement of the model's hands) with what is observed (the movement of the hands on the original clock). In economics, of course, models are not physical things. Rather they are mental constructs based on assumptions, concepts, and relations among variables. They also abstract from a multitude of possible forces to concentrate on the minimum number necessary for explanation, and their properties are their own and not properties of that which is the object of explanation. Nevertheless, they function in much the same way as in the Einstein-Infeld example.

The Walrasian model is properly understood in these terms. It primarily focuses on the notion of equilibrium and is built up by making assumptions about the preferences, endowments, and technologies of individual agents, *i.e.*, consumers and firms. Its purpose is to explain and clarify the determination of observed prices and quantities, and the simultaneous, interacting behavior of real agents in the economy. To achieve this purpose requires, among other things, that an investigation of the questions of existence, uniqueness, and global stability of equilibria in the model be successfully undertaken. An obvious way to comprehend the urgency of such an inquiry is to recall a common method of interpreting equilibrium in a single, real-world market in isolation: Suppose one

[3] Ways of overcoming Kirman's conclusion for the case of nontâtonnement adjustment rules, which permit trading out of equilibrium, have been suggested by Fisher [5] and Smale [15]. Other nontâtonnement methods that might be transferable from the single-market context in which they are developed are surveyed by Hahn [7, pp. 788-791]. Still additional approaches located somewhat beyond the confines of the standard Walrasian framework are described by Bénassy [2].

were to observe that market at a particular moment of time. In that case, one would see that so much of the market's commodity was traded at such-and-such a price or, in other words, one would observe a single point in commodity-price space. Subsequent observation at a later moment would yield a second point. In building a model to explain how these points came to be seen, the economist could, for example, assume (i) that there exist two distinct market demand curves each passing through one, but not the same, observed point, and (ii) that there exists only one market supply curve passing through them both. Then, since each observed point is identified as a market equilibrium point in the model,[4] each could be explained as analogous to, or as if it were, the outcome of the interaction of supply and demand as characterized by implicit rules of price adjustment. The economist could also assert that the movement from the first point to the second occurred because of a "shift" in demand. Clearly, equilibrium must exist in the model for this explanation to work. If, moreover, the equilibrium in the model were not unique, then the explanation would be incomplete; it would allow the observed point to be identified with many equilibria, each with its own properties, and thus the reason for the movement between the two points would become clouded. Lastly, when the observed point changes from the old to the new, the equilibrium in the model would have to adjust accordingly. But if the latter equilibrium were not globally stable, then whatever dynamics there were in the model could prevent the new equilibrium from being reached and, in that circumstance, the explanation given for the observed movement from the one point to the other would break down. An alternative interpretation of reality would be to locate observed points along time-paths that converge to equilibria in the model rather than to identify them specifically as those equilibria. But in either case the questions of existence, uniqueness, and global stability have to be explored because that is the only way to be sure that the model can be linked to the real world. And this is so apart from the issue of price determination, and without even questioning the realism of the assumptions and the relevance of conclusions derived from them.

A similar argument applies to the full exchange model considered here with many goods and consumers. Moreover, it is not possible to empirically corroborate or falsify any of its assumptions. This is because the economist does not have a laboratory in which he has sufficient control over the relevant parameters. If, for example, particular observations of a consumer's behavior are inconsistent with the properties of demand derived from utility maximization, since the observations themselves are necessarily taken across time, the inconsistency could always be attributed to changes in the individual's preferences from one observation to the next. Since he cannot control what goes on in peoples' minds, *i.e.*, prevent a person's preferences from changing, the economist does not have the ability to rule out such a possibility.

It is clear that without the ability to empirically corroborate or falsify, the exchange model can, in the final analysis, only be accepted on faith. Moreover,

[4]Note that demand curves, supply curves, and equilibrium points cannot exist in reality. They can only be present in models. Similarly, to prove that equilibrium exists and is unique and stable in a model can never imply that unique and stable equilibria exist in the real world.

from this perspective, one model that explains observed prices and quantities in the world of exchange is no better than any other.

2. The General-Equilibrium System and Price Determination

The general equilibrium system at issue here is based on the notion of perfect competition. In it, consumers are assumed to believe that their behavior has no impact on market prices, and therefore treat those prices as fixed parameters over which they have no control.

Let there be $I > 1$ goods and $K > 1$ persons. Denote the quantities demanded of good i by person k with the symbol x_{ik} and take x_{ik}^0 to be that person's endowment of good i. The excess demand of good i by person k is $q_{ik} = x_{ik} - x_{ik}^0$. Set p_i as the price of good i and $p = (p_1, \ldots, p_i)$. Write $q_{ik} = E^{ik}(p)$ for the excess demand function of person k for good i, where $i = 1, \ldots, I$, and $k = 1, \ldots, K$, and denote the market excess demand function for good i by $q_i = E^i(p)$, where

$$q_i = \sum_{k=1}^{K} q_{ik}, \quad E^i(p) = \sum_{k=1}^{K} E^{ik}(p),$$

and $i = 1, \ldots, I$. Set $q = (q_1, \ldots, q_I)$ and $E(p) = (E^1(p), \ldots, E^I(p))$ so that

$$q = E(p). \tag{1}$$

Take the E^{ik} to be derived from constrained utility maximization in the usual way, where the endowment vector $x_k^0 = (x_{1k}^0, \ldots, x_{Ik}^0)$ has at least one of its components positive for $k = 1, \ldots, K$, where, for every i, at least one $x_{ik}^0 > 0$ as k varies, and where all utility functions are suitably continuous, differentiable, increasing, and strictly quasi-concave. It follows that the functions E^{ik} and E^i are continuous, homogeneous of degree zero, and satisfy, respectively, the budget equations

$$\sum_{i=1}^{I} p_i E^{ik}(p) = 0, \quad k = 1, \ldots, K,$$

and Walras' law

$$\sum_{i=1}^{I} p_i E^i(p) = 0, \tag{2}$$

throughout their domains. General equilibrium in this model is known to exist. But equilibrium prices can only be obtained as the simultaneous solution of (1), and Walras' law (2) implies that not all of these equations are independent of each other. Hence it is necessary to either solve only for price ratios or introduce a normalization. The approach taken here is to normalize according to

$$\sum_{i=1}^{I} (p_i)^2 = 1. \tag{3}$$

Now the dynamic price-adjustment rule defining the operation of markets in a Walrasian model of this sort is often taken to be

$$\frac{dp}{d\tau} = \theta E(p), \tag{4}$$

where τ represents (continuous) time,

$$\frac{dp}{d\tau} = \left(\frac{dp_1}{d\tau}, \cdots, \frac{dp_I}{d\tau} \right),$$

and $\theta \neq 0$ is a scalar constant. This adjustment rule is usually assumed to represent a tâtonnement process in that trading is not permitted until equilibrium is reached. The latter process and its representation in (4) are continued in the present discussion. More will be said below about the process itself. It will be convenient to assume further that the E^{ik}, and hence the E^i, are continuously differentiable.

Two additional concepts are employed in subsequent argument: First, aggregating demand quantities and endowments over individuals, let

$$x_i = \sum_{k=1}^{K} x_{ik} \qquad \text{and} \qquad x_i^0 = \sum_{k=1}^{K} x_{ik}^0,$$

$x = (x_1, \ldots, x_I)$, and $x^0 = (x_1^0, \ldots, x_I^0)$. A *community utility function* is a function defined on the market-quantity commodity space $\{x : x \geq 0\}$ which, when maximized subject to the aggregate budget constraint $p \cdot x = p \cdot x^0$, where the dot denotes inner product, yields the market excess demand functions (1). The community utility function, should one exist, does not necessarily, by itself, carry any welfare implications.

Second, goods i and n are called *gross substitutes* provided that $i \neq n$ and, with E_n^i indicating the partial derivative of E^i with respect to the n^{th} price, $E_n^i(p) > 0$, for all $p > 0$. Although $E_n^i(p) > 0$ at $p > 0$ does not imply $E_i^n(p) > 0$ at that p, the possibility of having, say, $E_n^i(p) > 0$ and $E_i^n(p) < 0$ simultaneously is excluded by the assumptions imposed below.

It should be observed that this definition of substitutes is different from the usual one. The latter is characterized with respect to the ordinary – not excess – market demand functions. One definition neither implies nor precludes the other. Moreover, goods i and n would be called *gross complements* when $i \neq n$ and $E_n^i(p) < 0$, for all $p > 0$. But in what follows, this form of complementarity – not that with respect to ordinary demand functions – is ruled out. Under present assumptions, the following two results can be proved:[5]

Theorem 1 In the model of an exchange economy described above with dynamic (4) and normalization (3), if a community utility function exists and if $\theta > 0$, then the equilibrium path that exists is unique and globally stable.

[5] Proofs may be found in Katzner [10, Sect. 10.2].

Theorem 2 In the model of an exchange economy described above with dynamic (4) and normalization (3), if all pairs of distinct goods are gross substitutes, if $\lim_{p_n \to 0} E^n(p) = \infty$ for all positive values of $p_1, \ldots, p_{n-1}, p_{n+1}, \ldots, p_I$, and all $n = 1, \ldots, I$, and if $\theta > 0$, then the equilibrium path that exists is unique and globally stable.

It is interesting that the proofs of these theorems can be broken down into parallel parts. In the case of Theorem 1, the steps are first, if a community utility function exists, then the weak axiom of revealed preference holds in the following form: For all $p' = (p'_1, \ldots, p'_I) > 0$ and $p'' = (p''_1, \ldots, p''_I) > 0$ such that $E(p') \neq E(p'')$, if

$$\sum_{i=1}^{I} p'_i E^i(p'') \leqslant \sum_{i=1}^{I} p'_i E^i(p') ,$$

then

$$\sum_{i=1}^{I} p''_i E^i(p'') < \sum_{i=1}^{I} p''_i E^i(p') .$$

Second, if this weak axiom holds and if $\theta > 0$, then the competitive equilibrium that exists is unique and globally stable. With respect to Theorem 2, it is first shown that if all pairs of distinct goods are gross substitutes and if $\lim_{p_n \to 0} E^n(p) = \infty$ for all positive values of $p_1, \ldots, p_{n-1}, p_{n+1}, \ldots, p_I$, and all $n = 1, \ldots, I$, then

$$\sum_{i=1}^{I} \overline{p}_i E^i(p) > 0 \tag{5}$$

for all $p \neq \psi\overline{p}$, where the scalar $\psi > 0$ and $\overline{p} = (\overline{p}_1, \ldots, \overline{p}_I) > 0$ is the equilibrium price vector that exists and is now unique. Then it is demonstrated that, if this conclusion is in force and $\theta > 0$, then the unique competitive equilibrium that exists is globally stable. It turns out that the satisfaction of the weak axiom in the proof of Theorem 1 implies the conclusion relating to (5). And it is the latter, in conjunction with Liapunov's so-called "second method," that yields global stability.

Theorems 1 and 2 each constitute a separate theory or explanation of price determination since each indicates how equilibrium prices are determined from all appropriate out-of-equilibrium positions. What is wrong with these theorems? Why are they unacceptable to Kirman and other economists? As indicated at the outset, there are two main criticisms: lack of generality, and inconsistency with methodological individualism. Consider the former first.

3. Lack of Generality

It is certainly true that Theorems 1 and 2 apply to highly specialized cases. Requiring either that a community utility function exist or that all goods be gross substitutes deprives these theorems of any claim to generality. Furthermore, because it is known that, as a consequence of constrained utility maximization (under the assumptions of continuity, differentiability, increasingness, and strict quasi-concavity imposed above), the E^{ik} possess properties beyond those previously stated (such as the analogue of Slutsky negative definiteness and symmetry of ordinary demand functions), one might ask if any of these additional properties carry over to, or impose other restrictions on, the E^i. That is, when the E^i are an outgrowth of the constrained maximization of individual utility functions, do they necessarily, by dint of those maximizations, exhibit characteristics beyond continuity, homogeneity, and Walras' law that apply generally, regardless of the particular utility functions involved? Were such properties to be present, they might point the way to more general propositions relating to the determination of prices. But Debreu [3] has proved a result that forces the answer to this question to be negative.[6]

The Debreu result essentially establishes that constrained utility maximization by all individuals does not, at least in the typical model of an exchange economy described here, generally imply anything at all about market excess demand functions beyond the already-established properties of continuity, homogeneity, and Walras' law. To obtain further restrictions on market excess demand functions would therefore require the imposition of additional postulates like, for example, the supposition that individual utility functions take on special forms. Moreover, the uniqueness and global stability of equilibria in Walrasian models of exchange economies often turn on the presence of certain kinds of such further restrictions. In any case, it follows that without additional postulates beyond the continuity, differentiability, increasingness, and strict quasi-concavity of utility functions, exchange models do not have sufficient assumption content to permit the derivation of propositions that establish uniqueness and global stability. Uniqueness and global-stability analyses, then, can only proceed by adding extra hypotheses. Thus full generality with respect to the avoidance of additional restrictions is beyond our grasp.

However, the issue of how "close" to full generality it is possible to come is still an open question. And to give up before answering it is to foreclose on the possibility of finding conditions of "reasonable" generality. Moreover, the search for an answer need not focus solely on individuals' utility functions. For in addition to looking for more general classes of utility functions that lead to global stability, it may be possible to modify the price adjustment rule (4) itself so as to make global stability fully dependent on its properties alone. In that case, further restrictions on the utility function would not be needed. Indeed, Flaschel [6] has shown this to be so with respect to local stability when price adjustment rests, in addition to excess demands, on the rates of change of those excess demands. Thus, for example, greater adjustment in prices might occur where the potential movement away from equilibrium is faster. The details, however, are not pursued here.

[6] Similar theorems in different contexts have been demonstrated by Sonnenschein [16], [17] and Mantel [13].

Furthermore, it should also be pointed out that, if global stability is to be pursued through the imposition of restrictions on utility functions, then a certain case in which all utility functions are Cobb-Douglas, although not very general, turns out to be sufficient for explanatory purposes. To see why, note first that, when all utility functions are Cobb-Douglas, if $\theta > 0$, then the hypotheses of Theorem 2 are satisfied.[7] And if, in addition, all utility functions are identical, then the conditions of Theorem 1 are also met.[8] Hence two possible statements of price determination are obtained. But in addition to that, when all utility functions are (not-necessarily-identical) Cobb-Douglas, all positive price-quantity patterns that are observable in a real-world exchange economy are explainable as the outcome of the general equilibrium model. For with

$$u^k(x_k) = \sum_{i=1}^{I} \alpha_i^k \ln x_{ik}, \qquad k = 1, \ldots, K,$$

where u^k is the utility function for person k, the α_i^k are constants such that $\sum_{i=1}^{I} \alpha_i^k = 1$, and $\alpha_i^k > 0$, for $i = 1, \ldots, I$, and $k = 1, \ldots, K$, the individual ordinary (as opposed to excess) demand functions h^{ik} can be shown to be

$$x_{ik} = h^{ik}(p, m_k) = \alpha_i^k \frac{m_k}{p_i}, \qquad \begin{array}{l} i = 1, \ldots, I, \\ k = 1, \ldots, K, \end{array}$$

where $m_k = p \cdot x_k^0$. Now if $\overline{x} = (\overline{x}_1, \ldots, \overline{x}_K) > 0$ and $\overline{p} = (\overline{p}_1, \ldots, \overline{p}_I) > 0$ are any observed collection of price-quantity data, then choosing x_k^0 such that $\overline{p} \cdot \overline{x}_k = \overline{p} \cdot x_k^0$, and setting $\overline{m}_k = \overline{p} \cdot x_k^0$ and

$$\alpha_i^k = \frac{\overline{p}_i \overline{x}_{ik}}{\overline{m}_k},$$

results in

$$\overline{x}_{ik} = \alpha_i^k \frac{\overline{m}_k}{\overline{p}_i}, \qquad \begin{array}{l} i = 1, \ldots, I, \\ k = 1, \ldots, K. \end{array}$$

In spite its the lack of generality, then, the model relevant for Theorem 2 explains everything that can be observed. Of course, with Cobb-Douglas utility functions, there are no basic goods of which the individual needs a positive minimum quantity for subsistence. That is, such functions permit the consumer to demand quantities of goods like water that are insufficient for him to physically to survive. But even though such demands are possible in Cobb-Douglas (and other) worlds, they are unlikely to be observed in reality. Therefore, from the perspective of empirical sustainability (not necessarily from the vantage point of what might be thought to be reasonable or realistic), general-equilibrium explanations of price determination based on Cobb-Douglas utility functions are no worse than general-equilibrium explanations derived from any other collection of utility functions. This, by itself, is sufficient to ensure the viability and usefulness of the Walrasian general-equilibrium exchange system.

[7] See Katzner [10, Sect. 10.2].

[8] *Ibid.*

4. Inconsistency with Methodological Individualism

Economists have always intended that the Walrasian model, as described here, be encompassed within the tradition of methodological individualism, which generally understands individuals, with given preferences and endowments, and firms, with given technologies, to enter the market process as autonomous entities. Among other things, this tradition means that all assumptions should be made on individuals and their preferences, and all market-level entities and their properties should be built up from individual-level entities and their properties. But, in the words of Hahn [8, p. 137], the market-price-adjustment rule described in (4) is a market-level prescription and Theorems 1 and 2 generally do not constitute "... a theory of price formation based on the rational calculations of rational agents." Koopmans [12, p. 179] put it this way: "If, ... [in accordance with (4)], the net rate of increase in price is assumed to be proportional to the excess of demand over supply, whose behavior is thereby expressed? And how is that behavior motivated?" The standard interpretation of (4) is that it characterizes the activity of a market-level "auctioneer." In particular, it is assumed to indicate the information that that auctioneer has, namely, the aggregate excess demand quantities in every market at each positive price vector, and how he responds to that information when making price adjustments. No justification is provided for this response in terms of the rationality of individual agents. The theories so derived are unacceptable because, in this respect, they are inconsistent with the tenets of methodological individualism.

Before examining the issue in greater detail, it is worth pointing out that the Walrasian system is often meant to explain the determination of prices and the allocation of resources at a particular moment of time as previously indicated. Describing the evolution of the economy over long periods of time is not within its purview. It follows that those elements of the economic world that modify very slowly, and require great expanses of time to do so, may be taken as part of the fixed background within which Walrasian models are constructed. Apart from preferences and technologies, the most important of these elements for present purposes is the institution of the market. Thus the attempt to construct explanations of market behavior in terms of the behavior of individual agents, that is, the search, at the level of the individual agent, for a definition of the operation of markets and for uniqueness and global stability conditions, must necessarily take place within a given institutional, market framework. The possibility that market institutions themselves might have been built up in the past as a consequence of agent behavior is relevant for the theory of the development of markets[9] — not for the characterization and analysis of Walrasian models.

It should also be emphasized that the assumption of Cobb-Douglas utility functions in an exchange model that is intended to explain an observed configuration of prices and quantities at a particular time, in effect expresses all

[9] See, for example, Schotter [14].

hypotheses of Theorems 1 and 2 in terms of individual preferences alone. And in this respect, it brings the Walrasian model, except for (4), into compliance with the requirements of methodological individualism. Subsequent discussion, then, considers the possibility of adjustment rules that restore the foundation of methodological individualism to the Walrasian exchange model in its entirety. The presentation that follows provides a simple illustration of how this might be done.

The typical story that goes with dynamic adjustment rules like (4) depicts the Walrasian tâtonnement and the price determination it implies in terms of the market-level auctioneer mentioned above. According to that story, the auctioneer announces a vector of prices, maximizing agents (who are price takers and, to be general, are either consumers or firms) respond with statements of their excess demand quantities which are then summed, and if market excess demand vanishes everywhere, the stated trades are then consummated. Otherwise, the auctioneer announces a new vector of prices as dictated by the adjustment rule, and the process continues. Only when zero excess demand in all markets is achieved, is trade permitted to take place. Of course, the auctioneer of this story is neither a consuming nor a producing agent in the economy. He is, rather, a fictitious being whose sole purpose is to guide the operation of the markets and, as such, is part of the given institutional structure that characterizes the markets themselves. And since the activities of the auctioneer reflect price behavior that is not explainable in terms of the actions or decisions of individual agents, like equation (4) that the story accompanies, the auctioneer is also inconsistent with methodological individualism.

The story describing the operation of markets and the determination of market prices to be developed here is based on the idea that individual agents, although retaining their price-taking characteristics, are the ones who (simultaneously) propose prices. As before, trade is not permitted until equilibrium is achieved, perfect competition prevails everywhere, and individuals, in spite of their role as price proposers, still believe that their behavior (including that of announcing prices) has no impact on the final determination of market prices. Each agent, then, announces his own vector of suggested market prices to all other market participants. The latter respond as price takers by stating their desired excess demand quantities, derived from appropriate maximization, at those prices. In this way, each agent answers the price announcements of all other agents.[10]

Consider a representative agent participating in this process. Were trade actually to take place at the prices announced by that agent, the agents who responded to the price announcement would consummate their trades first, and the agent making that announcement would accept the residue of remaining demand and supply of each commodity. But if the announced prices were not equilibrium prices, then the announcing agent would find that he is unable to buy and sell what he wants at those prices. Since the announcing agent

[10] A clearing-house may be included that collects and disseminates all of the relevant information relating to these announcements and responses.

can deduce this without the occurrence of trade from the statement of excess demands by the individuals responding to his price-vector announcement, on the next round of announcements he changes his suggested price vector according to an agent-specific adjustment rule. This rule, though generally different for different agents, always reflects the following: When the agent is unable to buy as much as he would like of a particular good at the price he has announced, he raises his announced price; when he cannot sell as much as he would like, he lowers it. When the agent is able to buy more than he would like of a particular good at the price he has announced, he lowers his announced price; when he can sell more than he would like, he raises it. Proceeding in such a manner, the agent continues to modify his suggested price vector on succeeding rounds until it coincides with the market equilibrium price vector or, in other words, the price vector at which the residual the announcing agent would be required to accept is identical to that which he wants. With all agents doing the same thing, and with equilibrium unique, change would cease everywhere when all participating individuals arrive at the same equilibrium vector. At that point, since no agent would announce a new price, all agents would know that equilibrium has been reached, and trade would take place.[11] Henceforth, to distinguish it from the auctioneer story described earlier, this story will be referred to as the *agent-price-adjustment story*.

It is evident that every agent in the agent-price-adjustment story is, at the same time, both a price taker and his own "auctioneer," and that price-adjustment rules accompanying the story are necessarily postulated at the individual level. Although related to maximizing behavior in a manner to be subsequently indicated, these adjustment rules as such are not objects of choice derivable from agent preferences and maximization *per se*. Instead they emerge independently, in the same way as agent preferences and maximization, from current dispositions and understandings developed in light of a long evolution of historical experiences, and, again like agent preferences and maximization, are taken as given at the start of the analysis. Indeed, for present purposes, such price-adjustment rules along with agent maximization are lumped together in what might be regarded as an expanded "postulate of rationality," and the price-determining behavior the adjustment rules describe is considered as part of the "rational calculations of rational agents." In any case, the hypothesizing of price-adjustment rules like these clearly falls within the framework of methodological individualism.

[11] If there were a mechanism, such as a random draw, to determine which agent would be *the* price anouncer, then, as long as that person maintained his belief that his price announcements do not influence the final outcome, it would not be necessary to require that all agents announce suggested prices. Price announcements by the designated price announcer would suffice. In this case, the designated price announcer would function as a market-level auctioneer except that his price announcement would be based on the possibility of improving his own utility position and the process of adjustment to equilibrium would be fully consistent with methodological individualism.

In addition, the process of convergence could be shortened by having the first agent to reach the equilibrium price vector identify it as the equilibrium price vector for everyone. Since the other agents know that this is where they will end up, they immediately move to it.

Pursuing the characteristics and implications of the agent-price-adjustment story still further, and returning the focus of attention exclusively to the world of exchange, consider, for a moment, agent k (who is not necessarily the designated price announcer of n. 9 above). Set $\widetilde{q}_k = E^k(\widetilde{p}^k)$ and

$$\widehat{q}_k = \sum_{\kappa \neq k} E^\kappa(\widetilde{p}^k),$$

where $E^\kappa(\widetilde{p}^k) = (E^{1\kappa}(\widetilde{p}^k), \ldots, E^{I\kappa}(\widetilde{p}^k))$, and $\widetilde{p}^k = (\widetilde{p}_{1k}, \ldots, \widetilde{p}_{Ik}) > 0$ is a specific price vector announced by agent k. On the one hand, $\widetilde{q}_k$ is the desired excess demand vector of k at prices $\widetilde{p}^k$ and, as such, satisfies his budget constraint $\widetilde{p}^k \cdot \widetilde{q}_k = 0$ where, recall, the dot denotes inner product. On the other, $\widehat{q}_k$ is the vector presented to k by the markets at prices $\widetilde{p}_k$ and, therefore, $-\widehat{q}_k$ is the vector that k would have to accept if trade took place at $\widetilde{p}_k$. Clearly, if $\widetilde{q}_k = -\widehat{q}_k$, then the vector $\widetilde{p}_k$ announced by agent k is an equilibrium price vector.

Suppose $\widetilde{q}_k \neq -\widehat{q}_k$. Since, for each κ, $E^\kappa(p)$ satisfies κ's budget constraint for all $p > 0$, it follows that

$$\widetilde{p}^k \cdot [-\widehat{q}_k] = -\sum_{\kappa \neq k} \widetilde{p}^k \cdot E^\kappa(\widetilde{p}^k) = 0.$$

Hence, although $-\widehat{q}_k$ might reach beyond the limits of agent k's initial endowment (*i.e.*, might require k to sell more than he has of particular goods), it nevertheless also satisfies his budget constraint. Were agent k's utility function defined at $\widehat{x}_k = -\widehat{q}_k + x^0_k$, then, it would be necessary that

$$u^k(\widetilde{x}_k) > u^k(\widehat{x}_k),$$

where $\widetilde{x}_k = \widetilde{q}_k + x^0_k$. Now the agent-price-adjustment mechanism essentially requires that agent k raise his announced prices of those goods for which, at $\widetilde{p}^k$, market excess demand in the vector $\widetilde{q}_k + \widehat{q}_k$ is positive (quantity demanded is greater than quantity supplied), and lower his announced prices for those goods such that market excess demand is negative (quantity demanded is less than quantity supplied). To the extent that these two kinds of price changes call forth, respectively, (i) increases in supply where k is a buyer and decreases in demand where k is a seller, and (ii) increases in demand where k is a seller and decreases in supply where k is a buyer, agent k, as will be illustrated momentarily, has reason to hope that the components of $\widehat{q}_k$ will be altered in such a manner as to raise his utility from $u^k(\widehat{x}_k)$. In this sense, the agent-price-adjustment rule is grounded in the individual's efforts to increase his utility, and the "theory of price determination" associated with it arises from the pursuit of self-interest in the same way as the individual's decisions concerning quantities of goods to buy and sell.

Figure 1 illustrates, in a two-good case, why agent k has reason to hope that the price adjustment described above might increase his utility. In that diagram it is assumed that at his initially announced price vector (reflected by

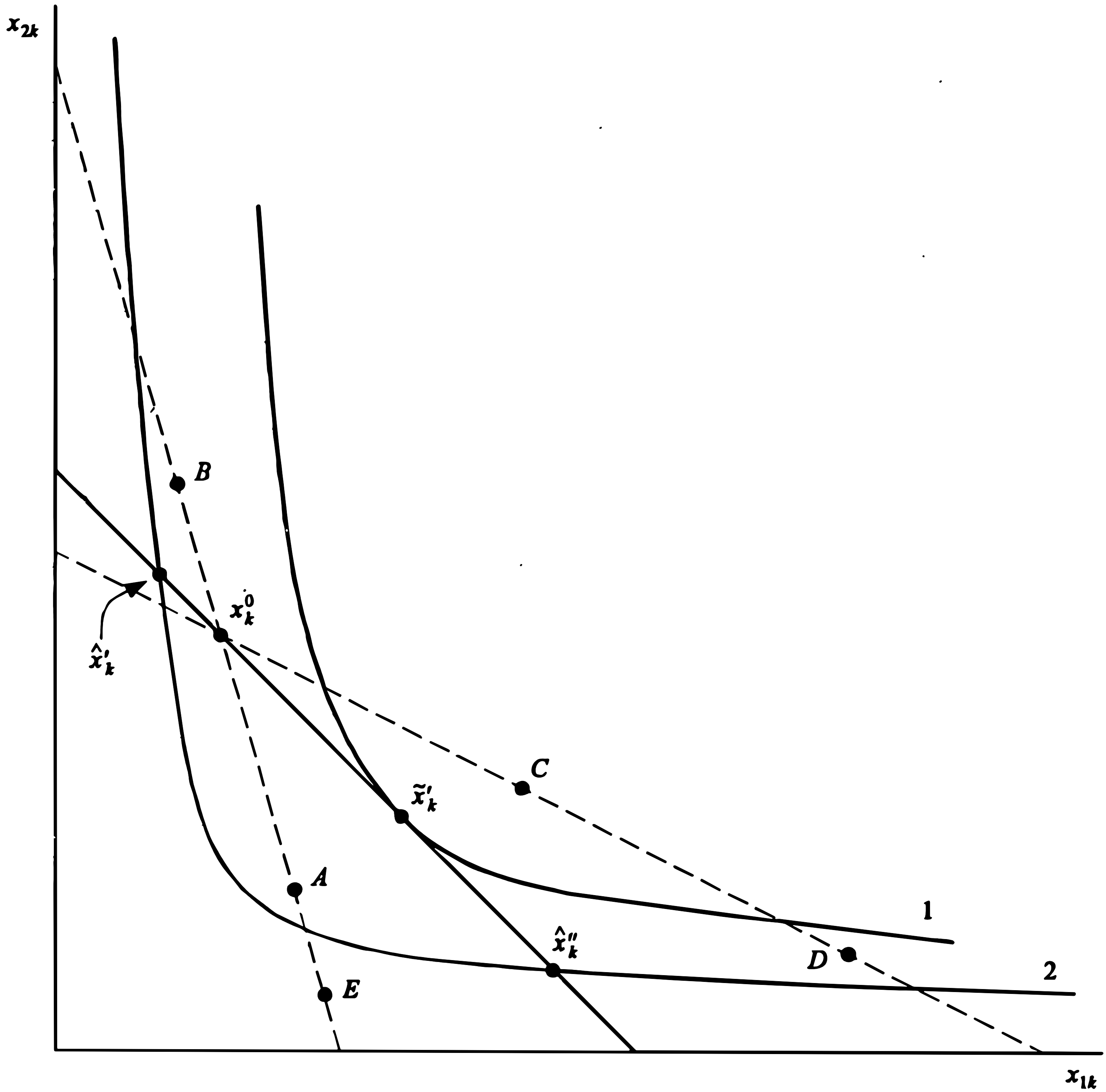

Figure 1 The agent-price-adjustment rule and person k.

the solid budget line through the initial endowment x_k^0), k's constrained utility-maximizing position is at $\widetilde{x}_k$ on the indifference curve labeled 1. Suppose the quantity vector he would have to accept from the markets at these prices is $\widehat{x}'_k$ on indifference curve 2. Then the market excess demand for good i, or the difference $(\widetilde{x}_{ik} - \widehat{x}'_{ik})$ between the amount k desires and the amount market i wants him to take at his initially announced prices, is positive when $i = 1$ and negative when $i = 2$. Since agent k wishes to be a buyer of good 1 and a seller of good 2, and since, at $\widehat{x}'_k$, he would neither be buying the former nor selling the latter, by raising his announced price for good 1 and lowering that of good 2 (to achieve, say, the steeper, dashed budget line through x_k^0 in Figure 1), he can hope that the markets will change what they want from him to a point such as A (which could also lie on the other side of x_k^0 in the diagram) where he would be buying (or at least selling less of) good 1, selling (or at least buying less of) good 2, and increasing his utility from $u^k(\widehat{x}'_k)$. However, even if the markets were to take him to a point like B, where he would be selling less (or possibly even more) of good 1 but buying more of good 2, his utility would still be higher than $u^k(\widehat{x}'_k)$. Of course, A or B may or may not be utility maximizing for k and, hence, may or may not represent the equilibrium. And whether he can actually arrive at intermediate or equilibrium points like A or B depends on how the remaining market participants react to his new price announcement. But in any case, agent k will always proceed with the new price announcement because, since trade takes place only after equilibrium is reached, he knows he will wind up at a point that is at least as good as x_k^0 where he started. Similarly, if agent k were required, on the basis of his initial price announcement, to accept a vector of quantities on the other side of $\widetilde{x}_k$, for example $\widehat{x}''_k$ in Figure 1, he would, according to the agent-price-adjustment rule, lower his announced price of good 1 and raise his announced price of good 2 (to produce the flatter, dashed budget line through x_k^0), hoping to obtain points such as C or D, which also increase his utility, this time from $u^k(\widehat{x}''_k)$.

Thus, in a two-good world, market reactions may be such that when the agent raises his announced price of a good, the residual excess demand forced upon him is reduced, and when he lowers it, the residual excess demand increases. That is, the market answer may move him in a direction such that, if not too large, his utility rises. But that answer could still turn out to be substantial enough, even in the "right" direction, to leave him with a lower utility, such as at point E in Figure 1, than that associated with the initial market response. Therefore, although the agent-price-adjustment rule considered here cannot guarantee that the agent, through his price-announcement activity, is always able to call forth an immediate market reaction that will increase his utility, he is still able to proceed in the hope that he can. Even with $n > 2$, although the agent may not succeed in procuring an immediate response from the market that increases his utility, it is still clearly in his interest to continue to participate in the adjustment process as described here. For, as has been pointed out earlier, as long as a unique equilibrium exists that is globally stable, even if the agent appears to be losing ground at any particular juncture, eventually, when equilibrium is achieved, he will be no worse off than when he began. That is, his utility cannot be lower than $u^k(x_k^0)$. And it may be considerably higher.

To move to a concrete specification of agent-price adjustment rules in Walrasian models of exchange economies, let $p^k = (p_{1k}, \ldots, p_{Ik}) > 0$ vary over possible price announcements by agent k. The agent-price-adjustment story says that the change in announced price p^k varies directly with the difference between agent k's desired trades at p^k (previously denoted by $\widetilde{q}_k$) and those required in response to the desires of the remaining market participants at p^k (previously written $\widehat{q}_k$). The latter difference at the price vector $\widetilde{p}^k$ considered above is

$$\widetilde{q}_k - [-\widehat{q}_k] = \widetilde{\widetilde{q}},$$

or

$$E^k(\widetilde{p}^k) - \left[- \sum_{\kappa \neq k} E^\kappa(\widetilde{p}^k) \right] = E(\widetilde{p}^k),$$

where $\widetilde{\widetilde{q}}$ is the market-excess-demand vector at $\widetilde{p}^k$. In general, then, agent k changes p^k directly with the market-excess-demand function $E(p^k)$. Thus, letting θ_k be a known, nonzero constant for each k, one collection of price-adjustment rules that might accompany the agent-price-adjustment story is

$$\frac{dp^k}{d\tau} = \theta_k E(p^k), \quad k = 1, \ldots, K, \tag{6}$$

where, recall, τ denotes time. Observe that (6) is a system of equations that, for each k, is formally identical to, but has a different interpretation and significance than, (4). Observe also that, because, as suggested in previous discussion, agent k knows $E(p^k)$ from the responses of the remaining agents to his announcement of each p^k, he has sufficient information to change p^k in line with (6). Moreover, since it is based on market excess demand functions, the impact of the Debreu result impinges, with all of its force, on the analysis of the global stability of equilibria under (6).

It should be pointed out, however, that due, in part, to the formal equivalence of (6) and (4), the extra sufficient conditions at the level of the individual ensuring uniqueness and global stability in Theorems 1 and 2, including the special circumstance in which all utility functions take on the Cobb-Douglas form, apply here as well. Under any of these sets of extra conditions, then, starting at a collection of K price vectors $p^k > 0$, where $k = 1, \ldots, K$, each different, say, from the unique, equilibrium price vector $\overline{p}$, the individual behaviors of the K agents will eventually lead everyone to $\overline{p}$. Equilibrium in this particular Walrasian exchange model with dynamic (6) is therefore unique and globally stable. And both (i) the additional conditions that ensure uniqueness and global stability, and (ii) the price adjustment mechanism in markets, derive, unlike the Walrasian tâtonnement of Section 2 above, from the properties of individual agents and their behaviors. Such a Walrasian model, then, fully meets the requirements of methodological individualism.

It is certainly true that the replacement of the standard auctioneer story and (4) by the agent-price-adjustment story and (6) in the theory of price determination of Walrasian systems still leaves something to be desired in so far as generality is concerned. Of course, to what extent greater generality can actually be achieved is, at present, unknown. But it is clear that past criticisms of the ability of the general-equilibrium system to explain price determination that arise from its lack of generality and its apparent inconsistency with methodological individualism are not as serious as they might, at first, have seemed. Economists should restore general equilibrium theory, or a more "acceptable" or more "realistic" version of it to a position of relevance and significance in their microeconomic work.

References

1. Arrow, K.J. and G. Debreu, "Existence of Equilibirum for a Competitive Economy," *Econometrica* 22 (1954), pp. 265-290.

2. Bénassy, J.P., "Nonclearing Markets: Microeconomic Concepts and Macroeconomic Applications," *Journal of Economic Literature* 31 (1993), pp. 732-761.

3. Debreu, G., "Excess Demand Functions," *Journal of Mathematical Economics* 1 (1974), pp. 15-21.

4. Einstein, A. and L. Infeld, *The Evolution of Physics* (New York: Simon & Schuster, 1938).

5. Fisher, F.M., "On Price Adjustment without an Auctioneer," *Review of Economic Studies* 39 (1972), pp. 1-15.

6. Flaschel, P., "Dressing the Emperor in a New Dynamic Outfit," *Annals of Operations Research* 37 (1992), pp. 33-49.

7. Hahn, F., "Stability," *Handbook of Mathematical Economics*, v. 2, K. J. Arrow and M. D. Intriligator, eds (Amsterdam: North Holland, 1982), pp. 745-793.

8. ——, "Auctioneer," *The New Palgrave*, v. 1, J. Eatwell, M. Milgate, and P. Newman, eds. (London: Macmillan, 1987), pp. 136-138.

9. Katzner, D.W., *Time, Ignorance, and Uncertainty in Economic Models* (Ann Arbor: University of Michigan Press, 1998).

10. ——, *An Introduction to the Economic Theory of Market Behavior*: *Microeconomics from a Walrasian Perspective* (Cheltenham: Elgar, forthcoming).

11. Kirman, A., “The Intrinsic Limits of Modern Economic Theory: The Emperor Has No Clothes,” *Economic Journal* 99 (Conference 1989), pp. 126-139.

12. Koopmans, T.C., *Three Essays on the State of Economic Science* (New York: McGraw-Hill, 1957).

13. Mantel, R.R., “On the Characterization of Aggregate Excess Demand,” *Journal of Economic Theory* 7 (1974), pp. 348-353.

14. Schotter, A., *The Economic Theory of Social Institutions* (Cambridge: Cambridge University Press, 1981).

15. Smale, S., “Exchange Processes with Price Adjustment,” *Journal of Mathematical Economics* 3 (1976), pp. 211-226.

16. Sonnenschein, H., “Market Excess Demand Functions,” *Econometrica* 40 (1972), pp. 549-563.

17. ——, “Do Walras’ Identity and Continuity Characterize the Class of Community Excess Demand Functions?” *Journal of Economic Theory* 6 (1973), pp. 345-354.

18. Walras, L., *Elements of Pure Economics*, W. Jaffé, trans. (Homewood: Irwin, 1954).