

**“Two children found four caterpillars”: Distributively and/or collectively**  
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Distributivity (Vendler, 1967)-- interpreting a predicate as applying to a set as individuals (distributively) or as a whole (collectively)--plays a crucial role in how children solve word problems in mathematics. We propose that the early bias toward collectivity observed in language acquisition (Roeper, 2007) carries over to early mathematics, with the consequence that simple quantified statements are not always interpreted the same by teachers and students; and not all interpretations lead to the “textbook” result.

We tested this claim with 56 English speakers, ages 3-10 years. Ten questions involving quantified expressions were presented orally to the children individually (and to adult controls through a web survey). Responses and explanations were recorded.

Consider this example: “Two children found four caterpillars. How many caterpillars did each child find? How many were there in all?” The standard textbook answer assumes a division problem, 4 caterpillars divided (not necessarily equally) by 2 children: children considered separately are “distributed”; caterpillars, considered together are “collective.” However, other equally grammatical interpretations are possible, in particular, collective/collective (children together and caterpillars together, 4-4) and distributive/distributive (children each and caterpillars each, 4-8).

Results:

Age	3-4 yrs	5-6	7-8	9-10
Response Type				
Collective/Collective No operations. The children (together) got 4 caterpillars total	67%	31%	<5%	
Distributive/Collective (Textbook ) Division: 2 children each; 4 caterpillars total	8%	15%	64*%	55*%
Distributive/distributive Addition: 2 children each with 4, 8 caterpillars total	8%	15%	14%	33%
Other	17%	32%	10%	11%

Consistent with our hypothesis, younger children often accepted the sentence at face value with no operation implied (C/C). A significant minority of older children interpreted the problem distributively, thus calling for addition rather than division. It appears, then, that semantic and pragmatic principles, in addition to math facts, must be learned (or taught) to help children arrive at the textbook response. (300)

#### Summary

We investigated how 56 English-speaking children, ages 3 to 10, interpreted the semantic property of distributivity (Vendler, 1967) in solving mathematics word problems. Most children under age 7 and many older children entertained non-conventional interpretations that produced unexpected, but plausible responses that would not match the answer key. (46)